Fostering Diagrammatic Reasoning in Science Education

Ilonca Hardy[1], Michael Schneider[1], Angela Jonen[2], Elsbeth Stern[1], and Kornelia Möller[3]

[1] Max Planck Institute for Human Development, Berlin, Germany
[2] University of Würzburg, Germany
[3] University of Münster, Germany

In a study with 56 third-graders, we tested whether reasoning with line graphs can be enhanced by representational activities within a curriculum on floating and sinking of objects in water. We hypothesized that representing mass and volume on the opposite arms of a balance beam allows the simultaneous consideration of both dimensions for a representation of density, and therefore will be particularly helpful for drawing inferences from the slopes of line graphs. In an experimental classrooms study, half of the students used the balance beam, while the other half worked with self-constructed representations. Five months after the instructional unit, students who had been familiarized with the quantitative interpretation of proportional information on the balance beam outperformed students who had worked with self-constructed representation in their interpretation of line graphs referring to density, but only marginally when referring to speed.

Keywords: Visual representations, interpretation of line graphs, transfer, science education

Graphs as Tools for Scientific Reasoning

It is now widely recognized that higher-order cognitive activities such as scientific reasoning are significantly facilitated by the competent use of visual representations, as there are graphs and diagrams (e.g., Boulter, 2000; Roth & McGinn, 1998). These tools are commonly employed to display relationships between variables; moreover, they can serve as reasoning tools as the inferences drawn from represented information allow the user to derive new conceptual insights. For instance, in a coordinate system two variables are represented on the axes and their relationship may be displayed by a line graph. The slope of this graph then may represent proportional concepts such as speed, density, or the degree of concentration of different mixtures of liquids. Reasoning about the relationship between the variables displayed in the coordinate system may then also instigate insight into the quantitative relations of ratios in proportional concepts. Apart from functioning as reasoning tools, the meaningful use of visual representations can support students' diagrammatic reasoning, as in students' ability to apply a variety of visual representations when drawing inferences in (familiar) scientific domains. In the following, we will argue that powerful learning environments in science domains should include opportunities for using graphs and diagrams as reasoning tools, thus fostering diagrammatic reasoning of students also in the long run.

In Germany, students encounter line graphs as a means for data representation already in early secondary school, while the meaning of core elements of graphs, such as the slope or the intercept on the y-axis, are usually only addressed in mathematics curricula in eighth grade when students learn to display linear functions in coordinate systems. However, as Mevarech and Kramarsky (1997) could show, core misconceptions of middle school students regarding graph elements even prevail after instruction. A study with German university students also suggests that the instructional support typically provided in schools is not sufficient for the competent use of graphs as scientif-

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ic reasoning tools. Stern, Aprea, and Ebner (2003) showed that even for university students of mathematics, the spontaneous use of graphs was not as common as one would expect given the widespread application of graphs in formal domains. Another piece of evidence for German students' deficits in graphical reasoning, even at the highest levels of education, was provided by the TIMS-III study, which focused on a pre-university sample (Baumert, Bos, & Lehmann, 2000). It turned out that 45% of German students in grade 12 who had chosen mathematics as a major subject did not know that answer (c) of the following problem was the correct one.

The acceleration of an object moving in a straight line can be determined by
(a) the slope of the distance-time graph
(b) the area below the distance-time graph
(c) the slope of the velocity-time graph
(d) the area below the velocity-time graph

The answer most frequently selected was answer (a), an error which indicates deficits in students' conceptualisation of acceleration as the rate of change of velocity per unit of time and/or in mapping this concept onto the appropriate variables of a coordinate system. For a mapping of acceleration, one needs to conceptualise the slope as representing the rate of change of the y-value in relation to the x-value. While some students may have had conceptual difficulties, mixing up speed and acceleration, others may have chosen answer (a) because this first-order relation (Gattis, 2002) is most easily identified in graphs. Due to the need to map science concepts onto the structural constraints of a coordinate system, students' mapping ability will not be furthered by graph instruction remaining entirely within an algebraic content (e.g., the transformation of formulas, tables, and graphs), nor by instruction about the concept of acceleration itself. Rather, students should be provided with ample opportunities to use graphs and other forms of visual representation for drawing inferences about science constructs, thus facilitating their mapping of information within meaningful contexts.

Given the pivotal role that graphs play in scientific reasoning and given the deficits of even well-educated students, there is good reason to re-think the curriculum in science and mathematics. Graphs should be regarded and employed as reasoning tools whenever students have to deal with content appropriate for graphical displays, and this should happen long before the transformation of algebraic formulas into graphs is part of the mathematics curriculum in secondary school. However, although there is evidence that already preschool children can be taught to read off values displayed in a coordinate system when the labelling of the axes is facilitated (Somerville & Bryant, 1985), it remains an open question how graphs may be meaningfully introduced at an early age. In an experimental training study, Koerber (2003) showed that proportional reasoning of 10-year-old children was considerably improved as a result of a two-afternoon training involving the use of graphs to display the proportional construct of juice mixtures. However, despite their significant increase in proportional understanding evident at the posttest, children working with graphs were outperformed by children using a representational form closer related to their everyday experience – the balance beam, which is akin to experiences with the sea-saw. While the interpretation of the slope requires the integration of quantitative information represented in the coordinate system, the interpretation of the balance beam is more intuitive since based on the equilibrium of the beam. Here, children may represent different amounts of orange and lemon juice with weights on each side of the beam, determining their concentration by balancing the beam. Two proportional ratios are then represented by an equilibrium (i.e., the beam remains balanced when the relation between two ratios remains the same).

Given the results of this study, we speculate that in order to further students' sense-making of graphs, one may need to first introduce them to a visual representation accessible to them on an experiential basis, such as the balance beam. Roth and Bowen (1994) differentiate between experience-near and experience-distant forms of representations. While manipulatives such as the balance beam and self-constructed forms of representation belong to the former category, graphs and equations may be viewed as examples of the latter due to the greater formalism of represented information. We assume that use of an experience-near visual representation will not only foster students' understanding of the represented science concepts but also their ability to make sense of an experience-distant representation such as the graph, thus enhancing their diagrammatic reasoning skills. Students' meaningful interpretation of graphs is likely if they have already been familiarized with their basic structural constraints, such as their two-dimensionality, their quantitative representation of variables, and their integration of two variables into a new one. This may be achieved through the use of a structurally similar, yet experience-near visual representation. We thus suppose that structural similarities will enhance students' interpretative access to graphs since students will be able to draw on both content and structural knowledge for interpretations.

In an experimental classroom study, we first addressed the question which of two experience-near forms of representation, a balance beam or self-constructed forms, is more helpful for developing students' conceptual understanding of "floating and sinking", including their understanding of the proportional relationship between mass and volume in the concept of density. In the follow-up study on diagrammatic reasoning reported here, which took place five months after the instructional unit, we additionally addressed the question of whether experiences with the balance beam, which is structurally similar to the graph, will help children in interpreting line graphs displayed in a coordinate system. Within the goal of fostering students' diagrammatic reasoning at an early age, it is especially pivotal that the long-term effects of a science curriculum be investigated. That is, we conceive of diagrammatic reasoning as a competence which is fostered by appropriate representational activities and which will be evident as a long-term consequence of science instruction. By having a delay between students' representational activities with experience-near representations and their use of graphs we allow children to consolidate their insights about the proportional relationship of mass and volume in the densities of objects – thus focussing on the structural relationships rather than the actual experiences with objects' floating and sinking observed during the instructional unit. Focusing on the graph as a widely applicable, yet experience-distant visual representation, we want to contribute to an understanding of how young children's access to this representational form may be facilitated within curricula designed primarily for fostering students' understanding of science concepts. Thus, diagrammatic reasoning may be seen as an effect accompanying the meaningful involvement of students in representational activities during the investigation of science phenomena. By investigating diagrammatic reasoning with a delayed test, we can therefore estimate the significance of a science curriculum for students' cross-curricular competencies such as diagrammatic reasoning.

Using Visual Representations for Conceptual Understanding of Density

Density is a proportional concept underlying explanations of the phenomenon of objects' floating and sinking in water. For elementary school children, this type of reasoning is difficult because they tend to focus on only one of the two quantities of mass and volume, which need to be considered simultaneously in order to arrive at an estimate of an object's density. Although objects' densities may be accessible even to preschoolers on a qualitative level (Singer, Kohn, & Resnick, 1997), they often confound (felt) weight and density in their explanations. Apparently, children's concept of weight is grounded in the extent to which weight may be felt rather than regarding it as a characteristic of the material world. In addition, when making predictions about objects' floating and sinking, young children tend to focus on only one of the two quantities of mass and volume to be considered for material kind. For example, elementary school children frequently predict that a big tree trunk will sink while a small needle will float when immersed into water. A developmental sequence of students' conceptual understanding of density has been described by Smith, Carey, and Wiser (1985). Children move from misconceptions grounded in a one-dimensional focus on objects' weight, size, or form, including air as an active force, to a qualitative description of different objects' material (classification as light or heavy material). Here, objects of the same material are thought to behave the same when immersed in water. At this level, children cannot easily explain why a ship made of iron does not sink as does a solid iron block. Nevertheless, children perceive of density as a quality which makes objects "feel heavy or light" for their size. Finally, a more explicit conception based on scientifically accepted explanations of density may be reached, where two densities are being compared. These explanations do not require reference to the formula of density, or its translation into numbers and fractions, but are grounded in an understanding of the relationships between quantities in determining a quality of an object (density), which is explanatory for its behavior in water.

An experimental classroom study by Möller, Jonen, Hardy, and Stern (2002) showed that even third-graders can achieve a basic understanding of the scientific concepts of density and buoyancy force, especially if the learning environment is based on an appropriate sequence of activities and allows for students' conceptual restructuring in classroom discourse. At the same time, the study revealed some deficits in students' understanding of the concept of density even after instruction. One reason may be that it is difficult to make accessible an object's density on an experiential basis, for example by comparing the relative densities of two objects held in each hand, while water pressure and water displacement of objects, related to buoyancy force, may be experienced more directly by students. It is thus especially for the concept of density with its necessary consideration of two dimensions that the use of a visual representation may be helpful. By highlighting the (proportional) relationship between mass and volume through the use of a visual representation, we expect to encourage a more mature understanding of density both as a quality of an object (e.g., an object as light for its size) and as a means of predicting its floating or sinking in water.

The Balance Beam as a Tool for Representing Density

If learners meaningfully connect the structures of a visual representation to the situational variables represented and the phenomenon to be explained, the use of a representation may enhance conceptual change by building on, refocusing, and extending initial conceptions (see Clement, 2000; Sfard & McClain, 2002). The use of visual representations displaying the proportional relationship between two quantities may thus lead learners to attend to both quantities simultaneously. In an instructional unit on floating and sinking, visual representations may show the inadequacy of students' initial conceptions, for example their consideration of only one quantity when describing objects. Visual representations may also support students' construction of new explanations by allowing the prediction of objects' floating and sinking based on a comparison of densities.

Representations differ in the way they afford the integration of two quantities. On the balance beam, weights representing mass and volume can be put on both sides of a beam so that it will be in balance. Two ratios may be compared by observing the behaviour of the beam when putting on weights for the second ratio. When representing two objects of the same material, the proportional relationship between mass and volume, as in doubling or tripling the weights, is highlighted. Accordingly, each material is represented by a certain position on the beam, for which the beam remains in balance. In other words, children learn that this position stands for the material of the object independently of its absolute weight or volume. When using the balance beam, children learn to represent different kinds of material at different positions of the balance beam, such as depicted in Figure 1. Thus, the balance beam draws attention to the simultaneous involvement of mass and volume in determining material kind, and to their (constant) quantitative relation, or ratio.

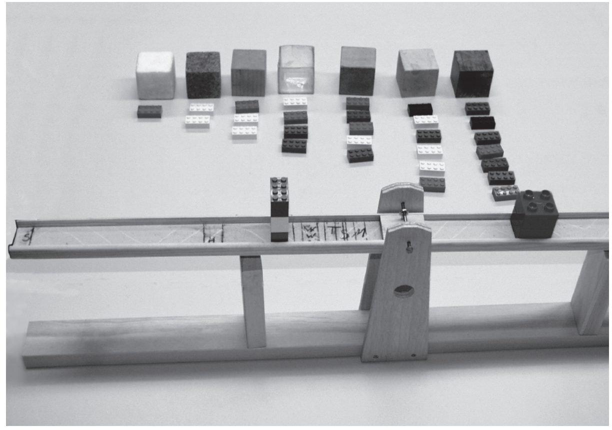

Figure 1. The balance beam with weights for mass and volume. Cubes of different material were assigned with different numbers of weights (left to right): Styrofoam, cork, wood, water, clay, stone, iron.

Student-Constructed Forms of Representation

In contrast to the use of provided, conventional representations, it is also possible to have students construct their own forms of representation. The use of representations that have been conceived in the course of cultural development, thus experience-distant forms of representation (Roth & Bowen, 1994), mostly needs to be introduced or modeled by teachers. This will involve a certain abstraction from students' own experience which does not necessarily reflect their own conceptualization of a scientific construct. In a self-constructed form of representation, an individual is able to explicate his or her understanding of a situation directly, thus adding to conceptual understanding since new strategies can be generated directly from students' existing conceptual knowledge of a phenomenon (Lehrer & Schauble, 2000; van Dijk, van Oers, & Terwel, 2003). Considering that students' knowledge of the essential variables of a situation is usually incomplete, the choice of a representational activity (use of a provided form versus construction of forms) may be essential for furthering conceptual understanding. In conventional forms of representation, quantities and the relationship between quantities are considered and represented in scientifically correct ways. In contrast, self-constructed representations may at first be scientifically incomplete. For example, students who only attend to the mass of objects in determining whether they will sink or float will likely also focus on this dimension in their visualizations – leaving unattended the dimension of volume. It follows that it is especially pivotal that teachers support students' development of their self-constructed representations into scientifically useful, general forms – a process Gravemeijer (1999) calls the development of a *representation of* (a specific situation) to a *representation for* (a model representing the structural relationships valid across a variety of situations). In order to support students' diagrammatic reasoning with new, more abstract forms of visual representation, it is especially pivotal to enable this development from context-bound representations to ones which are valid and useful for a variety of scientific concepts.

Gaining Insights by Transformations Between Representations

The transformation of information into different forms of representation (e.g., numerical, visual) is an effective method for the construction of flexible and transferable knowledge because the process of transformation requires learners to concentrate on pivotal aspects of the represented phenomenon (see Schwarz & Holton, in press). According to Greeno, Smith, and Moore (1993), transfer is based on the perception of affordances and constraints which are invariant in the learning situation and the transfer situation. Representational forms are one way of summarizing these structural invariants across different situations, thus helping the learner to extract those quantities and their respective relationships which are explanatory for the represented construct. Similarly, the structural invariants between different forms of representation, as in experience-near and experience-distant forms, may support students' sense-making of the represented content.

Based on these ideas, it can be expected that students' ability to make sense of line graphs depicting the different densities of objects can be particularly fostered if students first learn to quantitatively represent and integrate the dimensions of mass and volume on a balance beam. Although the balance beam and a coordinate system look quite differently, they provide similar affordances for representing proportional concepts such as density. The coordinate system provides two independent dimensions – the axes – which may be connected through coordinates representing particular instances of the two variables simultaneously. Similarly, in the balance beam, the weights put on either end of the beam represent the particular instances of two dimensions. In the coordinate system, the integration of the two variables is represented by the slope, whereas with the balance beam, the equilibrium, or the position on the beam in which an equilibrium is achieved, indicates the relationship between the two variables. Thus, in both cases the concept of density is inferred from quantitative information represented separately on two dimensions.

Whereas self-constructed forms of representations for the concept of density will also involve the representation of the two dimensions of mass and volume (see Experimental Classroom Study), this need not necessarily be done in a quantitative way. That is, intuitive visual representations of the concept of density will often involve the qualitative visualization of densities, as in the different shading of objects. Thus, self-constructed representations will likely miss the element of a quantitative comparison of mass and volume, involving the comparison of ratios, leaving them with less (context-independent) elements of structural similarity with line graphs than the balance beam. Despite of less structural similarity one may argue that the experience of constructing and interpreting meaningful visual representations within the context of density will further students' diagrammatic reasoning as a general competence. That is, the gradual development of *representations for* science constructs (Gravemeijer, 1999) may also involve an increasing ability of students to make sense of provided, conventional forms of representations (see also van Dijk et al., 2003). However, while there may be an advantage for the interpretation of other experience-near forms of representation, it is unlikely that these visualization activities will also foster students' ability to make sense of more abstract, experience-distant forms such as the graph.

Based on these elaborations we expect that students who worked with the balance beam during a curriculum on floating and sinking will outperform students who designed their own visual representations in the interpretation of line graphs both in the context of density and in the context of speed. In the context of density, students' interpretations of graphs not only need to consider graph features such as different slopes but also need to be embedded in a conceptual framework on floating and sinking, allowing correct predictions. In the context of speed, we focused on additional graph features such as the intersection of slopes, assuming that students' conceptual basis of (differences in) speed would allow for meaningful interpretations. We hypothesized that for both contexts, students who had worked with the balance beam would be able to recur to the structural similarities between the two forms which would help them to attend to the relevant features such as the coordinates or the slope for their interpretations. Before presenting results of the Diagrammatic Reasoning Study which took place as a follow-up measure, we will briefly report on the design and implementation of the preceding Experimental Classroom Study.

The Experimental Classroom Study

In this study, we investigated effects of representational activities on elementary school students' conceptual understanding of floating and sinking and their proportional understanding of different proportional concepts. A total of 98 third-graders in four classrooms participated in a curriculum of 11 lessons on floating and sinking, assimilated from the larger curriculum of Möller et al. (2002). The study varied students' use of visual representations for density, with two classrooms working with the balance beam and two classrooms using student-constructed representations. The study is described in detail in Hardy, Jonen, Möller, and Stern (2004). The curriculum initially focuses on the development of the concept of material kind, where students state hypotheses about the behaviour of different objects in water, test their hypotheses and reflect on their findings together with the teacher. They then compare material of the same volume but different mass and arrive at density as one difference between materials. Now, students proceed to represent the densities of different objects, either with the help of the balance beam or with self-constructed forms of representation. In both curricula, students work in pairs on representational tasks. The following sequence focuses on water displacement, where students work on learning stations focusing on water displacement as dependent on an object's volume. Finally, students represent the comparison of densities with displaced water in order to predict whether an object will float or sink. The two curricula differ only in the lessons based on representational activities-whereas students in the balance beam group learned to represent objects of different material on the balance beam, using weights to represent mass and volume, students in the group of self-constructed representations designed their own forms, using a variety of provided material such as cardboard of different colour and size, buttons, and crayons. In order to encourage the consideration of the two dimensions of mass and volume in the student-constructed forms, students were asked to find a way of visualizing two objects' differing "weight" and/or "size" in a way that other students could know which material was represented. During the course of the lessons, criteria such as the quantification of mass and volume emerged as important dimensions of a useful representation during teacher-student discourse. Thus, some of the students developed visual representations akin to Smith, Maclin, Grosslight, and Davis' (1997) cardboard matrix, where volume is represented by large squares of cardboard while mass is represented by small squares laid on top.

Results revealed a significant improvement in conceptual understanding of floating and sinking for students of both experimental groups to a similar extent, as assessed by a test in which both the correct rejection of misconceptions (such as the concentration on objects' size, form, weight, or the active role of air) and the adoption of scientific explanations based on density and buoyancy force were scored. For details of the test see Hardy et al. (2004) or Möller et al. (2002). In addition, we assessed students' gains in proportional understanding in the domains of density, speed, and juice mixtures. Tests required students to construct (proportional) ratios based on information about one ratio. For example, students were presented with information on the mass and volume of one block and had to compute the mass of another block of the same material, given its volume. Results on proportional reasoning in a pre-posttest design showed that both groups gained, albeit in different domains. While the balance beam group gained significantly in the domain of speed, the group of self-constructed forms of representations gained in the domains of density and juice mixtures.

Diagrammatic Reasoning Study (Follow-Up Study)

In the follow-up study described below, we investigated whether experiences with the balance beam during an instructional unit on floating and sinking will promote students' ability to make sense of line graphs in the contexts of density and speed.

Method

Participants

Five months after the end of the Experimental Classroom Study, a random sample of 56 students was drawn from the original sample of 98 students. Twenty-seven students (17 female, 10 male) were sampled from one of the two classes who had used self-constructed representations, and 29 students (16 female, 13 male) were sampled from one of the two classes who had worked with the balance beam. At the end of the Experimental Classroom Study, all children had been presented with the Test on Floating and Sinking and three tests on proportional reasoning, described above. There were no significant differences in age, sex, and achievement on these tests between those participants who were selected for the interview and those who were not. The two groups selected for interviews did not differ significantly on their mean posttest scores on the Test on Floating and Sinking and on the three tests of proportional reasoning.

Procedure

The interviews were conducted with single children in a room provided by each school, and without the interviewer knowing which child had worked with the balance beam or with self-constructed representations. Children were told that the interviews were a means of finding out "how children learn" and would not be taken into account in their school reports, so there was no need to be afraid of saying something wrong in the interview.

A total of 16 questions was asked. The first ten questions were dealing with the integration of mass and volume into the concept of density, displayed in slopes of a coordinate system, in the following referred to as *Densi-*

ty Graph Test. The goal of this test was to probe children's ability, after five months had passed, to compare different objects' densities and make predictions about floating and sinking when represented in a coordinate system as a linear graph. The subsequent six questions were aimed at evaluating whether or not the children were able to interpret graphs in another proportional context, as a ratio of distance and time, in the following referred to as *Speed Graph Test*. The goal of this test was to find out whether children could use the graph as a reasoning tool when interpreting the depicted information on the speed of different objects. In both contexts, the proportional relation between variables depicted on the axes needed to be considered in interpretations. In the Density Graph Test, students needed to anchor their interpretation of the line graphs in their conceptual understanding of floating and sinking, as in comparing the densities of water and object when predicting an object's floating or sinking.

The concrete course of the interview is shown in Table 1. Each child was presented with a coordinate system with the x-axis labeled "size of cubes" and the y-axis labeled "weight in grams" depicted in Figure 2. The children were presented with square blocks composed of different numbers of cubes, which were of the same size for all kinds of material. The blocks were made of wood, of metal, and there was a so-called water block, which was some transparent plastic material filled with water. Children were familiar with these blocks from the classroom curriculum on floating and sinking. Moreover, there was a block wrapped in some tin foil introduced as "secret material", and a wooden block wrapped in paper. Because ten-year-old children are overtaxed by the expressions of mass and volume, these dimensions were introduced as the weight and the size of the cubes. The weight of the blocks could be determined with the help of an electronic kitchen scales. The size of the blocks was determined by counting the number of cubes they were composed of. The children were also familiar with the convention of ascribing size 1 to the cubes made of each of these materials and to indicate the size of a square block by relating it to the size of the standard cubes. All the square blocks used had the sizes 1, 2, 3, 4 or 5.

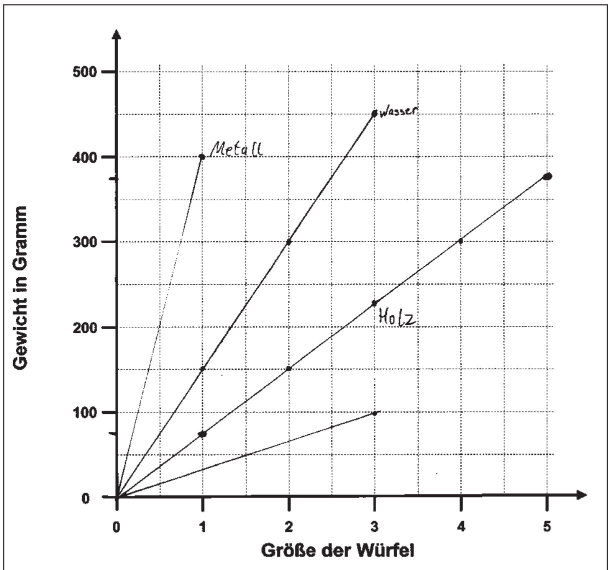

Figure 2. Sample graphs for density drawn during the interview. Y-Axis: Gewicht in Gramm = Weight in Gram; X-Axis: Größe der Würfel = Size of Cubes; Metall = metal; Wasser = water; Holz = wood.

The children were then asked the 10 questions referring to density and were told to find an answer by using the materials and to demonstrate and explain how they had found this answer. If a wrong answer or, after a delay of one minute, no answer at all was given, the interviewer once more presented the question. If, at this second attempt, the children still failed to come up with a solution, the interviewer told them what the correct solution would be and asked them to use the materials to demonstrate the way it could be reached.

The procedure of the interview for the Speed Graph Test was similar, but the materials and the coordinate system were replaced by distance-time coordinate systems with graphs representing the distances run by three cars. The questions now referred to these coordinate systems (for details see Figure 2 and Table 1).

Scoring

Two points were given if the correct solution was found, demonstrated and explained without any support by the interviewer. One point was given if children had at first failed to provide a solution or provided an incorrect one but had then, on being questioned by the interviewer, found the correct solution. All other cases were scored zero points. Thus, the maximum scores were 20 for the Density Graph Test and 12 for the Speed Graph Test. For each participant, the points obtained were divided by the maximum score, so the scores could be interpreted as solution rates.

Results

The range of item difficulties is .36 to .92. for the Density Graph Test and .23 to .86 for the Speed Graph Test, with an internal consistency of the scales as assessed by *Cronbachs Alpha* of .68 for the Density Graph Test and .43 for the Speed Graph Test. The correlation of the Density Graph Test and the Speed Graph Test with the posttest scores on the Test on Floating and Sinking is $r = .36$, $p < .01$ and $r = .39$, $p < .01$, respectively. Separate correlations for both groups did not reveal any differences. Correla-

Table 1
Overview of Interview Questions, Explanations, and Material

Material	Explanation/Question
• Diverse coordinate systems	• Have you ever seen something like this? (stock market prices, temperature curve) Now, let's see how this works…
• 1-unit cube of water (150g) • 2-units cube of water (300g) • 3-units cube of water (450g)	• I got a cube here. Do you know the material it is made of? (water) You know it from your classes with Ms Jonen. We now want to map this cube on this (show coordinate system). • To do this, we first got to know its size (1) and its weight (150g). • **Question 1: Have a good look at this. If we want to map the cube and its properties on this, we will have to locate a specific point and mark it. Can you show me this specific point?** • Show 2-units cube, determine values (300g), mark it • Show 3-units cube (450g), determine values, mark it • How are the points positioned? (straight line) connect points • Name graph "graph" • **Question 2: Does it make sense to extend the graph to reach point (0/0)? Why?**
• 2-units cube of wood (150g) • 3-units cube of wood (225g)	• Experimenter maps 2-units and 3-units cubes on the graph • How are these point positioned? (straight line) draw graph • **Question 3: Can you show, by using the graph (and not by calculating!), what would be the weight of a 1-unit cube made of wood?** • **Question 4: How can we determine, by using the graph, the weight of a 5-units cube made of wood?** • Extend graph • What is the difference between these two graphs? (slope) • Wood will float in water (demonstrate; classes with Ms Jonen) • We now have one graph for water (label) and one for wood (label). You can tell, by looking at them, that wood will not sink in water but float. Now, here is a really difficult question: • **Question 5: By what can you tell?**
• 1-unit cube of iron (400g)	• 1-unit cube: Experimenter marks point and draws graph • Will iron float in water? (no) • So, wood will float but iron won't. You can tell this by looking at the graph. • **Question 6: Do you have any idea how you can tell, by looking at the graph, which material will float and which will sink?** • Refer to slope and show that the material which, given the same size, has more weight will also have the steeper slope
• 4-units cube of wood (300g), wrapped	• Now we have a cube which is wrapped. We do not know which material it is made of. But we know its size and its weight. • **Question 7: How can you determine, using the graph, which material the cube is made of (water, wood, iron or something quite different)? Which is it?** • A 4-units cube made of wood weighs 300g. If I now add another 1-unit cube made of wood, I have got a 5-units cube. • **Question 8: Where in the graph can you read off how much more weight there is when a 1-unit cube is added to the 4-units cube?**
• 3-units cube of secret material (100g)	• We again only know the size and the weight of a cube. It is made of some secret material and we cannot tell what it is. But we know its size and its weight. • **Question 9: How can we tell whether or not the secret material will float?** • Two cubes having the same size, but made of different materials will be more or less deeply immersed in the water if you let them float (demonstrate). • **Question 10: Determine, by using the graph, which one will be more deeply immersed: a 1-unit cube made of the secret material or a 1-unit cube made of wood.**

Table 1 (continued)
Overview of Interview Questions, Explanations, and Material (Continue)

Material	Explanation/Question
• Distance-time coordinate systems	• Here we again have graphs. But their meaning is different from that of the previous ones. They now tell us something about the way three different cars are traveling. • **Question 11: How many hours has the red car been traveling?** • **Question 12: How many kilometers per hour is the green car traveling?** • **Question 13: Which of the three cars is traveling fastest?** • **Question 14: Please draw the graph for the distance that one of the cars has been traveling! It has been traveling 20 km in the first hour, and then it has been traveling 10 km in two hours.** • **Question 15: The two graphs intersect – what does this mean? What did the cars do then?** • **Question 16: The orange graph begins to go down after three hours – what does this mean?** (show what you mean)

tions of the two graph tests with the three tests of proportional reasoning (posttest scores) revealed different patterns for the two experimental groups. For the balance beam group, the correlation of the Density Graph Test with the proportional reasoning tests is $r = .42$, $p < .05$, for the context of speed, $r = .44$, $p < .05$, for the context of density, and $r = .37$, $p < .10$, for the context of mixtures. With the Speed Graph Test, the correlations are $r = .59$, $p < .001$, $r = .57$, $p < .01$, and $r = .61$, $p < .001$, respectively. In contrast, for the group of self-constructed representations, none of the correlations of the Density Graph Test with the proportional reasoning tests is significant with $p < .10$. In this group, the correlations of the Speed Graph Test with the proportional reasoning tests are $r = .42$, $p < .05$, for the context of speed, $r = .35$, $p < .10$, for the context of density and $r = .45$, $p < .05$, for the context of mixtures.

Table 2 shows the mean solution rates for the Density Graph Test and for the Speed Graph Test for both groups. T-tests revealed significant group differences for the Density Graph Test, $t(54) = 2.33$, $p < .05$, $d = 0.63$, but not for the Speed Graph Test, $t(54) = 0.34$. The two classrooms who had worked with the balance beam obtained higher scores (.72 and .68) than the two classrooms who had used self-constructed representations (.62 and .54), which indicates that the greater effect of the curriculum with the balance beam for an understanding of graphs is a stable feature.

Table 2
Means (Standard Deviations) of Solution Rates for Two Graph Tests by Experimental Group

	Representational form used 5 months ago		Total
Test	Balance beam	Self-constructed	
Density	.71 (.20)	.59 (.18)	.65 (.20)
Speed	.49 (.21)	.47 (.21)	.48 (.21)

While there were no gender differences for the Density Graph Test, the boys outperformed the girls in the Speed Graph Test (males: $M = .55$, females: $M = .43$, $t(54) = 2.10$, $p < .05$). A view to the solution rates of single items depicted in Table 1 indicated that in the Density Graph Test, the balance beam group gained higher scores for all items. In the Speed Graph Test part, the solution rates for Question 11–15 were almost alike for both groups, while in Question 16 the balance beam reached a clearly higher solution rate (.74) than the self-construction-group (.47), $t(54) = 2.51$, $p < .05$.

Discussion

In line with our hypothesis, students who had worked with the balance beam during a curriculum on floating and sinking outperformed students who had worked with self-constructed representations when interpreting line graphs depicting different densities five months after the instructional unit. Apparently, children could transfer the insights they had gained when using the balance beam for integrating the dimensions of mass and volume to help them make sense of line graphs in the context of the floating and sinking of objects. As the correlations with the posttest scores on three proportional reasoning tests indicate, these children seem to have interpreted density graphs in a quantitative way, as in determining the mass and volume of objects of different densities, more so than did children who had worked with self-constructed representations. Moreover, these children were able to use line graphs as reasoning tools when they had to determine the material of wrapped blocks and when they had to predict whether a "secret material" would float or sink in water. Interestingly, students in the group of self-constructed representations did show evidence of superior proportional understanding of density directly after the curriculum on floating and sinking; however, they apparently did not apply this knowledge when making sense of the line graphs depicting density. While the correlations between the Density Graph Test and the posttest scores on the Test on Floating and Sinking suggest that students also anchored their interpretations of line graphs in their conceptual understanding of floating and sinking to some degree, there was no hint for an aptitude-treatment interaction as correlations did not differ between groups.

Contrary to our hypothesis, however, both groups did not differ in their ability to interpret line graphs of the concept of speed. The correlational patterns of this test with the tests of proportional reasoning suggest that both experimental groups tended to use their knowledge of proportions when interpreting the line graphs for the concept of speed. For the group of self-constructed representations, this may explain the discrepancy between their achievement on the Density Graph Test and the Speed Graph Test. While children who had worked with the balance beam regarded line graphs as referring to proportions also in the context of density, the children with self-constructed representations only did so in the context of speed where the interview questions required less conceptual reasoning than did questions about density. Interestingly, it is especially on the last question of the Speed Graph Test, which requires reasoning about the slope, that the balance beam group showed superior results. This may be an indication that the questions in the Speed Graph Test did not tap into the type of diagrammatic reasoning competence that the balance beam group had developed through the instructional use of the balance beam. While the reading of graphs, as in the determination of coordinates is certainly important, it is especially the integration of conceptual knowledge with reasoning about (differences in) the slope that is asked from competent graph users. In addition, the significant gender differences suggest that the male students overall may have had a better conceptual understanding of the concept of speed, which may have had a stronger impact on answering the questions of the Speed Graph Test than the experience of practicing with the balance beam.

While achievement on both graph tests was certainly not at ceiling, it should be emphasized that this study provides evidence for young children's ability to make sense of graphs both in an area extensively dealt with in instruction (density) and in an area in which representational tools had not been employed during instruction (speed). Our results suggest that practicing the integration of mass and volume of different material on the balance beam helps children to make sense of line graphs in the same content area, even after a five-months delay. Thus, diagrammatic reasoning as a cross-curricular competence can be regarded as a long-term consequence of a science curriculum with appropriate representational activities.

Since students need to imbue the symbols used in abstract representations such as graphs with meaning, starting with representational activities closely related to students' experience, such as the balance beam, has proven to be advantageous. In contrast, the use of student-constructed representations did not enable students' reasoning with line graphs depicting densities to the same extent. Most likely, the visual representations constructed by students were too much context-bound as to allow the abstraction of general features of two-dimensional representations as requested for the interpretation of graphs. An interesting issue for further analyses therefore concerns the degree to which students' representation of quantitative features of two-dimensionality in their own representations can predict their achievement on a test of graph reasoning. While an instructional sequence that moves from experience-near forms of representations to experience-distant forms seems to enable students to make sense of graphs, it is especially the structural similarities such as the quantitative interpretation of visual representations displaying proportional concepts which fosters an early access to graphs. Clearly, students' experiences with visual representations, including line graphs, should be sought throughout the science curriculum so that they can discover the potential of these tools for scientific reasoning as early as possible.

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Ilonca Hardy

Max Planck Institute for Human Development
Center for Educational Research
Lentzeallee 94
14195 Berlin, Germany
hardy@mpib-berlin.mpg.de