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Chapter 7:

Associations of Magnitude Comparison and Number Line Estimation with Mathematical Competence: A Comparative Review

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Abstract

The magnitude comparison task and the number line estimation task are widely used in the literatures on numerical cognition, mathematical development and mathematics education. It has been suggested that these tasks assess a central component of mathematical competence and, thus, are useful tools for diagnosing mathematical competence and development. In the current chapter, we provide a comparative literature review of how strongly and under which conditions magnitude comparison and number line estimation are associated with broader mathematical competence. We found that both tasks reliably correlate with counting, arithmetic, and mathematical school achievement. Participants use different sets of solutions strategies in the two tasks, and the correlations with competence are higher for number line estimation than for magnitude comparison suggesting that the two tasks assess related but partly different aspects of mathematical competence. Number line estimation might be a more useful tool for diagnosing and predicting broader mathematical competence than magnitude comparison.

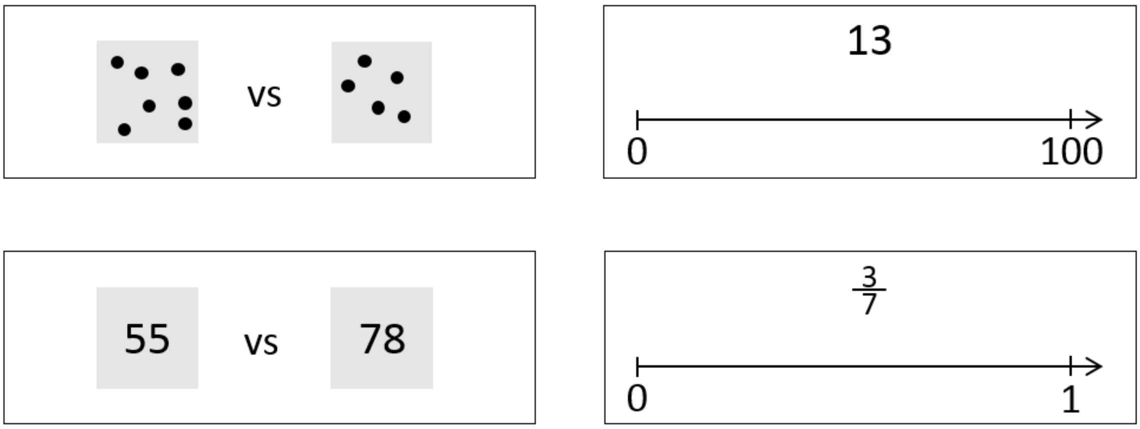
Introduction

This text is a chapter in the book *Cognitive Development from a strategy perspective: A Festschrift for Robert S. Siegler*. Bob Siegler drew many researchers' attention to the potential of the number line as a way to tap children's growing numerical knowledge. In many of his studies he also used the magnitude comparison task. All three co-authors of this chapter had the pleasure of working together with Bob on parts of this research program, either as PhD students or postdocs (Fazio, Bailey, Thompson, & Siegler, 2014; Rittle-Johnson & Siegler, 1998; Rittle-Johnson, Siegler, & Alibali, 2001; Schneider & Siegler, 2010; Siegler & Thompson, 2014; Siegler, Thompson, & Schneider, 2011; Thompson & Siegler, 2010; Torbeyns, Schneider, Xin, & Siegler, 2015; Vogel, Grabner, Schneider, Siegler, & Ansari, 2013). In the current chapter, we discuss questions related to this research. In addition to Bob’s inspiration to pay careful attention to children’s thinking and ways to empirically measure changes in their thinking, he has also taught us the importance of reviewing research critically and writing concisely. Nothing compares to the magnitude of Bob’s influence on us; the number of ways we profited from it cannot be overestimated. Hence, we focus on magnitude comparison and number line estimation as correlates and predictors of mathematical competence in this chapter.

Mathematical competence has many facets. For example, in the PISA 2015 framework (OECD, 2016), mathematical competence includes modeling situations mathematically, employing concepts, facts, and procedures to solve problems, and interpreting, applying, and evaluating mathematical outcomes in broad ranges of mathematical content areas and application contexts. Mathematical competence enables learners to use various notational systems, such as written language, numbers, algebraic expressions, pictures, and diagrams. It manifests itself differently in different age groups. For example, competence tests for Kindergarten children bear little similarity to competence tests for high school students. This multifaceted nature of mathematical competence makes it hard to develop tools for diagnosing the competence. Examples of mathematical competence tests are standardized mathematical development tests, numerical scales of intelligence tests, and school achievement tests. Almost all of these tests require considerable test time because they comprise several types of tasks in order to assess not just single facets, but broad mathematical competence.

However, two brief and easy to administer tasks, the magnitude comparison task and the number line estimation task (see Figure 1), correlate with a wide range of assessments of mathematical competence, even though these assessments differ in their content areas, application contexts, notational systems, appropriate age groups, and required solution processes. In the magnitude comparison task, participants decide which of two presented numerical magnitudes is larger. In the number line estimation task, participants indicate the positions of given numbers on an empty number line. The two tasks correlate widely with a range of diverse mathematical competence measures suggesting that the tasks might be particularly useful for investigating and diagnosing mathematical competence and its development over time. However, empirical studies with these tasks differ in many aspects and yielded partly heterogeneous results. In particular, some studies used the magnitude comparison task, others the number line estimation task, and only a few used both. To our knowledge, there is no published review comparing and integrating findings obtained with the two tasks. In the present chapter we therefore summarize the evidence on how strongly and under which conditions the two tasks are related to mathematical competence and can thus be used as tools for investigating and diagnosing this competence and its development.

Figure 1. *The four task versions most frequently used in studies: magnitude comparison with non-symbolic (top left) or symbolic (bottom left) numerosities, number line estimation with whole numbers (top right) and fractions (bottom right).*



The fact that magnitude comparison and number line estimation correlate with a wide range of diverse mathematical competence measures suggests that each of the two tasks assesses a facet of mathematical competence that is central to mathematical thinking. Indeed, several theorists have converged on the argument that such a fundamental aspect of mathematical cognition exists. For example, Dehaene (1997) concluded that humans and other primates use the intraparietal sulcus of the brain to represent the magnitude of numbers in an analog and continuous format. He labeled this representational system the Approximate Number System (ANS) and suggested that this "mental number line" gives learners a number sense, that is, an intuitive feeling for numbers and their interrelations that aids the acquisition of more advanced mathematical knowledge and skills. Spelke (2000) argued that infants and nonhuman primates are equipped with at least five different systems of core knowledge, one of them for representing magnitudes in analog and approximate form. It emerges early in human ontogeny and phylogeny and forms the base of complex cognitive achievements such as formal mathematics. Case (e.g., Case & Okamoto, 1996) posed the idea of a mental number line which, as a central conceptual structure, helps the learner to understand and integrate subsequently acquired mathematical knowledge. In his integrated theory of numerical development, Siegler (Siegler, 2016; Siegler & Lortie-Forgues, 2014; Siegler et al., 2011) described this development from infancy to adulthood in terms of learners progressively expanding the types and ranges of numbers whose magnitudes they are able to represent on a mental number line. This magnitude understanding "is concurrently correlated with, longitudinally predictive of, and causally related to multiple aspects of mathematical understanding, including arithmetic and overall math achievement" (Siegler, 2016, p. 341).

Drawing on these theories, many (but not all; cf. Dackermann, Huber, Bahnmueller, Nuerk, & Moeller, 2015b) researchers using the magnitude comparison task or the number line estimation task assume that these tasks directly or indirectly assess the mental representation of numerical magnitudes, that this representation enables semantic processing of numerical information, numerical magnitude understanding, and/or number sense, and that these processes are central to and thus correlate with a wide range of mathematical competence measures in various age groups and areas of mathematics. In the next two sections, we describe the magnitude comparison task and the number line estimation task and review the evidence on their relations with mathematical competence. In the discussion section, we will compare the findings obtained with the two tasks and relate them to the background outlined above. We do not review the extensive literatures on the (neuro)cognitive processes underlying the task solution (e.g., Vogel et al., 2013), instructional interventions for improving magnitude understanding in general (e.g., Laski & Siegler, 2007; Opfer & Siegler, 2007; Wilson, Dehaene, Dubois, & Fayol, 2009), or larger interventions or curricula using the tasks as one training component among others (e.g., Fuchs et al., 2013), which have extensively been covered elsewhere.

Magnitude Comparison

Task Description

In the magnitude comparison task, the participant decides which of two numerical magnitudes is larger. The magnitudes can be presented in non-symbolic form, that is, as numbers of shapes (see Figure 1, top left), or in symbolic form, that is, as Arabic numerals (see Figure 1, bottom left). In a few studies, number words or auditory stimuli have been used instead (cf. Dehaene, Dupoux, & Mehler, 1990; Halberda, Lya, Wilmer, Naiman, & Germine, 2012). The symbolic and the non-symbolic task version have been used with whole number-magnitudes, the former version also with fractions (Fazio, DeWolf, & Siegler, 2016; Schneider & Siegler, 2010). The two magnitudes can be presented simultaneously or, alternatively, only one is presented in each trial and has to be compared against a predefined standard. The task has been used in paper-and-pencil format as well as with computers. It has been used with and without time pressure on the participants. In non-symbolic magnitude comparison, time pressure serves to encourage the participants to use approximation instead of counting strategies (Halberda, Mazzocco, & Feigenson, 2008).

Participants' proficiency on the task has been coded in terms of solution rates (i.e. percentage correctly solved trials), solution times, measures of the distance effect, numerical ratio effects, and Weber fractions (Schneider et al., in press). Solution rates indicate the percentage of correctly solved trials. The distance effect is the logarithmic decrease of error rates and solution times with an increasing difference of the two numerosities to be compared. The numerical ratio effect is the logarithmic decrease of error rates and solution times with an increasing ratio of the two numerosities to be compared. Relatedly, the Weber fraction indicates the smallest numerical proportion that a participant can reliably compare.

A general problem with the non-symbolic version of the task is that the numerosity of the presented shapes is always confounded with at least one visual characteristic of the display (e.g., Gebuis & Reynvoet, 2012). For example, more black dots cover a greater proportion of the display area and lead to a darker average color of the display. It is possible to control for some of these confounding variables, but not for all of them simultaneously. This makes it hard to find out whether participants who solve the tasks did so by focusing on numerosities or on non-numerical visual cues.

Solution Processes and Strategies

Only a small proportion of studies using the magnitude comparison task with fractions and almost no study using the task with whole numbers directly assessed participants’ solution strategies, for example, as verbal self-reports. For whole numbers, reaction time patterns allow for indirect inferences about strategy use. The continuous relations between the numerical distances of the numerosities to be compared and the solution time or solution rates (i.e. the distance effect), shows that the participants use a continuous analog mental magnitude representation instead of comparing the two numerosities (Dehaene et al., 1990). The distance effect has been observed with children as well as with adults (Ansari, Garcia, Lucas, Hamon, & Dhital, 2005), with symbolic as well as with non-symbolic stimuli (Lonnemann, Linkersdörfer, Hasselhorn, & Lindberg, 2011) and with whole one-digit numbers, whole multi-digit numbers, one-digit fractions, and multi-digit fractions (Dehaene et al., 1990; Schneider & Siegler, 2010).

Other studies found evidence for additional cognitive processes involved in the task solution. For whole numbers presented in symbolic format, effects of decade-unit compatibility and language have been observed (Nuerk, Weger, & Willmes, 2005). When whole number magnitudes are presented in the non-symbolic format, participants use physical characteristics confounded with the numerosity, for example, area covered or average display color, as cues (e.g., Gebuis & Reynvoet, 2012; Leibovich, Katzin, Harel, & Henik, in press). For fractions, the situation is even more complex, as participants activate integrated analog magnitude representations only when numerators as well as denominators change between trials and thus carry meaningful information (Meert, Grégoire, & Noël, 2009; Schneider & Siegler, 2010). When possible, for example, with unit-fractions in which the numerator is always one (1/2, 1/3, 1/4 etc.), the participants instead use componential strategies, in which they process only numerators or only denominators (Bonato, Fabbri, Umiltà, & Zorzi, 2007). Participants not being able to recall the integrated magnitude of a fraction from long-term memory also resort to computational strategies, such as cross-multiplication of the numerators and denominators. Based on trial-by-trial assessments of strategy use, Fazio et al. (2016) give a detailed account of the types of fraction comparison strategies and their use in adults of different competence levels. The students used a large variety of strategies. Less mathematically skilled adults tended to lack the conceptual knowledge necessary for evaluating and choosing between these strategies.

Correlation

At least one systematic qualitative literature review (De Smedt, Noël, Gilmore, & Ansari, 2013) and three meta-analyses summarized the findings on the associations between magnitude comparison with whole-number magnitudes and other, broader measures of mathematical competence. There is currently no meta-analysis including fraction comparison. All three meta-analyses found similar correlations between non-symbolic magnitude comparison and mathematical competence. These were *r* = .22, 95% CI [.20, .25] based on 34 effect sizes (Fazio et al., 2014), *r* = .20, 95% CI [.14, .26] based on 47 effect sizes (Chen & Li, 2014), and *r* = .241, 95% CI [.198, .284] based on 195 effect sizes (Schneider et al., in press). Chen and Li (2014) showed that the correlation is lower but still significantly greater than zero for studies controlling the correlation for possible confounding influences of third variables, for example, intelligence. Only one of the three meta-analyses also included results found with whole numbers in symbolic format (Schneider et al., in press). Averaged over 89 effect sizes, the correlation between magnitude comparison with whole numbers and mathematical competence was *r* = .302, 95% CI [.243, .361]. The difference between the correlations found with non-symbolic and symbolic magnitude comparison was highly significant.

Moderator analyses in the latest and so far largest meta-analysis (Schneider et al., in press) showed that, averaged over non-symbolic and symbolic comparison, the correlation between magnitude comparison and mathematical competence is significant already in children younger than six years, that is, before the start of whole number instruction in primary school, and decreases only very slightly with age. The correlation was higher when comparison proficiency was coded as solution rate, solution time, or Weber fraction, and lower when it was coded as distance effect or ratio effect. The correlation was also strongly moderated by the competence measure used. It was highest for the Test of Early Mathematics Ability (Ginsburg & Baroody, 2003) (TEMA) , followed by mental arithmetic, written arithmetic, curriculum-based measures, and other measures.

To our knowledge, so far evidence on the correlation between fraction comparison and mathematical competence is sparse. One study with 6th- and 8th-graders found a significant correlation only for the US, but not for China or Belgium, partly due to ceiling effects in the Chinese children (Torbeyns et al., 2015).

Prediction

A number of studies demonstrated that the relation is not just correlational at a time point but also predictive over time. All of these studies used whole-number comparison. In the first study on this question, 47 Belgian elementary school children solved the symbolic magnitude comparison task and other tasks at the beginning of their first grade (De Smedt, Verschaffel, & Ghesquière, 2009). Magnitude comparison speed explained 26% of the variance in mathematical achievement a year later and still 10% of the variance after controlling for age, intellectual ability, and number reading speed. These findings have since been extended and differentiated. At least four other studies found similar predictive relations for preschool and elementary school children's non-symbolic magnitude comparison in the USA and China (Chu, vanMarle, & Geary, 2015; He et al., 2016; Libertus, Feigenson, & Halberda, 2013a; Mazzocco, Feigenson, & Halberda, 2011). However, at least one other study found no predictive relation in 49 elementary school children (Reigosa-Crespo et al., 2013), and one study (Libertus, Feigenson, & Halberda, 2013b) found that, after controlling for age and intelligence, non-symbolic magnitude comparison in four-year-olds only predicted informal but not formal mathematical competence (cf. Ginsburg & Baroody, 2003) two years later.

Other studies found that mathematical achievement tests predict magnitude comparison proficiency over time, even after controlling for intelligence and other possible confounding variables (e.g., Halberda et al., 2008; Libertus, Odic, & Halberda, 2012). This raises the question about the direction of the causal relation between magnitude comparison and mathematical competence.

Causality

The current evidence as to the causal effects of magnitude comparison training on broader mathematical competence is weak and preliminary. The published results, all obtained with the non-symbolic task version, suggest that trainings using only magnitude comparison do not reliably enhance symbolic arithmetic performance. Two experimental training studies with 132 Kindergarten children (Praet & Desoete, 2014) and 88 adults (Park & Brannon, 2014) found no statistically significant effect of a magnitude comparison training on arithmetic performance. One experimental study with 151 five-year olds found no statistically significant interaction effect between the training group (number line estimation, magnitude comparison, active control, passive control) and the test time (pretest vs. posttest) on arithmetic (Maertens, De Smedt, Sasanguie, Elen, & Reynvoet, 2016). In a randomized controlled trial with 96 first-graders, children who participated in a training of magnitude comparison skills were subsequently faster, but not more accurate, on symbolic arithmetic problems than those who were trained with non-numerical tasks, for example, brightness comparison (Hyde, Khanum, & Spelke, 2014, Exp.1). An experiment investigating the reverse causal ordering with 46 adults, found no effect of six 45-minute long arithmetic training sessions on non-symbolic magnitude comparison (Lindskog, Winman, & Poom, 2016).

Trainings combining non-symbolic magnitude comparison with non-symbolic arithmetic tasks (Hyde et al., 2014; Khanum, Hanif, Spelke, Berteletti, & Hyde, 2016; Park & Brannon, 2013) or combining symbolic or non-symbolic magnitude comparison with symbolic or non-symbolic number line estimation tasks (Honoré & Noël, 2016) found causal effects on symbolic arithmetic, but the contribution of magnitude comparison to these effects is unclear.

In sum, the magnitude comparison task correlates with and predicts mathematical competence. This shows consistently with whole numbers and fractions in symbolic format as well as with whole-number magnitudes in non-symbolic format, for different age groups, number ranges, and task versions.

Number Line Estimation

Task Description

In the most common version of the number line estimation task, the participants see a number and a number line. Only the startpoint and the endpoint of the line are marked and labeled with the respective number. The participants are asked to mark the position of the presented number on the number line. In the paper-and-pencil version of the task, they do so with a pen. In the computer version, they usually indicate the position by clicking the mouse there. The presented number is usually either a whole number or a numerical fraction in symbolic format (see Figure 1). Very few studies used the number line estimation task with non-symbolic stimuli, for example, dot patterns (Fazio et al., 2014; Sasanguie & Reynvoet, 2013) or shades of grey (Vogel et al., 2013). Unless stated otherwise, all subsequently reported findings were obtained with numbers in symbolic format. Few studies used an unbounded number line, that is, a line in which two points are labeled, for example, the positions of the zero and one, but not the endpoint of the line (Link, Nuerk, & Moeller, 2014). The typical number-to-position task is sometimes complemented by a position-to-number version, in which the participant translates a position on the line into a symbolic number (Siegler & Opfer, 2003).

Proficiency in number line estimation performance is most frequently coded as percent absolute error (PAE) with PAE = 100% * (correct number – estimated number) / numerical range of line. For example a PAE of 5 indicates that the mean difference between the correct and the estimated position is 5% of the range of the number line. Rarely, number line estimation proficiency has been coded as solution rate, that is, the proportion of trials, in which the distance between the correct position and the participants' solution lies within a predefined limit. A third measure is the linear R^2, which is the percentage of variance explained when the estimated positions are plotted against the correct positions and a linear regression is computed. Perfectly correct answer patterns would result in a regression line with a slope of 1 and an R^2 of 1. When estimating whole numbers, many children (Siegler & Opfer, 2003) as well as adults without formal mathematics instruction (Dehaene, Izard, Spelke, & Pica, 2008) place smaller numbers too far apart and larger numbers too close together on the line. The reasons for this are subject to an ongoing debate (Barth & Paladino, 2011; Dackermann, Huber, Bahnmueller, Nuerk, & Moeller, 2015a; Ebersbach, Luwel, Frick, Onghena, & Verschaffel, 2008; Opfer, Siegler, & Young, 2011).

Solution Strategies

The evidence on the solution strategies on the number line estimation task comes from error analyses, self-reports of strategy use, and eye-tracking studies. For whole numbers, children and adults tended to use counting strategies, landmark strategies, and proportional reasoning strategies for solving the number line estimation task (Peeters, Degrande, Ebersbach, Verschaffel, & Luwel, 2016; Siegler & Opfer, 2003). In counting strategies, the participants counted, for example, three units from left to right to find the position of the number three on the line. In landmark strategies, the participants used orientation points on the line, such as the middle or three quarters, as part of their strategy. Relatedly, in proportional reasoning strategies, participants found a position on the line by translating a numerical proportion (e.g., 75 is 3/4 of 100) into a visuospatial proportion (e.g., 75 lies at 3/4 of a 0-100 number line). By first-grade, children used counting strategies and landmark strategies and increasingly rely on landmark strategies and proportional reasoning strategies with age (Petitto, 1990; Schneider et al., 2008; Siegler & Opfer, 2003; Sullivan, Juhasz, Slattery, & Barth, 2011).

In number line estimation with fractions, students tended to use numerical transformation strategies, in which participants rounded, simplified, or otherwise transformed a fraction into an easier to estimate number (e.g., 4/17 into 1/4), and line segmentation strategies, in which the participants mentally segmented the line into halves, fifths, whole number units, or denominator units (1/7th) (Siegler et al., 2011). These strategies differed in the resulting accuracy of the estimates.

Correlation

Many studies investigated correlations between number line estimation and mathematical competence. However, to our knowledge, no systematic review or meta-analysis has been published so far. Single empirical studies found statistically significant correlations between number line estimation proficiency and counting (Östergren & Träff, 2013), mental arithmetic (Laski & Yu, 2014), written arithmetic (Torbeyns et al., 2015), algebra problems and word problems (Booth & Newton, 2012), standardized school achievement tests (Ashcraft & Moore, 2012), and school grades (Schneider, Grabner, & Paetsch, 2009). These correlations have been found on several continents (Torbeyns et al., 2015), with whole numbers as well as with fractions (Booth & Newton, 2012), and with kindergarten children, elementary school students, and students between 9 and 14 years (Siegler & Mu, 2008; Siegler & Pyke, 2013). We did not find any study with participants older than 14 years.

Most correlations were medium or large by the standards of Cohen (1992). The vast majority of studies found substantial correlations, for example, ranging from .34 to .46 (Ashcraft & Moore, 2012), from .57 to .69 (Cowan & Powell, 2014), from .24 to .68 (Lyons, Price, Vaessen, Blomert, & Ansari, 2014), from .43 to .67 (Östergren & Träff, 2013), from .45 to .55 (Schneider et al., 2009), from .29 to .84 (Siegler & Thompson, 2014), from .54 to .86 (Siegler et al., 2011), or from .40 to .56 (Wang et al., 2015). These findings seem to be of some generalizability as the cited studies included different number types, number ranges, measures, and children from various countries. In some exceptional cases, correlations were low and did not reach significance, but this was usually restricted to specific combination of measures (e.g., whole-number estimation on a line from 0 to 6257 with Algebra competence; Booth & Newton, 2012), was due to ceiling effects for some age groups and countries (e.g., Chinese 8th-graders' solution rate on fraction arithmetic problems; Torbeyns et al., 2015) or was found with nonstandard versions of the number line, for example, the unbounded number line with no label at the endpoint (Link et al., 2014). We are currently preparing a meta-analysis of these findings (Schneider et al., in prep). Preliminary analyses suggest that the average correlation between number line estimation and mathematical competence lies between .40 and .50.

Prediction

Several longitudinal studies investigated the predictive relations between whole-number number line estimation and later mathematical competence. They show that whole-number number line estimation measured in Kindergarten children predicted mathematical school achievement even after controlling for potentially confounding variables (Hornung, Schiltz, Brunner, & Martin, 2014; Östergren & Träff, 2013). These predictive relations persist during the elementary school years (Fuchs, Geary, Fuchs, Compton, & Hamlett, 2014; Gunderson, Ramirez, Beilock, & Levine, 2012; LeFevre et al., 2013; Sasanguie, Göbel, Moll, Smets, & Reynvoet, 2013). Number line estimation with whole numbers also predicts fraction competence over several years (Bailey, Siegler, & Geary, 2014; Vukovic et al., 2014). For example, out of attentive behavior, language, nonverbal reasoning, number line estimation with whole numbers, calculation fluency, and reading fluency, whole-number number line estimation was the best predictor of both fraction arithmetic and fraction concepts in a two-year longitudinal study with 357 children (Jordan et al., 2013). We do not know of any studies investigating whether number line estimation with fractions predicts mathematical competence or whether mathematical competence predicts number line estimation proficiency.

Causality

To our knowledge, there is no conclusive evidence on whether practice with the typical number line estimation task only causally affects arithmetic or more general measures of mathematical competence. As described in section 3.5, one experimental training study found no significant interaction between training condition (number line estimation, magnitude comparison, active control, passive control condition) and test time (pretest vs. posttest) on arithmetic (Maertens et al., 2016). One study found that visualizing the addends and sums of arithmetic problems on number lines increased the effectiveness of an arithmetic training phase (Booth & Siegler, 2008). However the learning gains were highest in a condition where the computer presented these visualizations as compared to the conditions where the children had to generate them by estimating the positions of the numbers. So the study highlights the potentials of number lines as visualizations, but not the potentials of number line estimation practice.

Randomized controlled trials found evidence for a causal effect of playing numerical board games on numerical magnitude understanding, counting skills, and arithmetic learning (Ramani & Siegler, 2008, 2011). On numerical board games, there are spaces that are counted, rather than a continuous number line. Improvements from playing with linear numerical board games were greater than ones found for various other numerical and non-numerical activities and persisted over at least two months. The development of these games was guided by the idea that they, similar to number lines, require the mapping of numbers onto a spatial dimension. In line with this interpretation, children learning with a linear version of the board showed greater learning gains than children playing with a circular version, because the former is more similar to a (mental) number line than the latter (Siegler & Ramani, 2009). Notwithstanding the structural and cognitive similarities between playing numerical board games and number line estimation, the two differ in that the number line is continuous and analog whereas the spaces on the board can be counted. Therefore, playing numerical board games usually does not involve estimation. Like the magnitude comparison task, the number line estimation task has been used as one component of several in more complex trainings and curricula targeting mathematical competence (Fuchs et al., 2013; Honoré & Noël, 2016; Kucian et al., 2011; Kuhn & Holling, 2014; Obersteiner, Reiss, & Ufer, 2013; Thompson & Opfer, 2016). In these cases, it is unclear to what extent the effects are due to the number line or to other intervention components. Causal effects of number line trainings on broader mathematical competence have not been conclusively shown yet. More randomized controlled trials on this issue are needed.

Magnitude Comparison and Number Line Estimation: Commonalities and Differences

Our literature review revealed commonalities and differences in how magnitude comparison tasks and number line estimation tasks are associated with mathematical competence (see Table 1 for a general overview). The commonalities are: First, the correlations between both types of tasks and mathematical competence are well investigated. A recent meta-analysis (Schneider et al., in press) included 284 effect sizes from 45 studies on the magnitude comparison task and the numbers seem to be similar for the number line estimation task as we found in our as yet unpublished meta-analysis on number line estimation (Schneider et al., in prep). Second, none of the two tasks can be considered a pure measure of mental magnitude representation. In non-symbolic magnitude comparison, numerosity and physical display characteristics are confounded (Gebuis & Reynvoet, 2012). The comparison of multi-digit numbers is biased by decade-unit compatibility and language effects (Nuerk et al., 2005). Fraction comparison proficiency partly reflects comparison strategy use (Fazio et al., 2016). Number line estimation performance partly depends on the chosen solution strategy (Peeters et al., 2016; Schneider et al., 2008; Siegler et al., 2011). Some of these strategies involve rounding, counting, landmark use, or proportional reasoning (Petitto, 1990; Siegler & Opfer, 2003). Third, both tasks correlate with measures of mathematical competence as diverse as counting, mental arithmetic, written arithmetic, and standardized tests of mathematical school achievement. Fourth, both tasks correlate with mathematical competence in children before the onset of formal schooling, during the elementary school years, and still after the elementary school years. For magnitude comparison, meta-analyses show that the correlation declines only slightly with age (Schneider et al., in press). The results seem to be similar for the number line task, but a meta-analytic integration of the findings is still lacking. Fifth, both tasks predict mathematical competence over months, and in some studies, years, even after controlling for other numerical and non-numerical predictors. Predictive relations have been shown with Kindergarten children, elementary school students, and older students. Finally, both tasks have been used as training tools in the context of more complex interventions that also included other components. These trainings, games, or curricula had positive causal effects on mathematical competence (for reviews, see Moeller, Fischer, Nuerk, & Cress, 2015; Siegler, 2016). Evidence on the question of whether simply practicing one of the tasks (with or without feedback) in isolation has a causal effect on arithmetic or even more advanced mathematical skills is sparse and inconclusive at best for both tasks. Given that positive results are more likely to be published than null results, so that unpublished null results of such training experiments might exist, the answer to the question is likely no. The reason for this might lie in the fact that humans engage in magnitude processing from infancy on (Feigenson, Dehaene, & Spelke, 2004), so that at least some of the component processes are highly overlearned already in kindergarten.

There are also some differences between the two tasks and how they relate to mathematical competence. First, the magnitude comparison task assesses an understanding of numerical magnitudes on an ordinal level, because it requires only larger/smaller judgments. The number line estimation task requires mapping of numerical magnitudes on a continuous spatial dimension and, thus, is thought to assess magnitude understanding on a ratio scale level. Second, because of this, the two tasks elicit different sets of solution strategies and biases. The number line, but not magnitude comparison, invites proportional reasoning and landmark strategies (Petitto, 1990; Siegler & Opfer, 2003). Conversely, bias due to incompatibilities between the digits of the presented numbers (e.g., in 91 vs. 28 the first number has the larger decade but the smaller unit than the second number) (Nuerk, Weger, & Willmes, 2001) seem to only play a role in magnitude comparison. Third, the meta-analytically averaged correlation with mathematical competence is .24 and .30 for non-symbolic and symbolic magnitude comparison (Schneider et al., in press), whereas many single studies report average correlations between .40 and .50 for the number line estimation task. This suggests that the number line estimation task is a better tool for diagnosing and predicting mathematical competence than the magnitude comparison task. A meta-analysis confirming this preliminary finding of our qualitative literature review is currently in preparation (Schneider et al., in prep). There are several explanations for this finding, which do not exclude each other. Contrary to magnitude comparison, number line estimation assesses magnitude understanding on a ratio scale level, it seems to have a somewhat higher reliability, and it can evoke counting and proportional reasoning strategies, which are important mathematical competencies beyond magnitude understanding.

In sum, the magnitude comparison task and the number line estimation task have many commonalities, but also some relevant differences. Their robust correlations with many facets of mathematical competence show that both tasks assess a central component of this competence. The correlations between the two tasks were rather low in some studies (Sasanguie & Reynvoet, 2013; Schneider et al., 2009) but high in others (Laski & Siegler, 2007; Siegler et al., 2011) and sometimes highly variable even within studies (Fazio et al., 2014; Torbeyns et al., 2015). Thus, they seem to assess at least partly different components of mathematical competence, the inter-relations of which might change due to developmental or environmental influences. This would also explain the effectiveness of interventions combining both tasks as training tools (Honoré & Noël, 2016; Kuhn & Holling, 2014).

A general problem in comparisons of the magnitude comparison task with the number line estimation task is that studies differing in which task they used can also differ in whether they used the symbolic or the non-symbolic presentation format, whole numbers or fractions, younger or older learners, how they coded proficiency on the task, and how they measured mathematical competence. Many different combinations of these study characteristics are possible (first graders' magnitude comparison with whole numbers in symbolic format and mental arithmetic; high school students' number line estimation with whole numbers in non-

***Table 1**. Overview over characteristics of the magnitude comparison task and the number line estimation task and their relations to mathematical competence.*

	Magnitude Comparison	**Number Line Estimation**
Scale level of assessment of magnitude understanding	Ordinal (smaller/larger judgment)	Ratio (difference of estimated and correct position)
Most frequently used stimuli	Non-symbolic and symbolic whole number magnitudes	Symbolic whole numbers and fractions
Participants use multiple strategies	Yes (shown for fractions only)	Yes
Possible influences on task performance (non-exhaustive list)	Magnitude representation; comparison strategies; visual display characteristics (for non-symbolic numerosities); decade-unit compatibility (for multi-digit numbers); language (for multi-digit numbers)	Magnitude representation; estimation strategies; landmark use; counting; proportional reasoning
Empirical evidence for the association with mathematical competence	284 effect sizes from 45 studies	More than 200 effect sizes from about 35 articles
Correlational evidence for an association with mathematical competence	Strong evidence for a medium correlation (r = .27)	Strong evidence for a medium to strong correlation
Age groups for which the association with mathematical competence was found	Kindergarten children; elementary school students; students older than 9 years; adults	Kindergarten children; elementary school students; students older than 9 years
Longitudinal evidence for an association with mathematical competence	Strong evidence for a predictive relation	Strong evidence for a predictive relation
Longitudinal relation remained present after controlling for confounding variables	Yes	Yes
Evidence from randomized controlled trials for a causal influence of practice with the task on broader mathematical competence	No	No
Task used as one component of several in trainings causally affecting mathematical competence	Yes	Yes

symbolic format and mental arithmetic; …). The current number of published empirical studies does not allow delineating all possible interactions of these variables (Schneider et al., in press), which remains as an important task for future studies.

Conclusion

The magnitude comparison task and the number line estimation task both assess central components of mathematical competence, as they both reliably correlate with and predict counting, arithmetic, and the results of standardized tests of mathematical school achievement during the kindergarten and school years and sometimes beyond. The two tasks elicit different sets of solution strategies and, in some cases, have low inter-correlations, suggesting that they assess related but not identical aspects of numerical magnitude understanding. The consistently found correlations with mathematical competence demonstrate that both tasks can be useful components of broader mathematical competence tests or serve as short screening instruments. The correlations with mathematical competence are medium for magnitude comparison and seem to be medium to large for number line estimation, suggesting that the number line estimation task might be a more useful tool for diagnosing and predicting broader mathematical competence. Future studies will have to more systematically investigate the underlying causal relations and the interaction effects of the choice of the task and other study characteristics, such as age groups, number ranges, and persentation types, on mathematical competence.

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