Linear-Time Computation of a Linear Problem Kernel for Dominating Set on Planar Graphs

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Abstract

Over the last years, reduction to a problem kernel, or kernelization for short, has developed into a very active research area within parameterized complexity analysis and the algorithmics of NP-hard problems in general [2]. In a nutshell, a kernelization algorithm transforms in polynomial time an instance of a (typically NP-hard) problem to an equivalent instance whose size is bounded by a function of a parameter. Nowadays, it has become a standard challenge to minimize the size of problem kernels. Consider the following examples:

1. For Feedback Vertex Set (given an undirected graph G and a positive integer k, find at most k vertices whose deletion destroys all cycles in G), there first has been an $O(k^{11})$-vertex problem kernel [3], improved to an $O(k^3)$-vertex problem kernel and finally to an $O(k^2)$-vertex problem kernel [5]
2. For Dominating Set on planar graphs (given an undirected planar graph G and a positive integer k, find at most k vertices such that each other vertex in G has at least one neighbor in the set of selected vertices), there first was a $335k$-vertex problem kernel [1], which was further refined into a $67k$-vertex problem kernel [4], both computable in $O(n^3)$ time.

From the viewpoint of practical relevance, however, also the running times of kernelization algorithms have to be optimized. We show an $O(k)$-size problem kernel for Dominating Set in planar graphs that is computable in $O(n)$ time.

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