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Informative tools for characterizing individual differences in learning: Latent class, latent profile, and latent transition analysis[☆]

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ABSTRACT

This article gives an introduction to latent class, latent profile, and latent transition models for researchers interested in investigating individual differences in learning and development. The models allow analyzing how the observed heterogeneity in a group (e.g., individual differences in conceptual knowledge) can be traced back to underlying homogeneous subgroups (e.g., learners differing systematically in their developmental phases). The estimated parameters include a characteristic response pattern for each subgroup, and, in the case of longitudinal data, the probabilities of transitioning from one subgroup to another over time. This article describes the steps involved in using the models, gives practical examples, and discusses limitations and extensions. Overall, the models help to characterize heterogeneous learner populations, multidimensional learning outcomes, non-linear learning pathways, and changing relations between learning processes. The application of these models can therefore make a substantial contribution to our understanding of learning and individual differences.

1. Introduction

Learning research often seeks to characterize patterns and pathways of learning or development. Many learning theories emphasize both qualitative and quantitative differences in learners' knowledge, skills, and strategies at a specific point in time. Furthermore, learning pathways are often discontinuous or non-linear: Learning can take place in stages, learning pathways can vary substantially between learners, and learning can interact with learner abilities and characteristics (e.g., Carey, 2009; Meiser, Stern, & Langeheine, 1998; Van der Maas & Molenaar, 1992). For example, conceptual knowledge research shows qualitatively different mental models between children, and demonstrates that children differ in their transitions between concepts over time (e.g., Carey, 2009; Kleickmann, Hardy, Pollmeier, & Möller, 2011; Schneider & Hardy, 2013; Smith, Carey, & Wiser, 1985; Vosniadou & Brewer, 1992). To fully characterize learning processes research therefore needs to account for both quantitative and qualitative individual differences at a specific measurement point as well as in change over time. Unfortunately, traditional analytical approaches have limited capabilities to accomplish these goals.

In this article, we outline a set of analytical techniques that are highly useful for this purpose: Latent class and latent profile analysis, and their longitudinal extensions, latent transition analysis. Latent class analysis (for categorical variables) and latent profile analysis (for continuous variables) are used to trace back the heterogeneity in a group to a number of underlying homogeneous subgroups, at a specific measurement point. These techniques have been applied in various domains of learning, for instance in adolescents' literacy (Mellard, Woods, & Lee, 2016), homework behavior (Flunger et al., 2017), and undergraduate science education (Romine, Todd, & Clark, 2016). In the longitudinal extensions of latent class and latent profile analysis, a transitioning component is added to reflect changes in learners' subgroup membership over time, representing potentially non-linear learning pathways. These models have been applied for instance to first-year university students' learning pathways (Fryer, 2017), and to the identification of English language learners at risk for reading disabilities (Swanson,

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2017).

The current paper aims to familiarize researchers in the domain of learning and individual differences with this family of techniques and to illustrate how they can make a substantial contribution to our understanding of learning and individual differences. Note that there are other introductions available, tailored to clinical research (Collins & Lanza, 2010), pediatrics (Berlin, Williams, & Parra, 2014), and developmental research (Kaplan, 2008; Lanza & Cooper, 2016). In the following, we will first elaborate on the usefulness of these models for learning research in comparison to the limits of more common analytic techniques. Then we will give more details about the four types of models that are central to the current paper: latent class analysis, latent profile analysis, latent class transition analysis and latent profile transition analysis. Next, we will discuss the current best practices in application of these techniques to empirical data, addressing several practical and statistical issues that researchers frequently encounter. The concluding remarks will summarize the usefulness of these approaches in learning research. While some basic knowledge of latent variable models may be helpful to understand the present article, the main goal here is to introduce the relevance of these models without expanding too far into the details of the statistical makeup.

2. Learners and learning: a person-centered approach

Inter-individual differences represent an important but complex issue for educators and learning researchers (Snow, 1986). Learners differ in their abilities, motivations, and preferences, which often interact while affecting their learning. Oftentimes, an "average" learning pattern is not an adequate description for many learners because this ignores the unobserved heterogeneity between learners. Assessing and modeling the heterogeneity that may arise from the complex interplay between abilities, motivations, and preferences is important for understanding how, and under which, circumstances learning takes place. In these cases, researchers may find latent class, latent profile, or latent transition analyses to be useful to more appropriately model the unobserved heterogeneity between and within individuals. These techniques constitute a powerful and informative toolbox to examine different subgroups of learners in cross-sectional data and different pathways of learning in longitudinal data.

By contrast, traditional analytical approaches from the general linear model such as ANOVAs, correlation and regression-based techniques, and factor analysis have serious limitations in appropriately characterizing heterogeneity and complex, non-linear learning patterns. These common analytical approaches are variable-centered, emphasizing the relations between variables (Bergman, Magnusson, & Khouri, 2003; Collins & Lanza, 2010). They assume that the relation between variables can be applied to all learners in the same way: in other words, that there is homogeneity in the nature of the individual differences (Bergman & Magnusson, 1997; Collins & Lanza, 2010). These linear techniques are thus restricted to quantitative individual differences, assuming that learners differ quantitatively in the amount of something, but not qualitatively (Lanza & Cooper, 2016; Sterba & Bauer, 2010). For example, in learning research it is common to perform statistical analyses on sum scores from learning measures. The use of sum scores implies the assumption of homogeneity in response patterns, and any heterogeneity – between individuals and within individuals – is primarily considered statistical noise. As a consequence, our understanding of learning processes is a general model that describes the average behavior of a sample. If qualitatively different subgroups exist within a population, they are not accurately represented by the general model.

One means of getting around the use of continuous measures to examine differences in learning processes is with arbitrary cut-off points (e.g., median-splits). The resulting groups often represent ability levels such as "high" and "low", and differences between the groups are then explored to infer learning differences. While the comparison of ability groups can provide a useful method of understanding implications of different abilities on learning outcomes, the use of arbitrary cut-off points is considered poor statistical practice: it is a-theoretical and introduces error which can result in a distorted picture of relations between variables (Altman & Royston, 2006; Irwin & McClelland, 2003; Maxwell & Delaney, 1993). Consequently, arbitrary cut-off points are never appropriate and should be avoided.

The most important limitation of variable-centered methods is their inability to deal with heterogeneity within and between individuals. Another constraint of *linear* variable-centered methods is their inability to accurately characterize non-linear and interactive patterns (Bergman et al., 2003). Consequently, the use of linear variable-centered analyses impedes our ability to test theoretical claims of learning that do not meet these assumptions, such as when heterogeneous patterns, discontinuous change, or interacting and changing relations between two or more learning processes are present. Although some non-linear variable-centered methods exist that allow analyzing some non-linear learning patterns and pathways, these are still limited to general patterns for the entire population. By contrast, person-centered approaches are not restricted to linear patterns and can model heterogeneity as well. Person-centered analyses place the emphasis on the individual, in order to account for heterogeneous patterns of variable interactions; "operationally, this focus often involves studying the individuals on the basis of their patterns of individual characteristics that are relevant for the problem under considerations" (Bergman & Magnusson, 1997, p. 293). In the present case, the problem under consideration is knowledge and learning.

The aim in learning research is rarely to describe a single learner, but rather to describe general patterns of learners' behavior and learning pathways. Understanding these patterns and pathways can enable educators to better understand why some learners are more successful with learning and some experience particular difficulties, or this understanding can be used to inform targeted learning interventions. The strength of person-centered approaches is that they can capture these different patterns and pathways, by identifying homogeneous subgroups of learners that exhibit similar patterns of characteristics (Bergman & Magnusson, 1997). Traditional clustering methods like *K*-means clustering (e.g., Kaufman & Rousseeuw, 2009) provide one approach to examine such subgroups. The family of model-based clustering methods that latent class and profile models belong to, however, have specific advantages over traditional cluster techniques. These models are more flexible, account for measurement error, and are able to handle longitudinal data (e.g., Magidson & Vermunt, 2002, 2004; Oberski, 2016; Vermunt, Tran, & Magidson, 2008). Most of the work on developing these models and estimation procedures has, in fact, been completed by statisticians and methodologists in the social sciences starting in the 1960s (e.g., Goodman, 1974; Lazarsfeld & Henry, 1968; McCutcheon, 1987; Wiggins, 1973). More recently, the approach has been adopted by psychology and education researchers to examine learners and learning processes.

2.1. What is a latent class or latent profile model?

The aim of latent class and latent profile models is to trace back heterogeneity in a population to a number of existing but unobserved subgroups of individuals, which are referred to as latent classes. The analyses are based on a set of observed variables that can be categorical and/or continuous. The classes are formed such that there is as much similarity within a class while at the same time as much differences between the classes as possible (Lanza & Cooper, 2016). The identification of these latent classes can be useful for characterizing qualitative differences between learners, which may be missed with traditional analytic approaches. For example, Fig. 1 depicts outcomes of two analytic approaches to the same example data with two variables: accuracy and response time on a particular measure. Note that the advantages of latent class and latent profile models are more pronounced

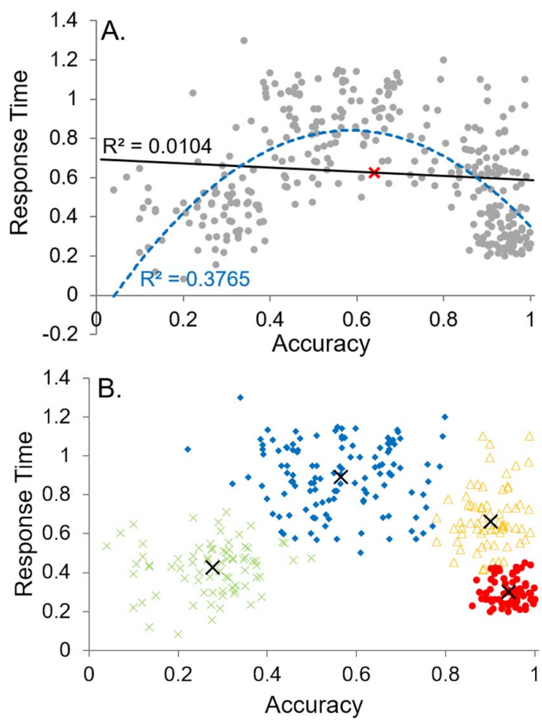

Fig. 1. Example data of accuracy and response time, subjected to (a) correlation analysis, and (b) latent profile analysis. In (a), the linear correlation is depicted by the black line; quadratic function indicated by the blue dotted line; means of accuracy and response time converge at the red X. In (b) the four subgroups identified by the latent class analysis are depicted by the green, blue, yellow, and red icons. Black crosses represent the means (accuracy and response time) for each subgroup. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

when there are more than two observed variables. However a two-dimensional space is more convenient for the present illustration purpose. Fig. 1a shows the linear and quadratic correlations between accuracy and response time; sample means are depicted by the red cross. The linear correlation and the means do not allow for drawing conclusions about patterns of learners' behavior. Although the quadratic curve does a better job in representing the data, it is still unable to reflect differences in dispersion. Conversely, Fig. 1b shows the same example data as an outcome of a latent profile analysis. Here, four latent classes are identified based on their profile of means across accuracy and response time: fast and inaccurate (green), slow and moderately accurate (blue), slow and accurate (yellow), and fast and accurate (red). The analysis suggests that these are meaningful patterns of learners' behavior.

It is important to note that the classes that are identified in latent class and profile analyses may represent qualitatively distinct learning stages, but that it is also possible that the behavior of different classes is an expression of the same underlying quantitative ability continuum, for instance when low, medium, and high proficiency classes are identified (Bouwmeester, Sijtsma, & Vermunt, 2004). In the latter case, latent classes can still be helpful for modeling non-normal ability distributions (Wall, Guo, & Amemiya, 2012) or providing statistically sound categorizations rather than using arbitrary cut-off points (e.g. median-split). It is also possible to identify quantitative and qualitative differences simultaneously. Consider for instance the following hypothetical example: In problem solving research, three learner classes are identified based on the responses (correct/incorrect) to six problems. In two classes the learners use the same strategy but with different success rates across all problems, while learners in the third class use another strategy which is very successful on some problems but quite unsuccessful on other problems. The first two classes differ primarily quantitatively in – overall – performance, while they both differ qualitatively from the third class. Furthermore, in the third class, qualitative intra-individual differences are present across the problems. In traditional linear models, most of this information would be obscured by focusing on sum scores only. The use of latent class and profile models is therefore a very general and flexible tool, since it allows us to analyze both qualitative and quantitative inter- and intra-individual differences simultaneously.

2.2. Latent class and latent profile models: new ways to understand learning

The ability to identify and describe different learner patterns or learning pathways can benefit both educational theory and practice. Latent class models provide the potential to integrate seemingly contradictory findings and/or theories. For example, researchers agree that magnitude representation abilities (e.g. accurately placing a number on a number line) support mathematical abilities, but there is no consensus on the role of symbolic and non-symbolic magnitude representation abilities (Chew, Forte, & Reeve, 2016). There are findings supporting theories claiming that non-symbolic abilities scaffold symbolic abilities, whereas there are also findings supporting theories claiming that symbolic abilities are independent of non-symbolic abilities. Taken together, these findings appear contradictory. A latent profile analysis on non-symbolic and symbolic abilities demonstrated that within a single sample, all interpretations suggested by different theories appeared (Chew et al., 2016), which allowed the construction of a comprehensive framework of the relations between non-symbolic, symbolic, and mathematical abilities. Similar approaches have been used to accommodate different theoretical perspectives in the relation between math anxiety, working memory, and math problem solving performance (Trezise & Reeve, 2014a, 2014b), the role of concrete materials in children's learning of abstract concepts (Ching & Nunes, 2017), and children's mental models of the earth (Straatemeier, van der Maas, & Jansen, 2008).

Latent class models can also provide insight into different patterns of strengths and weaknesses in learning processes, which can lead to the development of interventions that can be difficulty-defined or that target a specific learner profile. For example, latent profile analysis has been used to identify different types of at-risk children, based on their similarities in problem behaviors, such as socially and academically disruptive children and socially and academically disengaged children (Bulotsky-Shearer, Bell, & Domínguez, 2012). Subsequent analyses revealed a relation between at-risk profiles and learning trajectories, providing insight into how children with distinct combinations of problem behaviors have different academic growth patterns. These findings research can be used to inform researchers on the development of interventions specific to different forms of academic and social difficulties.

3. Model parameters in latent class, latent profile, and latent transition models

Table 1 presents an overview of the different latent class and latent profile models. These models are differentiated by the measurement level of the observed variables, also called indicators (categorical or continuous) and the number of measurement points (one or multiple).

Table 1
Overview of latent class, latent profile, and latent transition models.

	Cross-sectional design	Longitudinal design
Categorical indicators	Latent class model (LCM)	Latent class transition model (LCTM)[a]
Continuous indicators	Latent profile model (LPM)	Latent profile transition model (LPTM)[a]

[a] Also called: Latent Transition Model, Latent Markov Model, or Hidden Markov Model.

Models for categorical observed variables are called latent *class* models, whereas models for continuous observed variables are called latent *profile* models.[1] This distinction is not rigid because modern statistical software can handle categorical and continuous indicators simultaneously (e.g., Vermunt et al., 2008). For many relevant research designs it is however still common to exclusively use either categorical or continuous indicators.

Latent class and profile models are part of the family of latent variable models, which assume that a latent (unobserved) variable accounts for the relations between the manifest (observed) variables (Collins & Lanza, 2010). In these models, the model nomenclature depend on the type of observed variables (categorical or continuous), type of latent variable estimated in the model (categorical or continuous), and the number of measurement points (one or multiple). Latent variable models with a *continuous* latent variable are, for instance, factor analysis (for continuous observed variables) and item response models (for categorical observed variables). Continuous latent variable models are concerned with characterization of *variables* (e.g., the factorization of intelligence). The current paper focuses on models with a categorical latent variable, which are concerned with the characterization of *cases* (e.g., identifying different patterns of intelligence in individuals) and therefore belong to the person-centered approaches. The *categorical* latent variable, also called the latent class variable, represents the (unobserved) heterogeneity in the responses on a set of observed variables by a number of distinct subgroups of individuals. For instance, the four distinct groups of learners shown in Fig. 1b differ in their speed-accuracy behavior. Categorical latent variable models also go by the name of latent variable mixture models, since the overall data constitute a mixture of class-specific data.

The longitudinal extensions of these models include transitioning paths between the latent classes across measurement points, and are referred to as latent transition models. These models characterize learning pathways by estimating the probabilities with which learners move from one class to any of the other classes over time. For instance, it may be very likely for a student to move from a slow-but-accurate to a fast-and-accurate class in the course of learning, but very unlikely to make the change in the opposite direction. The distinction between models for cross-sectional and longitudinal designs is graphically displayed in Fig. 2. As is common in the representation of latent variable models, the latent variable (in circles) influences the responses on the observed variables (in rectangles). Specifically, the latent class variable accounts for the responses to a set of observed variables Y, which can be continuous and/or categorical. In a cross-sectional latent class or latent profile model (Fig. 2a) there is only one latent class variable, whereas in a latent transition model there is a latent class variable for each measurement point (Fig. 2b).

3.1. Model parameters in latent class and latent profile models for cross-sectional designs

To examine heterogeneity at a single measurement point, one needs to identify and characterize different subgroups. The subgroups or classes are represented by two types of estimated parameters: latent class parameters and class-conditional parameters.

Latent class parameters indicate the estimated proportion of individuals in each class, representing the class sizes or class prevalence. In the problem-solving example from Section 2.1, the size of the classes could be estimated for instance at 50%, 30%, and 20% for class 1, 2, and 3, respectively (note that they sum to 100%). The estimated *class-conditional parameters* characterize the classes. They form the measurement part of the model that accounts for the relation between the latent class variable and indicators (i.e., the arrows from the circles to the rectangles in Fig. 2). In that sense, the class-conditional parameters are similar to factor loadings in factor analysis. Like factor loadings, class-conditional parameters are used to interpret the classes. In the case of categorical indicators, the class-conditional parameters are the probabilities of a certain response (e.g., correct/incorrect) on each indicator (often an item on a test) for that class. In case of continuous indicators class-conditional parameters are the class-specific profile of means (and variances) for the indicators (e.g., mean number of correct responses, mean reaction-time). Note that the outcome of latent class or latent profile analysis does not include each individual's class membership. Instead, based on an individual's response pattern and the estimated class-conditional parameters it is possible to derive the probability that he/she belongs to each of the latent classes: this is known as the posterior classification probabilities.

3.2. Examples of latent class and latent profile analysis in learning research

Use of latent class and latent profile models in learning contexts has the ability to contribute to our understanding of learning. Estimation of classes and class membership can be used to characterize different types of learners, and their learning characteristics. In this section, we present examples of how latent class and latent profile analyses have been used in research into learning and individual differences. That is, we illustrate how latent class (Hickendorff, van Putten, Verhelst, & Heiser, 2010) and latent profile models (Abenavoli, Greenberg, & Bierman, 2017) in general, and their model parameters specifically, can make a contribution to our understanding of learning.

Using latent class analysis, Hickendorff et al. (2010) analyzed individual differences in sixth graders' adaptive use of mental and written solution strategies in solving nine multi-digit division problems such as "938:14 = _". In traditional, linear variable-centered approaches, each child's proportion of problems solved mentally would have been computed to identify quantitative differences between students. This would have obscured potential heterogeneity across problems, potentially concealing adaptive strategy choices (switching between strategies according to problem demands). Therefore, a latent class analysis was performed to characterize the naturally occurring individual differences in strategy use across the problems. The nine strategy use variables (binary: mental or written) served as indicator variables, resulting in a model with three classes. Fig. 3 presents the latent class parameters (class sizes) and the class-conditional parameters (the probability to use a mental strategy on each of the nine problems).

The differences between children in adaptivity of strategy choices appeared of qualitative rather than quantitative nature. Two subgroups were not adaptive in their strategy choices: a small group (class 1) quite consistently applying mental strategies, and a larger group (class 3) quite consistently applying written strategies. The other subgroup (class 2) demonstrated an adaptive strategy selection pattern by switching to mental calculation on less-demanding problems. Contrary to latent class analysis, in variable-centered approaches a priori assignment of division problems to particular problem types (mental or written) would be necessary to identify such a pattern. Relatedly, when researchers want to test specific hypotheses, for instance that problems 5 to 9 have similar response patterns in the current example, it is possible to impose restrictions on the model parameters (e.g., Finch & Bronk, 2011; Formann, 1985), such as equality restrictions on the class-conditional parameters for these five problems. Further analyses showed that there were very few children with low mathematics performance level in the 'adaptive' class, supporting theories that only high mathematically achieving students make adaptive strategy choices.

Using latent profile analysis, Abenavoli et al. (2017) investigated the variations in school readiness skills of high-risk, low-income children in kindergarten. They argued that instead of exclusively focusing on children's general level of school readiness skills, it is more informative to investigate children's *patterns* of school readiness skills. In

[1] The models are also known as Latent Class Cluster Models (Vermunt & Magidson, 2002), or Finite Mixture Models (Binomial for latent class and Gaussian for latent profile models; Oberski, 2016).

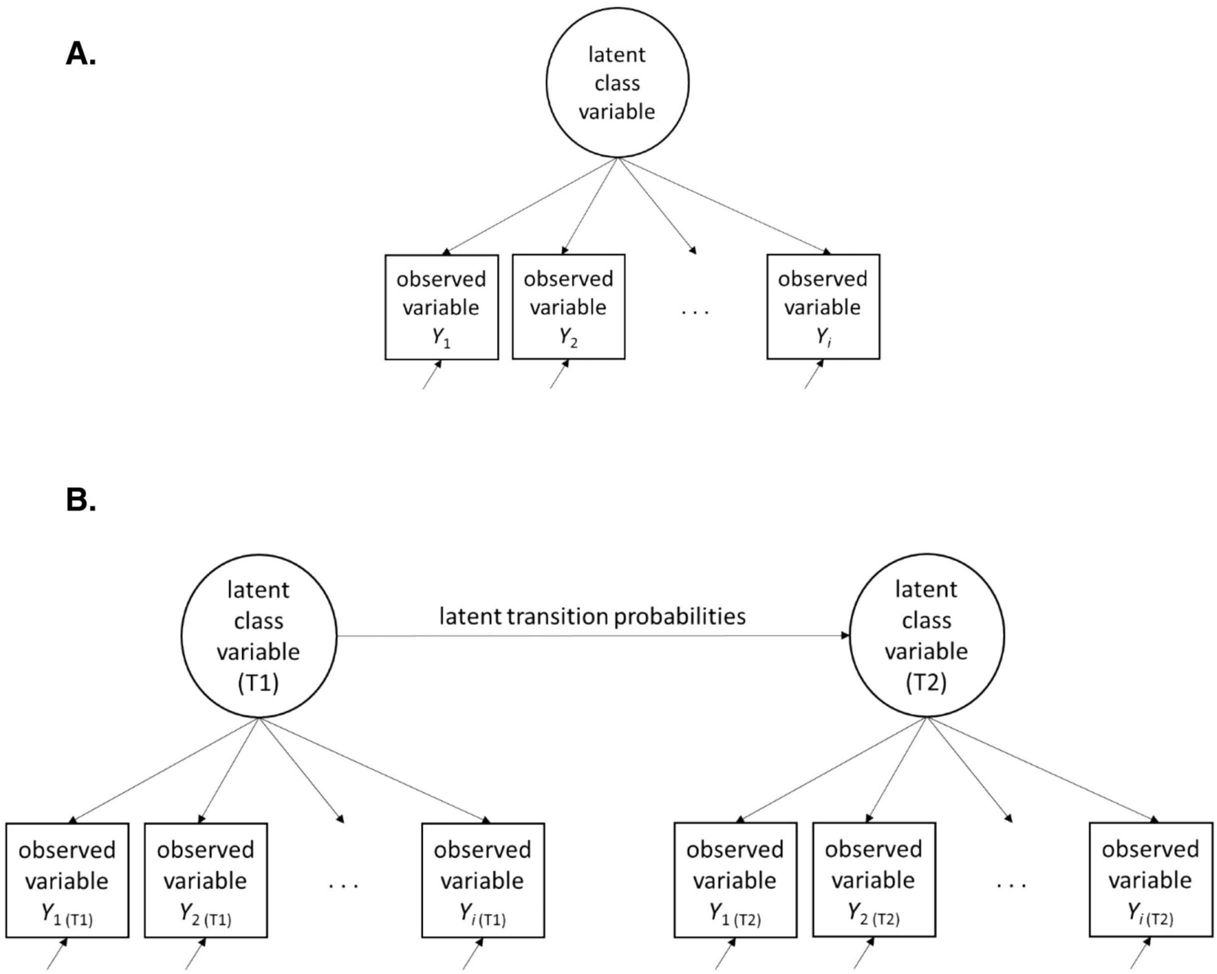

Fig. 2. Graphical representation of cross-sectional latent class or latent profile model (Fig. 2a) and latent transition model (Fig. 2b).

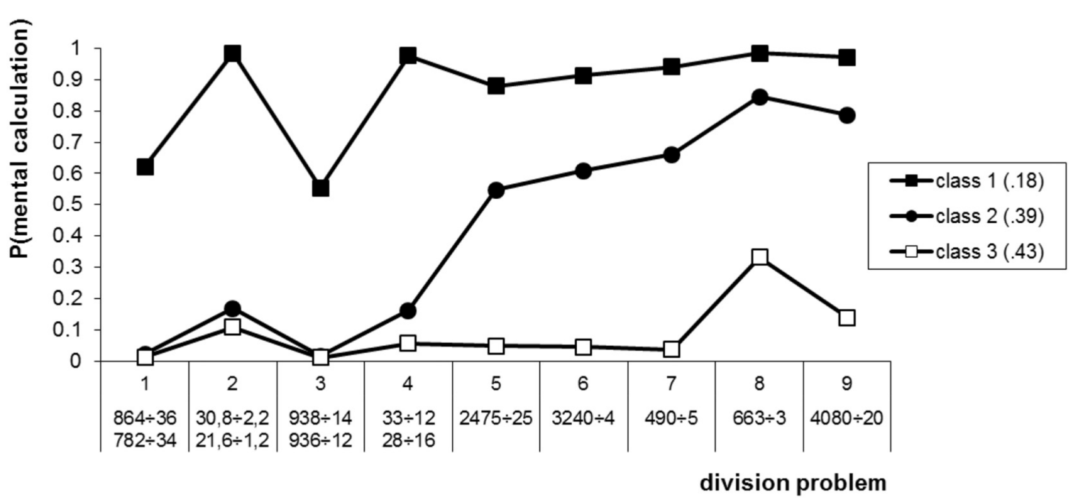

Fig. 3. Illustration of model parameter estimates from a latent class example on sixth graders' strategy choice (mental or written) in solving nine division problems (adapted from Hickendorff et al., 2010). The probabilities to use a written strategy are not displayed since they are redundant with two response categories per indicator.

a latent profile analysis using ten teacher-rated measures of children's academic ability, learning engagement, social-emotional skills, and aggressive-disruptive behaviors as indicator variables, four distinct classes were identified. The interpretation of each class's means profile was a well-adjusted profile (class size 42%) characterized by broad strengths across all readiness dimensions, a competent-aggressive profile (19%) with above-average academic ability but low social-emotional skills and elevated aggressive-disruptive behavior, an academically disengaged profile (22%) with risk in academic learning engagement and social-emotional skills but without aggressive-disruptive behavior, and a multi-risk profile (17%) characterized by risks across all readiness variables.

In summary, these examples illustrated how latent class and latent profile analysis can be used to understand *how* students' complete tasks (e.g., problem solving strategy), the differences between tasks in the solution strategies used (e.g., problem demands affecting written strategy-use), the nature of school-entry weaknesses children from high-risk contexts may display, and their implications for learning and education.

3.3. Model parameters in latent transition models for longitudinal designs

As Fig. 2b shows, a latent transition model is built up of consecutive latent class models for each measurement point and, additionally, class[2] transitions between measurement points.[3] Three types of model parameters are estimated in latent transition models. *Initial class probabilities* represent the class proportions at time $t = 0$; these are akin to the latent class parameters in latent class or profile models. *Latent transition probabilities* represent the probabilities of transitioning from a particular class at measurement point t to each of the classes at measurement point $t + 1$; these are unique to latent transition analysis. *Class-conditional parameters* are analogue to the class-conditional parameters from latent class or profile models (class-specific response probabilities in case of categorical indicators and class-specific means and variances in case of continuous indicators).

The application of latent transition analyses is illustrated by a study on conceptual change in the domain of children's development of the understanding of floating and sinking of objects in water (Schneider & Hardy, 2013). Using three continuous indicator variables, each assessed at three measurement occasions, a latent profile transition analysis identified five different knowledge profiles (Fig. 4a). For example, one profile had a high mean score for incorrect conceptions and low mean scores for partially correct and entirely correct conceptions, representing a group of children with a low proficiency level on this topic.

The latent transition part allowed for the analysis of how the children moved between the five classes over time, characterizing the developmental pathways. Theoretically there were 125 possible pathways (5*5*5) the children could follow across the measurement occasions. However, the majority of the children (63%) only followed seven transition pathways, illustrated in Fig. 4b. For instance, some children followed the 'ideal learning path' from the Misconceptions Profile at T1 to the Scientific Profile at T2 and T3, whereas others followed an 'enduring-fragmentation path' since they were in the Fragmented Profile at all three measurement points. These results indicate that there is a high degree of systematicity but at the same time also substantial heterogeneity in children's developmental pathways. Specific hypotheses about the transition paths children do and do not take could be rigorously tested by further restrictions on the latent transition parameters, for instance restricting all 'backwards' transition probabilities to zero. Moreover, they demonstrate that latent transition analysis can also be an effective data reduction technique: the heterogeneity of large numbers of individuals is reduced to a small number of relevant classes and transition pathways. Finally, the results indicate the predictive power of prior knowledge for subsequent conceptual learning and development, which may be used in educational practices, for example, to identify students needing special attention.

[2] In latent transition models, the latent classes are sometimes referred to as latent *states* to convey their temporary nature (e.g., Collins & Lanza, 2010), but in the current paper we continue to use the term latent class.

[3] Latent transition models measure changes over time using the Markov assumption: Change only depends on the prior class. They are therefore also referred to as Latent Markov Models (Kaplan, 2008; Vermunt et al., 2008). This is distinct from other longitudinal latent variable models where change is cumulative over time, such as panel models and latent growth models. *Panel models* examine the predictive effects of a variable at a single measurement point on that same and other variables at a subsequent measurement point (Ferrer & McArdle, 2003; Little, 2013) and is therefore limited in capturing individual differences in change (i.e., learning) over time (Ferrer & McArdle, 2003). In *latent growth models*, intercept and slope variables represent individuals' ability at a certain measurement point and its growth (i.e., learning) over time (Little, 2013). These models can characterize a continuous learning trajectory, but are limited in examining discontinuous learning or the complex, possibly changing interactions between multiple abilities over time.

4. A practical guide to latent class, latent profile, and latent transition analysis

In this section we give some guidelines for the application of latent class, latent profile, and latent transition models in general, important practical and statistical issues, and available software. Given the field is still emerging there are no consensus-based standards. Consequently, some of our points may be contentious. However, the following is an attempt on our part to describe what we believe to be the best practices for these models at the present time.

4.1. Research questions and data considerations

The preliminary step before conducting a latent class, profile, or transition analysis is to decide on the most appropriate model type for the research question and data. The nature of the expected individual differences is important: are they likely to be qualitative or primarily quantitative? Latent class or profile models are ideal if subgroups – qualitative differences – are expected; whereas if only quantitative differences are expected, the use of latent class or profile model should be reconsidered. It is also important to consider whether the sample size is appropriate. There is no definite rule for sample size in latent class or profile analysis. As discussed by Wurpts and Geiser (2014), usually sample sizes well into the hundreds are suggested. They argue, however, that the negative effects of a small sample size depend on the circumstances. Their research showed that using more and high quality indicators (strongly related to the latent class variable) or a covariate that is strongly related to class membership (see Section 4.2.3) may alleviate some of the problems frequently found with small sample sizes, but that samples below $N = 70$ were not feasible under virtually any circumstances. They recommend researchers to conduct an application-oriented simulation study (Muthén & Muthén, 2009) and/or a pilot study to identify the best indicators and covariates. Table 1 informs the choice between latent class, latent profile, or transition models, determined by whether the indicators variables are continuous, categorical, or both, and the number of measurement occasions.

4.2. Conducting a latent class or latent profile analysis

4.2.1. Determining the number of latent class

Once a latent class or profile analysis is deemed suitable, and the model type is selected, the first step is to fit a series of models with an increasing numbers of classes. This is done by starting with a one-class model (i.e. all individuals are in the same class), and then iteratively adding one additional class. Theoretically, the highest number of classes to be estimated depends on the nature of the data, particularly the number of possible response patterns. In practice the number of classes estimable is much smaller. Typically, maximum likelihood estimation is used to estimate the models.[4] As with all maximum likelihood procedures, starting values are a crucial aspect in estimating the model parameters (see Section 4.4). Once all stable models have been estimated, the best model is selected.

This model selection process is probably the most prominent and challenging issue, yet also central in drawing conclusions from these analyses. In comparing models with different numbers of classes, the decision for a certain number of classes is usually made based on a combination of statistical fit measures, parsimony, and interpretability based on the conceptual appeal of the model given substantive theory

[4] For more information on maximum likelihood estimation in latent class analysis, see Chapter 4 of Collins and Lanza (2010). Furthermore, Bayesian estimation has recently been applied which can for example be of advantage with moderate sample sizes. Interested readers are referred to Song and Lee (2012) and Ortega, Wagenmakers, Lee, Markowitsch, and Piefke (2012).

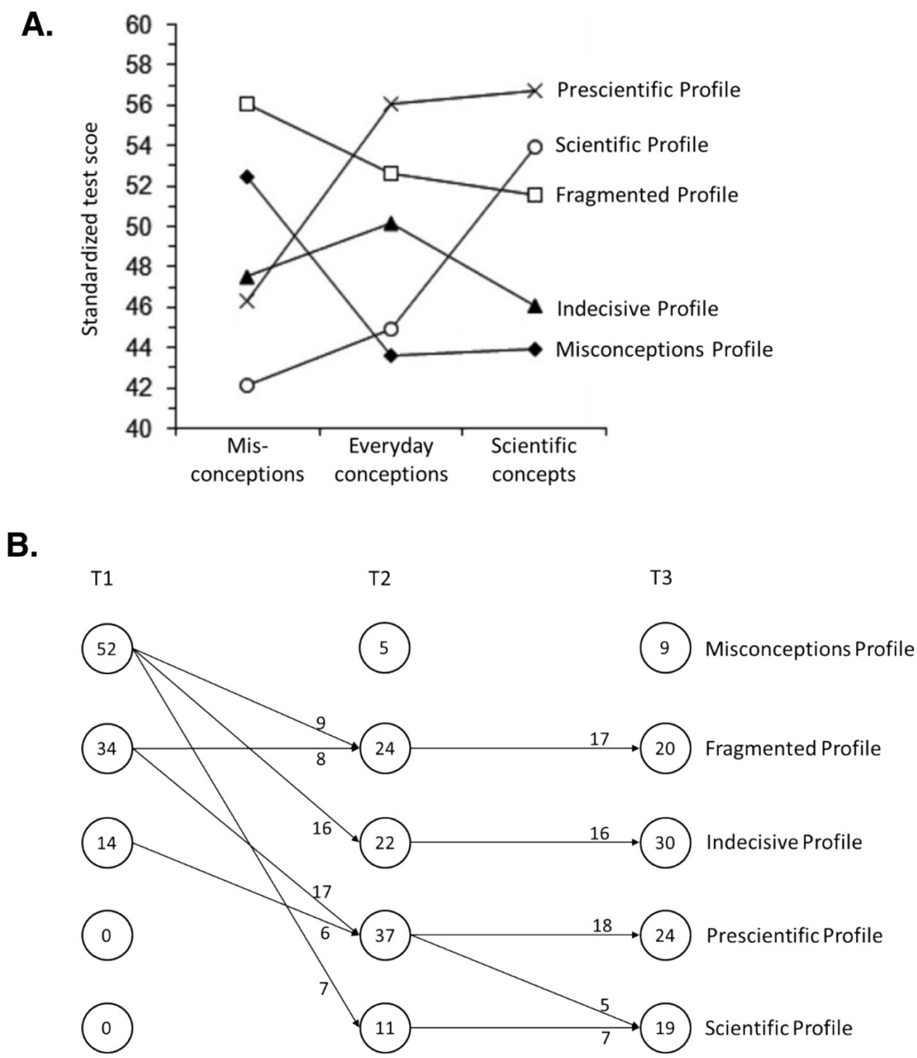

Fig. 4. Illustration of model parameter estimates from the latent profile (transition) analysis from Schneider and Hardy (2013). Fig. 4a: class-conditional mean profiles of the five knowledge classes; Fig. 4b: (most prevalent) transitions between the five classes across the three measurement occasions; numbers in circles = percentage of individuals per measurement point; numbers on arrows = percentages of individuals making this transition.

(Collins & Lanza, 2010). Nylund, Asparouhov, and Muthén (2007) have studied the performance of different statistical fit measures in latent class and latent profile analysis, and identified the bootstrap likelihood ratio test (BLRT) as the best-performing one, and the Bayesian Information Criterion (BIC) as second best. The BLRT allows examining whether including one more latent class significantly improves the model fit. If this is not the case, the more parsimonious model with fewer latent classes should be selected (Tekle, Gudicha, & Vermunt, 2016). Besides these relative model fit indices, absolute fit indices need to be considered as well. Model-likelihood *p*-values do not work well in latent class models due to the sparseness of the data (the extent to which the expected cell count in the indicators' contingency table is small). As an alternative, bootstrap procedures have been developed and implemented in different programs (Lanza, Dziak, Huang, Wagner, & Collins, 2015; Muthén & Muthén, 2015; Vermunt & Magidson, 2013).

The distinction between the increasing numbers of classes is depicted in Fig. 5. Fig. 5a depicts the same four-class solution to data shown in Fig. 1b. This is the best-fitting model. Fig. 5b–d show the classes identified in the two-class, three-class, and five-class solutions, respectively. The classes in the two-class and three-class solutions are mainly informed by the accuracies, and not the response times. In the five-class solution, the added class (grey triangles) is quite similar to the moderately-accurate-and-slow class (in blue diamonds). In the interest of parsimony and since the grey triangles do not substantially increase the conceptual information of the model, the four-class solution is deemed optimal.

4.2.2. *Model interpretation*

In the next step, the researcher interprets the selected model. Classes are typically labeled based on the patterns the class-conditional parameters convey, and typically tied to theory informing the research, as was done in the empirical examples from the previous section (Abenovali at el., 2017; Hickendorff et al., 2010; Schneider & Hardy, 2013). In latent profile analysis, testing which class means differ from each other or from the overall sample mean can aid the interpretation of the profiles. In general, classes can differ in *levels* (overall high, medium or low across the indicators) or *shape* (a pattern of high, medium, or low scores varying across the indicators) (Marsh, Lüdtke, Trautwein, & Morin, 2009). The prevalence of each class can be interpreted in light of how prevalent that learning pattern might be in a relevant population.

4.2.3. *Relations with external variables*

Often researchers are not just interested in characterizing the different classes, but also how they are predicted by other (cognitive, motivational, etc.) factors, or how the subgroups differ in their performance on a separate outcome variable. This is analogous to examining predictors of group membership using a multinomial regression, or how groups differ on a dependent variable using an ANOVA. There are two different approaches to investigate the effects of such external variables: incorporate them as covariates in the initial model estimation (one-step approach), or after the models have been estimated and subgroups have been established (three-step approaches).

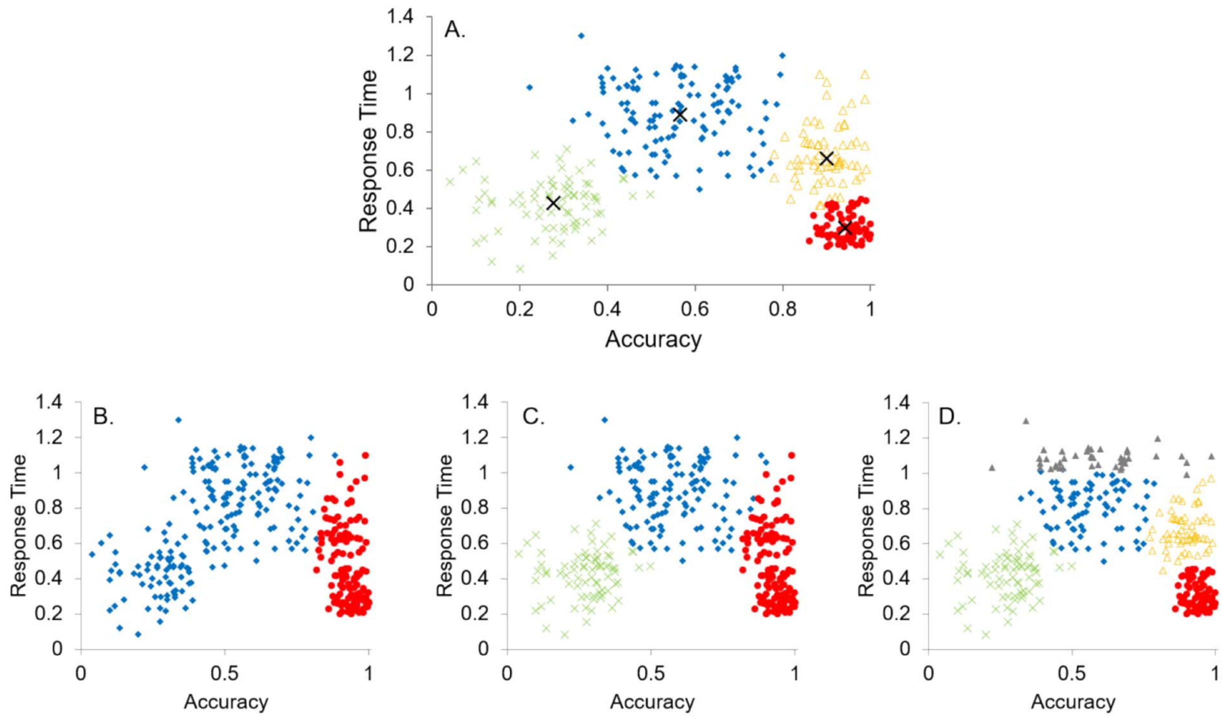

Fig. 5. Classes identified for the (a) optimal solution; (b) two-class solution; (c) three-class solution; (d) five-class solution, of the same data. Different colors indicate the different cluster assignments. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

In a one-step latent class or profile analysis, the covariate is incorporated in the initial model estimation, estimating the effect of the covariate on the class sizes. For instance, age or grade can be entered to the model as covariate, showing that classes characterized by more advanced cognitive behavior are more prevalent in older children than in younger ones (e.g., Paul & Reeve, 2016) allowing for a cross-sectional investigation of learning. Importantly, the one-step procedure allows the covariate to influence the class definitions: if the same model is estimated without the covariate, it is possible that a different solution in terms of class-conditional parameters (and hence interpretation of the classes) and/or class prevalence would be identified.

There are circumstances when the one-step approach is not appropriate. For example, it can be impractical if multiple covariates are of interest, as models would need to be re-estimated to test the significance of each covariate (Asparouhov & Muthén, 2014). Second, often researchers aim to first establish the existing subgroups, before they examine the effects of covariates on these groups. Third, the one-step approach is not designed to deal with distal outcomes (e.g., how cognitive ability classes differ in their ability to perform a math task). In these cases, a three-step approach should be applied. In step one a model is fit (without external variables). In step two individuals are assigned to classes based on their posterior class membership probabilities. These posterior classification probabilities are based on the responses an individual has given and the estimated class-conditional model parameters. For example an individual may have posterior classification probabilities 0.20, 0.70, 0.08, and 0.02 for the green, blue, yellow, and red class in Fig. 4b. Using a modal assignment procedure each individual is assigned to the class for which his/her probability is highest (e.g., Bray, Lanza, & Tan, 2015), which would be the blue class. In step three of the three-step approaches, the association between the assigned class membership and external variables is investigated.

The most straightforward way to conduct the third step is the *classify-analyze* approach (also referred to as a Standard Three-step Approach), in which each individual's assigned class is treated as a known (error-free) nominal variable in further analyses for categorical data (i.e., each individual is assigned to their most likely class based on their posterior probabilities, in the above instance the "blue" class). Since this latent class membership variable can be transferred to most statistical packages, a wide range of categorical data analyses can be used to examine the relation between latent class membership and other variables. A significant issue with this method, however, is that unless all individuals have a probability of 1 to belong to a class, error is introduced when individuals are assigned to classes (Bolck, Croon, & Hagenaars, 2004; Bray et al., 2015), which can attenuate the relations examined.

With the aim of reducing this error, a *bias-adjusted three-step analysis* has been developed. The latent class membership is treated as known (e.g. the above individual would still be assigned to the blue class), but weighted by classification error usually based on the posterior classification probabilities (there is also a substantial chance of 0.30 this individual belongs to one of the other classes). The individuals' weighted probabilities are then used to examine the relation between class membership and the covariates in question. This requires a statistical program capable of latent class three-step analyses. As developments in these methods continue, there are currently different bias adjustments across different programs. The details of these different methods are beyond the scope of this paper, but can influence the strength of the relations between the latent classes and covariates/outcomes (for more detailed information we refer to Bray et al., 2015; Vermunt & Magidson, 2016).

4.3. Conducting latent transition analyses

4.3.1. Determining the number of latent classes

The optimal number of latent classes in latent transition analyses is determined in the same way as for cross-sectional latent class or latent profile analysis, using a combination or statistical fit indices like the BIC and BLRT, combined with the conceptual appeal of the model solution. However, there is an important additional consideration in specifying a latent transition model: Whether the class-conditional parameters are constrained to be equal over time or not. If they are constrained, the response probabilities (categorical indicators) or means and variances (continuous indicators) for each class will remain the same, and the interpretation of the latent classes does not change over time. This is comparable to longitudinal measurement invariance in variable-centered methods such as confirmatory factor analysis. For example, in a model with three classes and two measurement points in which the class-specific means are constrained to be equal over time, class 1 will have the same profile of mean scores regardless of whether it is at measurement point 1 or 2. Conversely, in the same model specified without equality constraints over time, the three classes at the first measurement point may have different means and variances to the classes at the second measurement point. Thus, there are not three but six classes in total, which differ in their mean profiles. Consequently, this affects the interpretation of the classes: class 1 at the first measurement point can have different characteristics and meaning than class 1 at the second measurement point. Thus, in the absence of equality constraints over time, it is important to report the class-conditional profiles and class interpretations separately for each measurement point. It is also possible to statistically test the assumption of time-invariant class-conditional parameters.

If the class-conditional parameters are constrained to be equal over time, it is advisable to test the number of latent classes simultaneously for all measurement points (e.g., Schneider & Hardy, 2013), where it is possible that a class is 'empty' at a specific measurement point (for instance if learners develop new solution strategies between measurement points). If the class-specific parameters are unconstrained, the number of latent classes is usually determined separately for each measurement occasion.

4.3.2. Model interpretation

The latent classes are interpreted the same way as in cross-sectional latent class or profile analyses, using the class-conditional parameters. Additionally, for each latent class a set of transition probabilities show how likely it is that learners transition from this class into any of the other classes over time. For example, a probability of 0.2 for transitioning from class 2 to class 3 implies that one fifth of the learners who were in class 2 moved to class 3. Note that the transition probabilities are asymmetric: the probability of moving from class 2 to class 3 is different from the probability of transitioning from class 3 to class 2. The stronger the asymmetry of the transition probabilities, the clearer the developmental ordering of the classes. For example, if the probability of moving from class 2 to class 3 is 0.9, but the probability of moving from class 3 to class 2 is 0.2, then there is a clear developmental ordering in which most individuals develop from class 2 (e.g., having a specific misconception) to class 3 (e.g., holding the correct concept) but not vice versa. The transition paths need to be interpreted in terms of the underlying learning processes. As with the interpretation of the latent classes, this is done by the researcher based on theoretical considerations and is not a direct output of the model estimation process.

4.3.3. Relations with external variables

Like in latent class or profile models, in latent transition analysis covariates can be used to examine predictors of initial class size. Additionally, the effects of covariates on the transition probabilities (i.e., the change in class membership over time) can be tested, such as the impact of training type on the transitions between knowledge state frrom pretest to posttest in an intervention study. Both one-step and three-step approaches can be used. These analyses can be complex, with a number of constraints and assumptions within the model. Developments in this field are continuing. For more information on this technique, we refer to Bartolucci, Montanari, and Pandolfi (2015), Di Mari, Oberski, and Vermunt (2016), and Collins and Lanza (2010).

4.4. Practical and statistical issues

Some general practical issues researchers encounter in application of latent class, profile, or transition models are, first, that these models are usually applied in a rather exploratory way. This can be informative in some situations but a potential drawback in others. Researchers are usually left with various decisions during the modeling process. These include deciding on the number of classes, which might be compared with deciding on the number of factors in factor analysis. This exploratory nature may also call the generalizability of findings into question. Researchers usually try to overcome this issue and validate their findings by examining whether the identified classes show theoretically expected relations with external variables. Statistically a cross-validation procedure, for instance the split-half method, can be used to address the question of stability of results. Furthermore, it is possible to rigorously test specific hypotheses by imposing constraint on parameters (Finch & Bronk, 2011), such as equality constraints on the class-conditional probabilities for a set of variables, or setting certain latent transition probabilities to zero. A related issue is that interpretation of the latent classes may yield many different labels and interpretations across researchers. For instance, Wormington and Linnenbrink-Garcia (2017) found ten different goal profile types in a meta-analysis of 24 studies on students' achievement goals, making the comparison between the studies difficult. The use of latent class, latent profile and latent transition analysis to test theories should therefore be encouraged.

Second, latent transition models work best with relatively few measurement occasions: with a higher number of test occasions, results may reveal a number of heterogeneous profiles and transition pathways that are difficult to interpret (Bergman & Magnusson, 1997). A final limitation is that although latent variable models are very explicit in the model underlying the data, the trade-off is that they bring along various assumptions. For instance, in latent class models it is usually assumed that the latent class variable accounts for all relations between the observed variables (local independence assumption, e.g., Collins & Lanza, 2010). In latent profile models it is usually assumed that the indicators follow a specific – usually normal – distribution within each class (e.g., Oberski, 2016). In latent transition models it is assumed that the change over time only depends on one measurement occasion before, but not on the occasions before that (Markov assumption, e.g., Collins & Lanza, 2010). These potential drawbacks highlight that prior to any analysis, it is important to consider whether the chosen analysis is suitable, given the research questions, data, expected outcomes, and any follow-up analyses.

Some more specific statistical issues researchers may encounter in conducting these analyses include model identification, local optima in estimating the model parameters, and missing data. Model identification relates to the extent to which there is enough information in the data to yield unique parameter estimates (e.g., Collins & Lanza, 2010). Identification problems may appear particularly in models with many parameters, which is the case in models with many classes and/or a transitioning structure. Achieving identification may not be easy, and the only possible way to overcome identification problems is to either increase the amount of known information (which would require additional data) or to decrease the number of parameters, for instance by estimating fewer clusters or putting constraints or restrictions on certain parameters (e.g., by 'demanding' that for two very similar variables the class-conditional parameters are equal). A related issue is the tendency to yield solutions that are not the optimal one in terms of the model likelihood (local optima). This becomes more pervasive when models get more complex (more parameters) and model identification becomes poorer. Most programs have a built-in procedure to deal with local optima, by trying several random starting values, monitoring the likelihoods of each of the solutions, and reporting the solution with the highest likelihood. Finally, it is very common in empirical research to have missing data. Fortunately, when the missing data is in the observed variables and the missing-at-random assumption can be made (i.e., ignorable missingness given all available information), the maximum likelihood estimation procedure is sufficient because it can deal

Table 2
Overview of common software for latent class, profile, and/or transition analysis.

Software (reference)	Commercial	LCA	LPA	LTA	Further capabilities
Proc LCA & LTA (Lanza et al., 2015)	x	x	x	x	1-step, 3-step, LD
Latent GOLD (Vermunt & Magidson, 2013, 2016)	x	x	x	x	Suitable for nominal, ordinal, count, and continuous data; 1-step, 3-step, LD, time-intensive data
Mplus (Muthén & Muthén, 2015)	x	x	x	x	Suitable for nominal, ordinal, count, and continuous data; Bayesian estimation; 1-step, 3-step, LD, time-intensive data
LEM (Vermunt, 1997)		x		x	1-step, LD
poLCA (Linzer & Lewis, 2011, 2013)		x			1-step
MCLUST (Fraley et al., 2012)			x		
DepMixS4 (Visser & Speekenbrink, 2010)		x	x	x	1-step, time-intensive data
mixtools (Benaglia et al., 2009)			x		non-parametric models
Bayes/LCA (White & Murphy, 2014)		x			Bayesian estimation

Note. *LCA* indicates whether software can handle categorical data. *LPA* indicates whether software can handle continuous data. *LTA* indicates whether software can implement a latent transition structure.
1-step = one-step approach to including covariates into the model; *3-step* = three-step approach to including covariates into the model; *LD* = local dependence diagnosing and modeling; *time-intensive* refers to intensive longitudinal data collected at many measurement points.

with both complete and incomplete response patterns. In case there are data missing in covariates and the researcher wants to conduct a one-step analysis in which the covariate is included in the model estimation, however, this cannot be handled by the standard estimation procedures. In this case, the researcher would have to implement a missing-data strategy such as multiple imputation (Rubin, 2004).

4.5. Software implementation

There are several commercial and free software packages to perform latent class, latent profile, and latent transition analyses. An overview of the main characteristics of a selection of common software packages that are tailored towards these models is provided in Table 2. Commercial programs include Mplus (Muthén & Muthén, 2015, 2017), Latent GOLD (Vermunt & Magidson, 2013, 2016), and the SAS add-on procedures PROC LCA/PROC LTA (Lanza et al., 2015). Free programs are LEM (Vermunt, 1997) and various packages developed for R, a free software environment for statistical computing. These include the packages poLCA (Linzer & Lewis, 2011, 2013), MCLUST (Fraley, Raftery, Murphy, & Scrucca, 2012), depmixS4 (Visser & Speekenbrink, 2010), mixtools (Benaglia, Chauveau, Hunter, & Young, 2009), and BayesLCA (White & Murphy, 2014).

Notably, some of these packages allow relaxing some of the described statistical assumptions. In the PROC LCA/LTA-procedures, Mplus, Latent GOLD and LEM, deviations from local independence (see Section 4.4) can be diagnosed and modelled (Lanza et al., 2015; Muthén & Muthén, 2017; Vermunt, 1997; Vermunt & Magidson, 2013). The mixtools-package allows non-parametric modeling to circumvent assumptions of a normal distribution of indicator variables (Benaglia et al., 2009). In the BayesLCA package, Bayesian estimation is applied which does not depend on asymptotic assumptions and allows incorporating prior information into the model to ease parameter estimation and inference in small samples (White & Murphy, 2014). Bayesian estimation is also possible in the Mplus package, albeit currently only for cross-sectional (LCA/LPA) models without covariates (Muthén & Muthén, 2017). In terms of user friendliness, most packages require syntax input; Latent GOLD is the only package that can be fully handled via simple point and click. The syntaxes of all listed packages are however on a rather user-friendly level and can be learned relatively quickly, particularly because for all of these packages well-written manuals with example code are freely available. Haughton, Legrand, and Woolford (2012) provide a review of the programs Latent GOLD, poLCA, and MCLUST.

4.6. Further model extensions

The field is developing fast, and there are many further extensions of latent class, profile, and transition models. Some that might be relevant to learning researchers are briefly mentioned here. Importantly, alternative models and extensions represent additions to the basic idea that qualitative differences between students in learning and learning pathways do exist and are relevant, and should thus be modelled.

In multi-group latent class, profile, or transition models, the issue of measurement invariance across known groups like gender is investigated (e.g., Kankaras, Vermunt, & Moors, 2011). Regression mixture models, also called latent class regression models, aim to identify hidden subgroups with different regression models (e.g., Ding, 2006), whereas growth mixture models (GMMs) including latent class growth models aim to identify classes of individuals with similar change/growth trajectories on a quantitative outcome measured longitudinally (e.g., Jung & Wickrama, 2008). Multilevel structures can be incorporated when data have a hierarchical structure like children in classrooms (e.g., Fagginger Auer, Hickendorff, Van Putten, Béguin, & Heiser, 2016; Mutz & Daniel, 2013). Hybrid models combining continuous and categorical latent variables are for instance mixture item response theory models (e.g., Mislevy & Verhelst, 1990) or factor mixture models (Kleickmann et al., 2011; Lubke & Muthén, 2005), and there are further hybrids of variable-centered and person-centered models (Masyn, Henderson, & Greenbaum, 2010). Vermunt et al. (2008) provide a general framework for longitudinal mixture models, starting from the most general model: the mixture latent Markov model, in which the transition probabilities are allowed to vary over time and/or unobserved subgroups of individuals. Readers interested in related techniques are further referred to available books and edited volumes (Bartolucci, Farcomeni, & Pennoni, 2012; Collins & Lanza, 2010; Hagenaars & McCutcheon, 2002; McLachlan & Peel, 2000).

5. Conclusion

We aimed to show that latent class, latent profile, and latent transition models constitute a powerful, informative, and flexible toolkit for identifying qualitative intra- and inter-individual differences in the context of learning. To that end we provided a conceptual introduction and illustrations of the use and added value by example applications. The models central to this paper are a selection of models that we deem most relevant for learning research. The hope is that with this paper in hand, researchers are able to make the first steps into using these methods, when they are appropriate. Often times, frustration in the translation of research in education and psychology into practical implications stems from a misapplication of statistical measures that do not take into account the rich variability in students' experiences and development. By appropriately applying latent class, profile, and transition models to learners' understanding, characteristics, and/or development it may be possible to better serve our collective goal of understanding student behavior and learning, and the impact of instruction. Not all questions are better served by these tools, but another tool in the box is not a bad thing to have. We strongly believe that more frequent application of the demonstrated techniques will likely provide new and useful information about learning and individual differences.

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