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What is the Value of the Tax Shield of Debt?

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Titel: What is the Value of the Tax Shield of Debt?
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Abstract: More than a quarter-century ago, Modigliani and Miller (1963), Myers (1974) as well as Miles and Ezzell (1980) suggested that the value of tax shields be computed by discounting the debt tax savings to the present using a discount rate that reflects the risk of the debt tax savings. The difference between the various theories lies primarily in the assumed debt policy. In contrast to this literature, Fernandez (2004) derives the value of tax shields by computing the difference between the present value of taxes for the unlevered company and the present value of taxes for the levered company. Fernandez' results challenge existing valuation theories, as he claims that the value of tax shields he derives is generally valid and in fact the only way to compute the correct value of tax shields. This diploma thesis demonstrates that Fernandez' claim is not consistent with his set of assumptions. Further, it is shown that Fernandez' value of tax shields expands on the existent valuation theories by using an additional set of assumptions that includes a new debt policy, but it does not correct or contradict the standard valuation theories.

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Table of Abbreviations

α	discount rate reflecting the risk of the increase in net assets
APV	adjusted present value
β_D	debt beta
β_{DTS}	beta of the debt tax shield
β_E	equity beta
β_L	beta of the levered firm
β_U	unlevered asset beta
B	dollar amount of debt
CAPM	capital asset pricing model
CCF	capital cash flow
CFD	costs of financial distress
D	market value of debt
DEP	depreciation
D_L	market value of the levered firm's debt
D_P	market value of personal borrowed debt
ε_{t+1}	random variable of the free cash flow
E	market value of equity
EBIT	earnings before interest and tax
Ebv	book value of equity
ECF	equity cash flow
E_L	market value of the levered firm's equity
E_U	market value of the unlevered firm's equity
$E(\cdot)$	expected value operator
FCF	free cash flow
g	growth rate
GL	gain from leverage
G_L	present value of taxes paid by the levered company
G_U	present value of taxes paid by the unlevered company
INV	investment (change in net working capital plus capital expenditures)
K	a constant
λ	percentage share
L	constant leverage ratio (D/V=L)
LBO	leveraged buyout
M	pricing kernel
ME	Miles and Ezzell

MM	Modigliani and Miller
ϕ	random variable of the increase of net assets
PV	present value
PVTS	present value of the tax shield of debt
ρ	unlevered cost of capital
r_A	expected asset return
r_D	costs of debt/ expected return of debt
$\overline{r_D}$	fixed yield
r_{Demand}	demand interest rate
r_E	costs of equity/ expected return to equity
r_f	risk-free interest rate
r_P	market risk-premium
r_{Supply}	supply interest rate
τ_C	marginal corporate tax rate
τ_{PB}	personal income tax rate applicable to income from bonds
τ^i_{PB}	individual investor's marginal personal tax rate on income from bonds
τ_{PS}	personal income tax rate applicable to income from common stocks
$Taxes_L$	taxes paid by the levered constant perpetuity
$Taxes_U$	taxes paid by the unlevered constant perpetuity
TS	tax shield from debt
V	firm value
V_L	value of the levered firm
VTS	value of tax shields
V_U	value of the unlevered firm
WACC	after-tax weighted average cost of capital
$WACC_{BT}$	before-tax WACC
X	random amount

1 Introduction

The value of the tax shield of debt has gained considerable attention in recent years in real world applications as well as in the academic literature. The recent boom in the private equity industry has substantially increased the number of highly leveraged transactions, such as leveraged- or management buyouts, in order to finance acquisitions. In these transactions, leverage is used as a significant source of value added. Kaplan (1989) as well as Newbould, Chatfield and Anderson (1992) show that the tax benefits generated by the use of debt are an important source of wealth gain in highly leveraged transactions. Kaplan (1989), for instance, estimates the median value of the tax benefits associated with management buyouts to be in the range of 21 percent to 143 percent of the premium paid to pre-buyout shareholders. In addition, Graham (2000) estimates tax savings of debt, for a typical firm in his sample, to be 20 percent of pretax income annually and the value of the tax shield of debt to be about 10 percent of firm value. The debt tax shield therefore represents a significant component of firm value and hence it is important to calculate the correct value of the tax shield of debt.

In the financial literature, the ongoing debate about the value of debt tax shields has been reinforced in recent years by Fernandez (2004a), who claims that his results are generally valid and in conflict with the results of existing literature. If Fernandez were correct, his paper would be of great importance and would have far-reaching consequences in the field of corporate finance.

This paper tries to shed light on how to correctly value the tax shield of debt. Specifically, I examine the assumptions that Fernandez (2004a) makes and analyze whether Fernandez' (2004a) approach of valuing the debt tax shield is correct under broad conditions as he claims. Furthermore, I look at which valuation method should be used to find the correct value of the debt tax shield and whether all valuation methods lead to the same result when appropriately applied.

It should be noted that this paper does not address the debate on the optimal capital structure. That is, I do not raise the question of which capital structure will maximize the value of the debt tax shield; rather, I focus on what the value of the debt tax shield will be if a certain debt policy is given.

The remainder of the paper proceeds as follows. Chapter 2 provides the building blocks by introducing the basic valuation model of Modigliani and Miller (1958), explaining the mechanics of the debt tax shield and reviewing three commonly used discounted cash flow methods. Chapter 3 discusses in detail the different theories on how to value the tax shield of debt. In addition, the three valuation methods are compared and analyzed when used together with the different valuation theories. Chapter 4 presents the approach of Fernandez (2004a) followed by the counterstatement of Cooper and Nyborg (2006b). In Chapter 5 the assumptions of Fernandez (2004a) are then analyzed in detail. Chapter 6 concludes.

2 Basic Model and Valuation Methods

This chapter provides the building blocks for the paper's analyses on the value of the tax shield of debt. These building blocks are then used in Chapters 3 through 5 in the paper's main analyses on the value of the tax shield of debt.

2.1 Basic Valuation Model: Modigliani and Miller (1958)

Modigliani and Miller (1958), hereafter MM (1958), demonstrate in their seminal paper on capital structure that corporate financial decisions are irrelevant for firm valuation in perfect capital markets. To derive this result, they make, either explicitly or implicitly, the following nine assumptions (Copeland, Weston and Shastri 2005, 559):

(A1) *The investment policy of the firm is given and constant:* The assets of the firm are expected to generate constant operating cash flows in perpetuity. It is important to notice that the cash flow stream is completely unaffected by changes in capital structure.

(A2) *Firms can be divided into risk classes:* Firms in the same risk class are assumed to have perfectly correlated cash flows. Hence, the cash flow streams of two firms in the same risk class differ at most by a scale factor. Therefore, investors will require the same expected return on any two assets within a given risk class.

(A3) *No taxes:* There are neither corporate income nor wealth taxes on corporations, nor personal taxes on individual investors.

(A4) *No transaction and bankruptcy costs:* Capital markets are frictionless and no costs of financial distress occur in the event of bankruptcy.

(A5) *Symmetric information:* Corporate insiders and outsiders have the same information.

(A6) *No agency costs:* Managers always try to maximize shareholders' wealth and hence only invest in value-increasing projects.

(A7) *Absence of arbitrage opportunities:* Two assets with an identical payoff structure must sell in equilibrium at the same price.

(A8) *Individuals can borrow and lend at the risk-free rate.*

(A9) *The capital structure of a firm consists only of risk-free debt and risky equity.*

Under assumptions (A1) to (A9), MM's (1958) famous Proposition 1 states that the market value of the levered firm is the sum of the market value of the firm's debt and the market value of its common equity, which is equal to the market value of the unlevered firm. Thus, in equilibrium, it must hold that

$$V_L = E_L + D_L = V_U = E_U \,, \tag{1}$$

where V_L is the market value of the levered firm, E_L is the market value of the levered firm's equity, D_L is the market value of the levered firm's debt, V_U is the market value of the unlevered firm and E_U is the market value of the unlevered firm's equity. Note from equation (1) that the value of a firm is independent of how the firm is financed. Thus, it follows that corporate financial policies do not add value in equilibrium.

To establish their Proposition 1, MM (1958) use one of the very first arbitrage pricing arguments in finance theory. They assume the presence of two firms in the same risk class and with identical cash flows. Firm U is capitalized with equity only and firm L has both debt and equity outstanding. Suppose that the value of the levered firm (V_L) is larger than that of the unlevered firm (V_U). Then, instead of buying the levered firm's equity (E_L), an investor could combine the unlevered equity (E_U) with borrowing on personal account (D_P) in the same proportion as the capital structure of the levered firm.[1] In other words, instead of buying $E_L = (V_L - D_L)$, the investor buys $(E_U - D_P) = (V_U - D_P)$. Since both firms have identical cash flows, both strategies offer the same payoff in every state of the world. Hence, the investor could duplicate the return of the levered equity at lower cost. Hence, in the absence of arbitrage opportunities, V_L must equal V_U in equilibrium.

Conversely, if the levered firm is relatively undervalued, purchasing an equal percentage share (λ) of the levered firm's equity and debt, $\lambda(E_L + D_L) = \lambda(V_L)$, would cost less than the same percentage of the all-equity firm, $\lambda(E_U) = \lambda(V_U)$. However, the two

[1] Assumptions (A8) and (A9) imply that investors can borrow on their own account as cheaply as the levered firm, and thus D_P equals D_L for a given amount of debt.

strategies would entitle an investor to exactly the same cash flow. Thus, in equilibrium, the two firms must sell for the same price, that is, V_L must equal V_U. These two arbitrage arguments lead to the conclusion that capital structure does not influence firm value, because investors can "undo" the effect of any changes in capital structure (Modigliani and Miller 1958).

An example that is often used to illustrate the intuition behind Proposition 1 is to compare the size of a pie with the value of a firm. The size of the pie is independent of how the pie is sliced.[2] So too, the value of the firm is independent of how the firm's financial policy divides its operating cash flows among different claimants (debtholders and equityholders). Ultimately, firm value equals the present value of operating cash flows (Brealey, Myers and Allen 2006, 471).

The basic principle underlying Proposition 1 is the Value Additivity Theorem. Under the assumption that capital markets are complete, this theorem says: "Assume no arbitrage possibilities exist. Then the price of a security whose payoffs are a linear combination of other assets must be given by the same linear combination of the prices of the other assets" (Varian 1987). That is, the value of the whole is equal to the sum of the values of its parts. Hence, the value of a firm in a given risk class should depend only on its operating cash flows and this value equals the sum of the values of the firm's equity and debt. The value does not depend on whether the firm is financed more heavily by debt or equity (Varian 1987).

At first sight, MM (1958) seems to be no more than a theoretical economic model with unrealistic assumptions that is irrelevant for real-world applications. Concerning this skepticism, Miller (1988) replied: "Looking back now, perhaps we should have put more emphasis on the other, upbeat side of the 'nothing matters' coin: showing what *doesn't* matter can also show, by implication, what *does*." During the course of this paper I relax several of the assumptions above in order to examine their effect on the tax shield of debt. Right away, by applying the modern asset-pricing approach, (A2) can be relaxed without changing the conclusion of the model. The modern asset-pricing approach requires firms within a given risk class to have only the same systematic risk, not perfectly correlated cash flows. Since the cost of capital depends on the systematic risk of a firm, firms with the same cost of capital must be placed in the same risk class (Ross 1988).

[2] Merton Miller usually compared the firm value with the price of a pizza, which is also unchanged by how the pizza is sliced (Copeland, Weston and Shastri 2005, 563). This example has the advantage that the price of a pizza can increase, while the size of a pie can hardly grow. It is left to the "taste" of the reader, which illustration he prefers to choose.

2.2 Introducing Corporate Taxes and the Mechanics of the Tax Shield of Debt

I now turn to the question of how corporate taxes affect the implications of the MM (1958) model, thereby relaxing assumption (A3). A simple example will illustrate the general mechanics, as well as the calculation, of the tax shield of debt.

For ease of discussion and to maintain consistency with the bulk of the literature, a classical tax system (such as that found in the United States) is assumed. A classical system taxes corporate and personal income separately. The key feature of a classical system is the tax-deductibility of interest payments at the corporate level, so that interest is paid out of income before taxes. In contrast, equity payouts are not tax-exempt and are paid from residual corporate income after taxes. Corporate income is taxed at the marginal corporate tax rate τ_C (hereafter corporate tax rate), which is assumed to be constant over time. Personal income on dividends, capital gains, and interest is taxed upon receipt by investors (Graham 2003). For now, however, I assume individual investors are not taxed. Further, all taxes throughout are assumed to be proportional, if not differently stated.

Table 1, which shows a simplified income statement of Microsoft Corporation, illustrates the advantage of debt financing when taxes are introduced. The left column is calculated by applying the de facto capital structure of Microsoft in 2007 with no debt outstanding and a corporate tax rate of 30 percent. In the right column, I assume that $20 billion of equity is retired and replaced with a perpetual risk-free bond issued at par value with an interest rate of 5 percent. The change in capital structure reduces the tax bill of Microsoft by $300 million and increases the total cash flow to debt- and equityholders by the same amount, as the government "subsidizes" interest payments by allowing the firm to deduct interest payments as an expense.

Table 1
The Tax-Deductibility of Interest Expenses

	Microsoft Corporation 2007 ($ millions)	
	No Debt	Debt ($20bn bond @ 5%)
EBIT	20,101	20,101
Interest Expense	0	1,000
EBT	20,101	19,101
Taxes(30%)	**6,030**	**5,730**
Net Income	14,071	13,371
Cash flow to debtholders	0	1,000
Cash flow to equityholders	14,071	13,371
Total cash flow to debt- and equityholders	**14,071**	**14,371**

Under the assumption that Microsoft earns enough taxable income to cover interest payments[3], the tax shield from debt (TS) generates a cash flow stream that is equal to the corporate tax rate times the risk-free interest payment,

$$\textit{Cash flow from TS} \;=\; \tau_C * r_f * D\,, \tag{2}$$

where r_f is the risk-free interest rate and D is the market value of debt. Recall that at issuance, the market value of debt is not necessarily equal to the amount borrowed or to the nominal value. Moreover, the market value of debt might change during the bond's life. This is important to note, because the literature does not consistently use the same variable to compute the tax savings from debt.[4] For simplicity and in order to remain consistency among the different approaches in the literature, it is assumed throughout, if not differently stated, that the market value of debt is equal to the nominal value and therefore also equal to the book value of debt.

If the cash flow stream from equation (2) could be expected to occur every year in perpetuity, a new valuable asset would result. The question then becomes: What would the value of this asset be, that is, what is the value of the tax shield of debt? The value can be found by discounting the cash flows from the future annual tax savings. In order to derive a reasonable value, one needs to know two things: First, the characteristics of the distribution of the cash flows created by the tax shield (depending on the variables corporate tax rate,

[3] If the current interest deductions cannot be offset against taxable income in any given year, US tax law allows a firm to carry them backward or forward against past or future taxable income or to merge with another firm that can utilize the deductions.

[4] Brealey, Myers and Allen (2006, 470) and Ruback (2002) use the amount borrowed, Copeland, Weston and Shastri (2005, 560) use the principal value and Modigliani and Miller (1963), Myers (1974), Miller (1977) and Taggart (1991), for instance, use the market value of debt.

interest expense and taxable income), and second, the appropriate discount rate reflecting the time value and the riskiness of the cash flows.

2.3 Valuation Methods

In this section I review three basic discounted cash flow methods for valuing companies that are commonly used in practice and which explicitly or implicitly include the value of the tax shield of debt.[5] It is important to note, as Bertoneche and Federici (2006) and Fernandez (2007a) show, that the different valuation methods give the same result for total firm value as well as for the value of the debt tax shield, as long as the valuation methods rely on the same hypotheses and do not implicitly include any additional assumptions. Indeed, Fernandez (2007a) notes: "This result is logical, as all the methods analyze the same reality under the same hypotheses; they differ only in the cash flows taken as a starting point for the valuation."

2.3.1 The Adjusted Present Value Method

All valuation methods that try to capture the tax advantage of debt can be represented by the Adjusted Present Value (APV) method (Cooper and Nyborg 2007). The term "adjusted present value" applies because V_U --the value of an unlevered firm, which implies that the firm is all-equity-financed-- is adjusted for the present value (PV) of the side effects of other investment and financing options in order to derive the value of the levered firm (Myers 1974). Possible side effects are the value of the debt tax shield, subsidized financing, costs of financial distress and issue costs. The APV method relies on the principle of value additivity, since it splits a company into pieces, values each piece and then adds them up. The basic APV formula is as follows (Brealey, Myers and Allen 2006, 521):

$$\text{APV} = V_U + \text{sum of PVs of side effects} = V_L. \tag{3}$$

Note that all valuation methods throughout this chapter assume the only side effect to be the tax shield of debt. I refer to this as "the world of MM with corporate taxes", since Modigliani and Miller were the first to assume that all the effects of the financing decision pertain to the tax shield of debt. Thus, we have

[5] The Equity Cash Flow (ECF) method, which computes the value of equity by discounting the expected cash flows to equityholders (ECF) at the cost of equity, r_E, is not discussed in detail since the focus is on cash flow methods that yield estimates of total firm value.

$$V_L = V_U + PVTS = APV, \tag{4}$$

where PVTS is the present value of the tax shield of debt. In this model, V_U represents the effect of investment decisions while PVTS captures the effect of financing decisions.

Since by assumption (A9) the firm is financed only by debt and equity, the following equation must hold:

$$E + D = V_L = V_U + PVTS, \tag{5}$$

where D and E are the market values of debt and equity, which sum up to the firm value. It is important to note that expressions (4) and (5) are completely general with respect to the values of V_U and PVTS, and thus further assumptions are needed to specify these values (Miles and Ezzell 1980).

2.3.2 The Free Cash Flow Method

The Free Cash Flow (FCF) method derives the value of a firm by discounting the expected Free Cash Flows, period by period, at the after-tax weighted average cost of capital (variables without time subscripts (t) are dated time 0):

$$V = \sum_{t=1}^{T} \frac{E(FCF_t)}{(1 + WACC_t)^t}, \tag{6}$$

where V is the firm value, E(·) the expected value operator, FCF the Free Cash Flow and WACC the after-tax weighted average cost of capital. WACC is defined as:

$$WACC_t = r_{D,t}(1 - \tau_C)\frac{D_{t-1}}{V_{t-1}} + r_{E,t}\frac{E_{t-1}}{V_{t-1}}, \tag{7}$$

and r_D and r_E are the costs of debt and equity,[6] respectively. Usually, these costs are derived using the capital asset pricing model (CAPM),

$$r_D = r_f + \beta_D * r_P \text{ and} \tag{8}$$

$$r_E = r_f + \beta_E * r_P, \tag{9}$$

where r_P is the market risk-premium, β_D the debt beta and β_E the equity beta (Fernandez 2007a).

[6] All formulas throughout assume that the expected return on debt and equity, the cost of debt and equity and the debt beta and equity beta are defined consistently.

Free cash flow (FCF) is the cash flow in excess of the amount required to fund all projects that have positive net present values, and is defined in the standard way:

$$FCF_t = EBIT_t(1-\tau_C) + DEP_t - INV_t\,, \tag{10}$$

where EBIT is earnings before interest and tax, DEP is depreciation and INV is investment (change in net working capital plus capital expenditures). The tax is calculated by assuming an all-equity-financed company. Thus, FCF does not include the tax shield of debt (Copeland, Weston and Shastri 2005, 510).

Since the debt tax shield is excluded from FCF calculations, the tax deductibility of interest is treated as a decrease in the cost of capital using the *after-tax* weighted average cost of capital. Thus, the FCF method incorporates the benefit of the tax shield into the tax-adjusted discount rate, which is reflected by the after-tax cost of debt $r_D(1-\tau_C)$ (Luehrman 1997).

The value of the debt tax shield is automatically included without adding a separate component of value as in the APV method. Thus, the FCF method does not give an explicit value for PVTS. However, under the assumption that capital markets are complete and the Value Additivity Theorem holds, the PVTS can be inferred by the following calculation (Cooper and Nyborg 2006a):

$$\text{PVTS} = V_L - V_U. \tag{11}$$

Hence, PVTS is the difference between the levered and the unlevered firm's value.

2.3.3 The Capital Cash Flow Method

According to Ruback (1995, 2002), the Capital Cash Flow (CCF) method derives the firm value by discounting the expected CCFs, period by period, at the unlevered cost of capital ρ:

$$V = \sum_{t=1}^{T} \frac{E(CCF_t)}{(1+\rho)^t}\,, \tag{12}$$

where the unlevered cost of capital is equal to the expected asset return (r_A):

$$\rho = r_A = r_f + \beta_U r_P\,. \tag{13}$$

Thus, the unlevered cost of capital only depends on the risk-free rate, the risk premium and the unlevered asset beta (β_U), which captures operating risk only, that is, ρ depends only on operating risk, with the effects of financial leverage removed. Hence, ρ is the firm's cost of capital given all-equity financing and therefore when applying the CCF method the discount rate does not have to be recomputed as capital structure changes (Ruback 1995).[7]

CCF is defined as all after-tax cash flows available to capital providers. In a capital structure with only ordinary debt and common equity, CCF equals the cash flow available to equityholders plus the cash flow to debtholders. In other words, CCF equals FCF plus the debt tax shield. Since the debt tax shield is included in the CCF, the discount rate is before-tax and corresponds to the riskiness of the assets (Ruback 2002).

[7] Note that Ruback (1995, 2002) claims that equation (12) holds in general. This is not correct as will be shown in section 3.5, because the CCF method uses implicit assumptions.

3 Valuation Theories for the Tax Shield of Debt

In this chapter, the main theories for valuing the firm value and the debt tax shield are presented and analyzed. In addition, the results of the three valuation methods according to the different valuation theories are compared and discussed.

3.1 The Value of the Tax Shield if the Level of Risk-Free Debt is Constant and Perpetual

As I note above, additional assumptions are needed to derive specific values for V_U, PVTS and V_L. Modigliani and Miller (1963), hereafter MM (1963), introduce the first theory for deriving the firm value and the value of the tax shield of debt.

MM (1963) assume the *expected* free cash flow to be constant and permanent. This implies that although the actual cash flow each period is risky, once each period's cash flow has been received the value of the firm is always the same, hence the company has zero expected growth (Cooper and Nyborg 2006a). Note that FCF is equal to $EBIT(1-\tau_C)$ under MM (1963) since the firm is assumed to be a perpetuity and therefore depreciation equals investment each year to keep the same amount of capital in place (Copeland, Weston and Shastri 2005, 561). The firm's risk-free debt is perpetual and constant at the level D. This implies that the future levels of D are known with certainty at time $t=0$. Further, the company always earns enough taxable income to obtain the full tax benefit of interest deductions and the corporate tax rate remains constant.

Based upon the above, the expected capital cash flow to debt- and equityholders is

$$E(CCF) = (E(EBIT) - r_f D)(1-\tau_C) + r_f D\ , \tag{14}$$

which can be written as the sum of two components, namely, a stochastic stream $E(EBIT)(1-\tau_C)$, and a non-stochastic stream, $\tau_C r_f D$. The first component is the expected FCF, which is equal to the cash flow to the unlevered firm. Therefore, it is discounted at ρ, a rate that appropriately reflects the risk of the expected cash flow to the unlevered firm. The second component, the tax savings from debt, is riskless and thus it is discounted at the risk-free rate, r_f. Therefore, the value of a levered firm can be given by (Modigliani and Miller 1963):

$$V_L = \frac{E(FCF)}{\rho} + \frac{\tau_C r_f D}{r_f} = V_U + \tau_C D \,. \tag{15}$$

Equation (15) implies that firms should finance their operations with 100 percent debt, because the marginal benefit of debt is τ_C, which is often assumed to be positive. Additionally, since τ_C is assumed to be constant, firm value increases linearly with D (Graham 2003). Note that equation (15) captures only the tax benefit of debt but contains no offsetting costs of debt.

Equation (15) corrects an inaccuracy in MM (1958), where "the tax advantage of debt was due solely to the fact that the deductibility of interest payments implied a higher level of after-tax income for any given level of before earnings" (Modigliani and Miller 1963). Under formula (15), an additional gain appears, since the non-stochastic tax savings from debt is discounted at a lower rate than the stochastic stream $E(EBIT)(1-\tau_C)$. Given that leverage creates the non-stochastic cash flow stream, $\tau_C r_f D$, the variability of *total* cash flows is reduced by leverage (Modigliani and Miller 1963).

The approach of MM (1963) has fairly restrictive and unrealistic assumptions, which lead to the implications above. However, the purpose of their paper was to show the tax advantage of debt financing. MM did not claim that their assumptions were realistic (Koller, Goedhart and Wessels 2005, 720). Indeed, MM (1963) point out that equation (15) gives only an upper bound on the value of the firm.

3.2 The Value of the Tax Shield if Debt is Risky and Debt Policy Fixed

In this section, I additionally relax assumption (A9), which asserts that only risk-free debt exists, by introducing risky debt. Debt is defined as risky in the sense that the value of debt can change over time if the yield is fixed and the cost of debt changes. Note that this assumes interest rate risk but no default risk of a bond. Further, debt is perpetual and fixed at a dollar amount (B) with a fixed yield (Ruback 2002). This implies that the future *dollar amounts* of debt outstanding are known with certainty and thus debt policy is fixed in t = 0. Note that the dollar amount of debt outstanding (B) is not assumed equal to the market value of debt (D) over time. Ruback (2002) computes the cash flow stream from debt tax savings as follows:

$$\textit{Cash flow from TS} = \tau_C \overline{r_D} B \,, \tag{16}$$

where $\overline{r_D}$ is the fixed yield on the risky debt. The question that arises is: what is the appropriate discount rate for this cash flow stream?

In a CAPM framework, the discount rate for debt tax shields (r_{DTS}) should depend on the beta of debt tax shields (β_{DTS}) (Ruback 2002):

$$r_{DTS} = r_f + \beta_{DTS} * r_P \tag{17}$$

Ruback (2002) shows that, under the assumptions given, the beta of the debt tax shield equals the beta of the debt.[8] The debt tax shield has the same amount of systematic risk as the debt.[9] This implies that the appropriate discount rate for debt tax shields is the expected return on the debt. Thus, the following equation holds:

$$V_L = \frac{E(EBIT)(1-\tau_C)}{\rho} + \frac{\tau_C \overline{r_D} B}{r_D} = V_U + \tau_C D, \tag{18}$$

where r_D is the expected return on debt and the value of debt is defined as

$$D = \frac{\overline{r_D} B}{r_D}. \tag{19}$$

Note that the cash flow stream from equation (16) is non-stochastic. However, the PVTS is subject to risk since it varies as the cost of debt or equivalently the value of debt changes. Note also that equation (18) is perfectly consistent with equation (15): if debt is assumed to be riskless, then the debt tax shield will also be riskless and should be discounted at the risk-free rate. Thus, equation (18) is also called the "extended MM approach", where the tax savings from debt have the same beta as the debt and are therefore discounted at r_D (Cooper and Nyborg 2006a).

Myers (1974) proposes a generalized approach to equations (15) and (18) to allow for finite and uneven expected operating cash flow streams. In particular, if future debt levels are currently known with certainty, Myers suggests discounting the tax savings, $\tau_C r_D D$, at the cost of debt (r_D), since he argues (similar to Ruback 2002) that the risk of the tax savings arising from the use of debt is the same as the risk of the debt. Hence, the value of a levered firm whose useful life ends at time T can be given by:

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[8] For the formal proof, see Appendix A.

[9] If debt is assumed to be fixed in *value*, then, as Ruback (2002) points out, the beta of debt tax shields is zero and has no systematic risk regardless of the debt beta.

$$V_L = \sum_{t=1}^{T} \frac{E(FCF_t)}{(1+\rho)^t} + \sum_{t=1}^{T} \frac{\tau_C r_D D_{t-1}}{(1+r_D)^t}, \tag{20}$$

Equations (18) and (20) continue to imply that a firm should take up as much debt as possible. However, empirical evidence contradicts this analysis, since firms are seldom highly leveraged. Miller (1988) puts it this way: “We seemed to face an unhappy dilemma: either corporate managers did not know (or perhaps care) that they were paying too much in taxes; or something major was being left out of the model.” To solve this empirical paradox, either other offsetting costs must be associated with issuing debt, or the tax benefit of debt must be lower than expected.[10]

3.3 Other Effects on the Tax Shield of Debt

3.3.1 Introducing Costs of Financial Distress

The implications in sections 3.1 and 3.2 that a firm should be financed with 100 percent debt are somewhat extreme. In this section, I additionally introduce an offsetting cost of debt term by relaxing assumption (A4), which asserts that no costs of financial distress (CFD) exist. Financial distress occurs when cash flow is not sufficient to cover current obligations to creditors. CFD include direct costs like legal and administrative costs of bankruptcy as well as indirect costs like moral hazard, agency, monitoring and contracting costs, which can destroy firm value even if formal default is avoided (Myers 1984).[11] To determine *expected* CFD, which are important to the calculation of V_L, the costs mentioned above need to be multiplied by the probability of distress, which increases with additional borrowing. Thus, V_L can be broken down into three components (Brealey, Myers and Allen 2006, 476):

$$V_L = V_U + \text{PVTS} - \text{PV (expected CFD)}. \tag{21}$$

Expression (21) is also known as the Static Tradeoff Hypothesis, in which the incentive to finance with debt increases with the corporate tax rate and firm value increases with the use of debt up to the point (the optimal debt ratio) where the marginal cost equals the marginal benefit of debt (Graham 2003). However, empirically the expected CFD appear

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[10] Under the assumption of asymmetric information and costs of financial distress, Myers (1984) suggests a third way to explain the empirical paradox: the Pecking Order Theory, according to which the firm prefers internal to external financing and debt to equity if it issues securities. The Pecking Order Theory assumes the debt tax shield to be a second-order effect. For reasons of space, however, I do not treat this theory in detail.

[11] For a comprehensive analysis of these indirect costs, see Jensen and Meckling (1976) and Myers (1977).

to be very small compared to the apparently large tax benefits derived from equations (18) or (20) (Andrade and Kaplan 1998). Thus, the PVTS may not lose its importance by the offsetting cost of debt term and the question remains why corporations do not use the PVTS more aggressively. Empirical studies by Wald (1998) and Rajan and Zingales (1995) reinforce this question even further. They show across a range of countries including Japan, Germany, the United Kingdom and the United States that the most profitable firms with high amounts of taxable income to shield borrow the least.

Since the CFD do not appear to fully explain the empirical paradox, I turn now to the second explanation for why the tax benefit of debt might be lower than expected by introducing personal taxes.

3.3.2 Introducing Personal Taxes and the Miller Model

In a classical tax system, including personal taxes, the after-personal-tax value of \$1 to debt investors is \$1(1-τ_{PB}) and to equity investors is \$1(1-τ_C)(1-τ_{PS}), where τ_{PB} is the personal income tax rate applicable to income from bonds and τ_{PS} is the personal income tax rate applicable to income from common stock.[12] The net tax advantage of \$1 of debt payout, relative to \$1 of equity payout, is

$$(1-\tau_{PB}) - (1-\tau_C)(1-\tau_{PS}). \tag{22}$$

If expression (22) is positive, debt interest is the tax-favored way to return capital to investors and the firm has a tax incentive to issue debt instead of equity (Graham 2003).

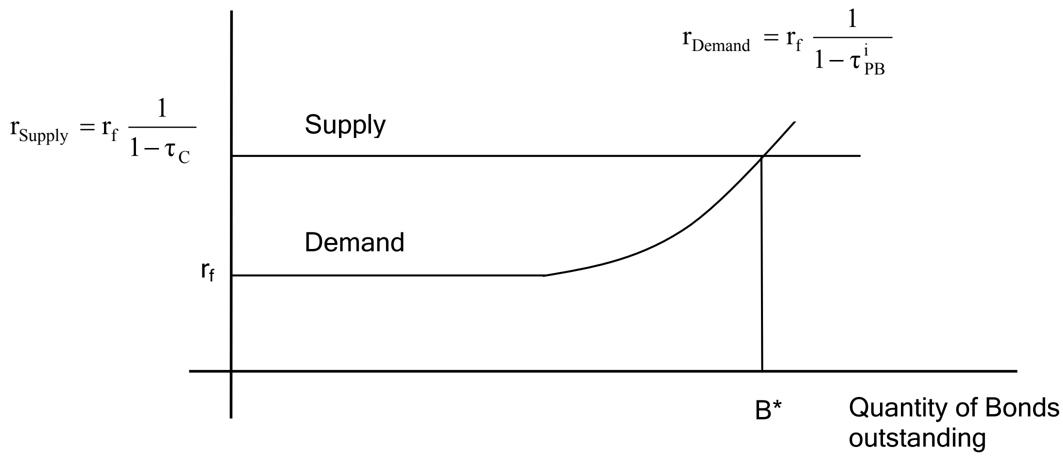
In his well-known model, Miller (1977) claims that since the empirical paradox cannot be convincingly explained by introducing costs of financial distress "the tax advantage of debt financing must be substantially less than the conventional wisdom suggests." Miller (1977) argues that in a world with fully deductible interest expenses, personal taxes can eliminate the tax advantage of debt without the need for costs of financial distress. Miller shows that when personal and corporate income taxes are taken into account, the value of a levered firm is given by:

$$V_L = V_U + \left[1 - \frac{(1-\tau_C)(1-\tau_{PS})}{(1-\tau_{PB})}\right] D, \tag{23}$$

[12] τ_{PS} is assumed to be a blended dividend and capital gains tax rate.

where $\left[1 - \frac{(1-\tau_C)(1-\tau_{PS})}{(1-\tau_{PB})}\right] D = GL$ is the gain from leverage. The intuition of the term in brackets is that as long as the after-personal-tax income from bonds, $(1-\tau_{PB})$, is higher than that from common stock, $(1-\tau_C)(1-\tau_{PS})$, there is a gain from corporate leverage (Miller 1977). Note that when $\tau_{PB} = \tau_{PS}$, or when there are no personal income taxes at all, the gain from leverage is τ_CD, the same result as in equation (15) under the original MM (1963) model with corporate taxes. Thus, Miller (1977) expands the model of MM (1963) by introducing personal income taxes. Miller assumes in his model that τ_{PS} equals zero, all firms face the same corporate tax rate, τ_C, and personal income tax is *progressive*. Moreover, regulations by tax authorities prevent taxable investors from eliminating their tax liabilities.[13] In such a world, the market equilibrium for corporate debt would be that pictured in Figure 1:

Figure 1
Equilibrium in the Market for Corporate Debt



Source: Miller (1977)

where r_{Supply} is the supply interest rate, r_{Demand} the demand interest rate, B* the equilibrium quantity of aggregate corporate debt and τ^i_{PB} the individual investor's marginal personal tax rate on interest income (Miller 1977).

The horizontal supply curve of bonds is perfectly elastic because the marginal tax benefit of debt is constant for all firms and thus all offer the same pre-tax interest rate, r_{supply}.

The demand curve is initially horizontal, representing demand by tax-free investors, but eventually slopes upward since personal income tax is progressive and the demand

[13] To maintain continuity with MM (1963), Miller (1977) assumes perpetual and riskless debt.

interest rate has to keep rising to attract investors from higher and higher tax brackets. The intuition is that the higher personal tax to income from bonds relative to equity causes investors to demand higher pre-tax returns on debt, other things being equal (including risk), in order to equalize after-tax returns from debt and equity (Graham 2003).

The market equilibrium, defined by the intersection of the two curves, has an equilibrium interest rate of $r_f \frac{1}{1-\tau_C} = r_f \frac{1}{1-\tau^i_{PB}}$. Observe that the tax benefit for corporations vanishes entirely because the higher pre-tax return for corporate debt required by the marginal investor just offsets the advantage of corporate debt financing. Thus, in the Miller model, the gain from leverage, GL, equals zero and a firm's value should be independent of its capital structure (Miller 1977).

This explanation works only if all firms face the same corporate tax rate, which is an unrealistic assumption given the large non-debt tax shields (e.g., depreciation and investment tax credits) that differ across firms (Myers 1984).[14] Even if one disagrees with Miller's result that GL = 0, equation (23) can be seen as a general version of Miller's argument. As long as τ_{PS} is less than τ_{PB}, the gain from leverage will be less than $\tau_C D$ in the original version of MM (1963) (Miller 1977).

Several empirical studies (e.g., Graham 2000, Kemsley and Nissim 2002) find evidence of a positive tax benefit when firms use corporate debt, whereas Fama and French (1998) fail to find any increase in firm value for debt tax savings. The empirical study by Kemsley and Nissim (2002) estimates for the US that the tax savings from debt net of personal taxes is approximately 40 percent of debt balances, which was close to the marginal corporate tax rate at that time.[15] Recently, Dyreng and Graham (2007) find in an event study of the Canadian income trust market that each additional dollar of debt increases firm value by \$0.38, which is not statistically different from the statutory corporate tax rate of Canadian firms. Despite these findings, the empirical value of the debt tax shield remains an open question since it is difficult empirically to distinguish between the impact of leverage on value and the impact of factors such as profitability, with which leverage is associated (Cooper and Nyborg 2004). Cooper and Nyborg (2007) suggest using the full corporate tax rate, τ_C, as a reasonable assumption for the net tax savings from

[14] DeAngelo and Masulis (1980) present a model where the debt tax shield decreases in nondebt tax shields. This implies that the supply of debt curve can become downward sloping. Under this assumption, the tax benefit of debt still adds value for some high tax rate firms.

[15] Note that the US Congress passed in mid-2003 a law that largely reduced the tax rate on both dividends and capital gains to 15 percent for individual investors (Graham 2003). Hence, the tax savings from debt net of personal taxes might be lower today for the US than at the time when Kemsley and Nissim (2002) conducted their empirical study.

debt when valuing the debt tax shield, as long as the tax system is a classical tax system in which τ_{PS} equals approximately τ_{PB} and the company can use all its future interest expenses to save tax. Therefore, it is reasonable to use τ_C as a good approximation in the standard valuation methods in order to calculate the tax savings from debt under a classical tax system.

3.4 The Value of the Tax Shield if the Leverage Ratio is Constant

In this section, it is assumed that equation (4) still holds, that is, the value of the levered firm is equal to the market value of the unlevered firm plus the value of the tax shield of debt. Hence, no costs of financial distress exist and there are no personal taxes. I introduce the new assumption that the debt policy follows a constant leverage ratio so that the market value of debt is a constant proportion of stochastic firm value. This is in contrast to the previous sections where all future debt levels were assumed to be known with certainty so that the firm had preset levels of debt.

When we assume that the value of a firm follows a random walk[16] over time, because each period's expected free cash flow also follows a random walk, one can argue that it is inconsistent to assume that the future debt levels are known with certainty in t=0 (Taggart 1991).[17] Miles and Ezzell (1980) are the first to address this inconsistency. They show that the only assumption that is generally consistent with the standard use of a *constant* WACC in the FCF method, regardless of future patterns of FCFs, is the assumption that the firm maintains a constant debt-to-value ratio in market values (D/V). Hence, Miles and Ezzell (1980, 1985), herafter ME, assume the firm rebalances its capital structure at the end of every year to maintain a constant leverage ratio. This implies that only the first-period debt level is known and the first-period debt tax shield is non-stochastic. After the first period, if a constant leverage ratio is assumed, future debt levels (D) are stochastic since they are dependent on the stochastic future firm value (V) and consequently tax savings from debt, $\tau_C r_D D$, cannot be known with certainty. Thus, even though the firm might issue only riskless debt, the debt tax shields become a stochastic cash flow stream (Miles and Ezzell 1980).

Harris and Pringle (1985) assume that *all* debt tax shields, including that of the first period, are stochastic. Their approach is analogous to ME, as shown by Taggart (1991), if

[16] A random walk is defined as a series of price changes that are unpredictable and which are only subject to new information (see, for instance, Bodie, Kane and Marcus 2008, 340).
[17] Note that the assumption that firm value follows a random walk implies that the expectations about future cash flows are revised on the arrival of new information (Arzac and Glosten 2005).

the firm's cash flows are continuous and debt levels are adjusted instantaneously. As Harris and Pringle (1985) point out, their assumptions yield to valuations that differ only slightly from the original ME approach with discrete cash flows and debt level adjustments. Throughout the remainder of the paper, the Miles and Ezzell leverage policy with continuous rebalancing (hereafter, the ME debt policy) is assumed, which implies that all debt tax shields are assumed to be stochastic. This is done in order to avoid unnecessary complexity and to be consistent with the literature in the following chapters.

Taggart (1991) shows that if the ME debt policy is assumed, the cash flow streams from tax savings are as risky as the cash flow streams to the unlevered firm and should therefore be discounted at the same rate, ρ.

As equation (11) shows, the FCF method can be replaced by the basic APV method in order to derive an explicit value for PVTS. The following expression for the value of a levered firm whose useful life ends at time T and that only issues risk-free debt then obtains:

$$V_L = \sum_{t=1}^{T} \frac{E(FCF_t)}{(1+\rho)^t} + \sum_{t=1}^{T} \frac{E(TS_t)}{(1+\rho)^t}, \tag{24}$$

where $E(TS_t)$ is the expectation at time 0 of the tax savings from debt at time t, which is equal to $E(TS_t) = \tau_C r_f L[E(V_{t-1})]$, where $L = D/V$ is the constant leverage ratio and $E(V_{t-1})$ the expectation at time 0 of the firm value at time t-1. Expression (24) is also applicable to firms that are level perpetuities (Miles and Ezzell 1985 and Taggart 1991).

The main difference between the approach of MM (1963) or Myers (1974) with a fixed debt policy and the ME (1980) approach with a constant leverage ratio policy is due to the assigned risk and magnitude of PVTS.[18] For both theories, the discount rate for the FCFs is ρ, the unlevered cost of capital. The theories differ in the discount rate for the tax savings from debt. Since MM (1963) and Myers (1974) use the cost of debt, a higher value is assigned to PVTS than under the ME approach, which uses ρ.[19] Hence, the theories differ only in the assigned risk and value to the tax shield of debt (Ruback 2002).

The difference in the risk of PVTS is caused by the different assumptions about debt policy: future debt levels are known with certainty in the MM (1963) and Myers (1974)

[18] The assumptions in the MM (1963) approach lead to a constant leverage ratio as well, since MM assume a static amount of debt and a static firm value. However, the crucial point is that tax savings from debt are non-stochastic in the MM approach since future debt levels are fixed (Cooper and Nyborg 2006a).

[19] Note that under the ME debt policy the risk of the debt tax shield is equal to the risk of the FCFs. This implies the special case that if the FCFs have no systematic risk, ρ is equal to the risk-free rate, which is also the discount rate for the debt tax shields in this case.

approach. Consequently, the risk of the non-stochastic tax savings from debt is equal to the risk of debt. By contrast, the future debt levels are stochastic if a ME debt policy is assumed. Thus, the tax saving from debt in each period becomes stochastic and has risk equal to the operating risk of the firm (Cooper and Nyborg 2006a).

The debt policy-specific effect on the gain from leverage can also be seen in Figure 2. Consider two firms, both of which are level perpetuities; firm value is not *expected* to change for these firms. The first firm, called 'MM', follows the MM (1963) approach with permanent and risk-free debt, and thus its future debt levels are known with certainty. The firm called 'ME' follows the ME approach and hence faces uncertain future debt levels. Now assume both firms issue an additional dollar of perpetual debt. The marginal gain from leverage is represented by the term dV/dD, which indicates the value created at time t=0 by an additional dollar of perpetual debt issued at time t=0. Asset beta represents the firm's operating risk (Miles and Ezzell 1985).[20] The gain from leverage per additional dollar of debt for the firm 'MM' (yellow line) is derived from equation (15) and equals $dV/dD = \tau_C$ regardless of the asset beta. In contrast, the firm 'ME' with a proportional debt policy (blue line) derives a gain from leverage from equation (24) if V_{t-1} is not *expected* to change, that is:

$$\frac{dV}{dD} = \left[\frac{r_f}{\rho} \right] \tau_C . \qquad (25)$$

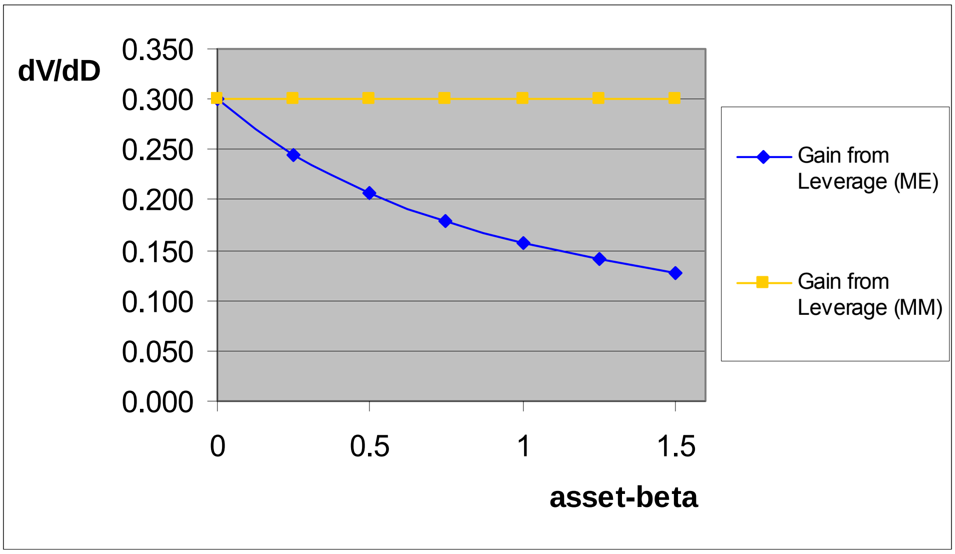
[21]

Note that in the figure, it is assumed that $\tau_C = 0.3$, $r_f = 0.055$ and $r_P = 0.05$.

[20] 'Operating risk of the firm' is equivalent to 'systematic risk of the firm's free cash flows' in a CAPM world. The former is used to better illustrate the fact that this risk is only dependent on the firm's operations and not on financing decisions.

[21] The formula appears to be slightly more complex in Miles and Ezzell (1985), since they assume the first-period debt tax shield to be non-stochastic.

Figure 2
The Relationship Between the Gain from Leverage and the Asset Beta



Based upon: Miles and Ezzell (1985)

Figure 2 shows that the marginal gain from leverage is only equal to τ_C if debt is assumed to be constant and non-stochastic under the MM (1963) approach or in the absence of operating risk under the ME approach. However, if operating risk is assumed to be positive the marginal gain from leverage is less than τ_C under the ME approach even in the absence of personal taxes. Moreover, dV/dD appears to be a decreasing function of the firm's operating risk, as measured by either the asset beta or ρ, if the ME debt policy is applied. This illustrates the dependency of the PVTS on the operating risk of the firm if the ME debt policy is assumed (Miles and Ezzell 1985).[22]

3.5 Which Valuation Method Should Be Used with Which Theory?

As noted above, the three valuation methods discussed in section 2.3 lead to the same results as long as the same assumptions or theories are applied and the valuation methods themselves do not make any implicit assumptions except their general hypotheses (Fernandez 2007a). Given a particular set of assumptions, however, it is of practical importance to know which method is easier to apply and less prone to error.

The advantage of the FCF method is its simplicity: the FCF method calculates the value of a levered firm in one step. The FCF method is most commonly applied in practice when the leverage ratio is constant and thus the risk of the debt tax shield is the same as the

[22] Arzac and Glosten (2005) derive a similar conclusion by working in a pricing kernel framework and applying the modern asset pricing approach.

risk of the firm. However, according to Taggart (1991), the FCF method and their basic equations (6) and (7) can be used under any assumption about the risk of the debt tax shield because the valuation of the debt tax shield is captured in the cost of equity, r_E. This implies that the FCF method can be used under any assumption about debt policy. The only variable that differs when the risk of the debt tax shield changes is the cost of equity. Nevertheless, the FCF method works best for firms that have a long-run target capital structure, which is in line with the assumption of a constant leverage ratio. In the case of a dynamic capital structure, however, the WACC must be adjusted each period to reflect changes in capital structure. These adjustments are prone to errors and the WACC is easy to misestimate. Hence, the PVTS might be incorrectly estimated too (Luehrman 1997).

Ruback (2002) demonstrates that when a constant leverage ratio is assumed, the CCF and FCF methods are equivalent to each other. The CCF method includes the debt tax shield in the cash flows whereas the FCF method incorporates the tax benefit of debt in the WACC.

Ruback (2002) further points out that the advantage of the CCF method, compared to the FCF method, is its simplicity whenever the capital structure changes over time and debt is forecasted in levels instead of as a proportion of firm value. Ruback argues that since the CCF method includes the debt tax shield in the cash flows, the discount rate does not change when leverage ratios change.[23] It is important to notice that the CCF method discounts all cash flows, including the tax savings from debt, at the unlevered cost of capital, which is only dependent on operating risk. However, since the tax savings from debt are discounted at ρ, the CCF method *implicitly* makes the assumption that the risk of the debt tax shield matches the operating risk of the firm (Cooper and Nyborg 2006a).

Ruback (1995, exhibit 1) demonstrates by way of example how to use the CCF method to value a firm that has undergone a leveraged buyout (LBO) and for which the schedule of future debt levels is given. By applying the CCF method in this example, the tax saving from debt is implicitly discounted by the unlevered cost of capital. Now, extending Ruback, assume further that the debt policy is fixed (i.e., future debt levels are known with certainty, which is very likely in an LBO transaction) and the tax savings from debt are realized when interest is paid. Then, we know from section 3.2 that the risk of the tax shields matches the risk of debt and consequently the appropriate discount rate must be the cost of debt. Since the CCF method uses ρ as the constant discount rate for valuing the debt tax shield, the CCF method is inconsistent with a fixed debt policy when interest expenses

[23] Ruback (1995) states: “The introduction of ... a changing capital structure has no impact on the expected asset return which is used to discount the Capital Cash Flows.”

can be fully deducted in every period. The reason is that, as noted earlier, unlike the other valuation methods the CCF method makes the implicit assumption that the risk of the tax shield is always equal to the firm's operating risk. As Fernandez (2007a) shows, the CCF method holds in general if a discount rate is used that is not always equal to the unlevered cost of capital but consistent with a fixed debt policy. Fernandez (2007a) uses the before-tax WACC, which he defines as:

$$WACC_{BT,t} = r_{D,t}\frac{D_{t-1}}{V_{t-1}} + r_{E,t}\frac{E_{t-1}}{V_{t-1}}. \tag{26}$$

However, the $WACC_{BT}$ varies over time when a fixed debt policy is assumed. Therefore, the CCF method would lose its simplicity.

In order to justify the CCF method introduced by Ruback (2002), even when the debt policy is fixed, proponents claim that the future tax savings from debt should depend on the level of future operating income. If the company does not have enough taxable income to shield, interest cannot be used to save tax immediately and hence the risk of the tax shield will be higher than the risk of debt. In such cases, proponents assume, as a simple approximation, that the risk of the tax shield of debt is equal to the operating risk of the firm. Given this assumption, the tax savings may be discounted at the unlevered discount rate without assuming a specific debt policy. Therefore, if the future tax status of the firm is uncertain because the firm could become non-taxpaying, the CCF method might be a good approximation in practice (Cooper and Nyborg 2007). Note that the reason for discounting the debt tax shield at the unlevered cost of capital in the CCF method is then the uncertain future tax status of the firm, which is independent of the firm's debt policy. In the FCF and APV methods, however, a proportional debt policy is assumed, which forces the debt tax savings to have the same risk as the firm's operating risk. It is further assumed that the firm will always be paying taxes. Hence, the debt tax shield is discounted at the unlevered cost of capital (Cooper and Nyborg 2006a).

If the leverage ratio is not assumed to be constant and the firm will remain taxpaying in the future, the approach that is less prone to error than the other methods is the APV method, using a discount rate for the debt tax shield that correctly reflects the firm's risk (Cooper and Nyborg 2007). Since the APV method lays out each component of value separately, it is exceptionally transparent. In particular, if the schedule of future debt levels is given initially, the APV method provides a technique to value each period's debt tax shield separately (Luehrman 1997). Consistent with this result, Koller, Goedhart and

Wessels (2005, 104) suggest the use of the APV method when capital structure changes significantly over time, as in an LBO or in the valuation of distressed companies.

4 Is the Value of Tax Shields equal to the Present Value of Tax Shields or not?

The previous sections have shown that each valuation theory relies on a specific set of assumptions about the firm's debt policy, assumptions that affect the value of the tax shield of debt. The debate about the value of tax shields is still ongoing, however, and has been reinforced by the development of a new approach by Fernandez (2004a). Fernandez (2004a) claims that "The value of tax shields is NOT equal to the present value of tax shields" and introduces a new theory on how to calculate the value of debt tax shields. As a reply to Fernandez (2004a), Cooper and Nyborg (2006b) argue, "The value of tax shields IS equal to the present value of tax shields" and claim that the standard valuation theories are still valid. Both approaches and their underlying assumptions are presented in this chapter.

4.1 The Value of Tax Shields is NOT equal to the Present Value of Tax Shields

Fernandez (2004a) derives the value of tax shields for two different cases: for perpetuities with no growth and for perpetuities with constant growth. Fernandez then argues that the value of tax shields depends only on growth characteristics of the firm and is independent of debt policies.

4.1.1 The General Idea

Fernandez (2004a) claims that the only way to obtain the *correct* value for the debt tax shield that holds under broad conditions is to compute *the difference* between the present values of two separate cash flows, each with its own risk. That is, the difference between the present value of taxes paid by the unlevered firm minus the present value of taxes paid by the levered firm.

The general idea underlying the model of Fernandez (2004a) can be explained by three simple expressions, where costs of financial distress are explicitly ignored. From equation (5), we know that if the Value Additivity Theorem holds, then the *after-tax* value of a levered firm is:

$$E_L + D = V_L = V_U + VTS. \tag{27}$$

Hence, the sum of the market values of debt (D) and levered equity (E_L) is equal to the value of the unlevered firm (V_U) plus the value of tax shields from debt (VTS).

Note that the term PVTS (Present value of debt tax shields) from previous sections is replaced by the term VTS (value of tax shields) to be consistent with the claim that "The value of tax shields is NOT equal to the present value of tax shields".

In addition to shareholders and bondholders, there is a third stakeholder in the company, namely, the government, which has a claim on taxes. If there are no costs of financial distress and FCFs are unaffected by changes in capital structure, the *before-tax* value of the unlevered firm must be equal to the *before-tax* value of the levered firm. Therefore:

$$V_U + G_U = V_L + G_L, \tag{28}$$

where G_U is the present value of taxes paid by the unlevered company and G_L the present value of taxes paid by the levered company. Equation (28) means that the total value of the firm, which is the after-tax value of the firm plus the present value of taxes, is independent of capital structure (Fernandez 2004a). Knowing from (27) that VTS = V_L - V_U, we get:

$$VTS = G_U - G_L. \tag{29}$$

Hence, VTS is the difference between two present values of two cash flows with *different risk* (Fernandez 2004a).

Fernandez (2004a) admits that calculating the *present value* of tax savings from debt, as suggested by the standard literature, is not necessarily wrong if the appropriate discount rate is used.[24] However, he claims that it is not feasible to evaluate *the riskiness of the difference* between two expected cash flows with different risk in order to derive the appropriate discount rate. Thus, Fernandez (2004a) claims that "perhaps the most important issue ... is that the term 'discounted value of tax shields' in itself is senseless." Hence, Fernandez' (2004a) approach challenges the conventional results in the literature. In particular, he argues that the standard valuation theories by MM (1963), Myers (1974), ME (1980), Harris and Pringle (1985) and Ruback (2002) "result in inconsistent valuations of the tax shield or inconsistent relation between the cost of capital of the unlevered and levered firm" (Fernandez 2004a).

[24] Note that the standard literature, as shown in the previous chapter, computes *first* the difference between the two cash flows, which is the debt tax saving, and *then* this difference is discounted at the appropriate rate in order to derive the present value of tax savings from debt.

4.1.2 The Value of Tax Shields for Constant Perpetuities

Fernandez' (2004a) only explicit assumption is a constant perpetuity, which implies an expected growth rate of zero (g=0), which is analogous to assuming that the firm is following a random walk with no drift indefinitely as in the MM (1963) setting. Fernandez (2004a) does not assume a specific debt policy since he claims that his result for the VTS is generally valid for constant perpetuities independent of debt policy.

In order to derive the VTS for a constant perpetuity from equation (29), one has to know the taxes paid by the unlevered and the levered company and the appropriate discount rates reflecting the risk of both of the cash flows.

The taxable income for an *unlevered* firm at time t can be derived from equation (10) by solving for EBIT:

$$EBIT_t = (FCF_t - DEP_t + INV_t)/(1-\tau_C). \tag{30}$$

Since the firm is assumed to be a constant perpetuity, it is known from section 3.1 that depreciation (DEP) equals investment (INV) to keep the same amount of capital in place. Thus, for a constant perpetuity, equation (30) simplifies to:

$$EBIT_t = (FCF_t)/(1-\tau_C). \tag{31}$$

Hence, the taxes paid every year by the *unlevered* constant perpetuity, $Taxes_{U,t}$, are:

$$Taxes_{U,t} = EBIT_t \tau_C = (FCF_t)\tau_C /(1-\tau_C). \tag{32}$$

In expectations, the taxes to be paid every year by the unlevered company, $E(Taxes_{U,t})$, are:[25]

$$E(Taxes_{U,t}) = E(FCF_t)\tau_C /(1-\tau_C). \tag{33}$$

On the other hand, the taxable income of the *levered* firm at time t can be inferred from the equity cash flow (ECF) formula:

$$ECF_t = (EBIT_t - r_D D_{t-1})(1-\tau_C) + DEP_t - INV_t + \Delta D_t, \tag{34}$$

[25] Fernandez (2004b) states that he neglected to use expected value notations for the sake of simplicity in Fernandez (2004a). I will include however the expected value operator E(·) into the formulas to be as specific as possible.

where $\Delta D_t = D_t - D_{t-1}$ is the change in the amount of debt outstanding at time t. Solving equation (34) for the taxable income of a levered firm, we get:

$$EBIT_t - r_D D_{t-1} = (ECF_t - DEP_t + INV_t - \Delta D_t)/(1-\tau_C)\,. \tag{35}$$

For the *levered* constant perpetuity, Fernandez (2004a) claims that *"..taxes paid by the levered company are proportional to ECF...."* Therefore, he obtains:

$$Taxes_{L,t} = (EBIT_t - r_D D_{t-1})\tau_C = (ECF_t)\tau_C /(1-\tau_C)\,, \tag{36}$$

where $Taxes_{L,t}$ are the taxes paid by the levered constant perpetuity at time t. Hence, in expectations, the taxes to be paid every year by the levered company, $E(Taxes_{L,t})$, are:

$$E(Taxes_{L,t}) = E(ECF_t)\tau_C /(1-\tau_C)\,. \tag{37}$$

In order to get the present values of the two expected cash flow streams from equations (33) and (37), one has to derive the appropriate discount rate reflecting the risk of each cash flow stream. As can be seen from equations (32) and (36), the taxes paid by the unlevered firm are proportional to FCF and the taxes paid by the levered firm are proportional to ECF under the assumptions that Fernandez (2004a) makes. Hence, Fernandez (2004a) concludes that in the case of constant perpetuities, the taxes of the unlevered firm have the same risk as the FCF and the taxes of the levered firm have the same risk as the ECF. Thus, the taxes of the unlevered firm must be discounted at the unlevered cost of capital, ρ, and the taxes of the levered firm must be discounted at the expected return to equity, r_E (Fernandez 2004a). Applying these findings it follows that:

$$VTS = G_U - G_L = \frac{E(Taxes_U)}{\rho} - \frac{E(Taxes_L)}{r_E} = \frac{E(FCF)\tau_C /(1-\tau_C)}{\rho} - \frac{E(ECF)\tau_C /(1-\tau_C)}{r_E} \tag{38}$$

and by knowing that $E(FCF)/\rho = V_U$ and $E(ECF)/r_E = E_L$ (the equity of the levered company in this case), we obtain:

$$VTS = (V_U - E_L)\tau_C /(1-\tau_C)\,. \tag{39}$$

As from equation (27), since $V_U - E_L = D - VTS$, it follows that (Fernandez 2004a):

$$VTS = \tau_C D\,. \tag{F16}$$

Thus, Fernandez (2004a) introduces a new approach for deriving a result that was found, for instance, by MM (1963) or Myers (1974) (see sections 3.1 and 3.2). The difference, however, is that Fernandez claims that this result is generally valid for constant perpetuities independent of the debt policy of the firm.

4.1.3 The Value of Tax Shields for Growing Perpetuities

Fernandez (2004a) further investigates a growing perpetuity with an expected constant growth rate of g (g>0), which is equal to a firm following a random walk with drift indefinitely. Note that for growing perpetuities the expected values of annual depreciation (DEP), investment (INV) and the change in the amount of debt (ΔD) are not equal to zero as in the previous case with constant perpetuities. Hence, it follows from equation (30) that the taxes expected to be paid every year by the *unlevered growing perpetuity* are:

$$E(Taxes_{U,t}) = [E(FCF_t) - E(DEP_t) + E(INV_t)]\tau_C /(1-\tau_C)\,, \tag{40}$$

whereas it follows from (35) that the *levered growing perpetuity* expects to pay:

$$E(Taxes_{L,t}) = [E(ECF_t) - E(DEP_t) + E(INV_t) - E(\Delta D_t)]\tau_C /(1-\tau_C)\,. \tag{41}$$

As can be seen from equations (40) and (41), it can no longer be assumed that the taxes paid by the unlevered or the levered company are proportional to FCF or ECF, respectively. Hence, the appropriate discount rates for the two cash flow streams above cannot be equal to the discount rates, ρ and r_E, from the previous section (Fernandez 2004a).

Fernandez (2004a) derives the following expression for the value of tax shields for constant growing perpetuities: [26]

$$VTS(g) = \frac{\tau_C \rho D}{\rho - g}\,. \tag{F28'}$$

Fernandez (2004a) claims that (F28') is generally valid for growing perpetuities and the only formula in the literature that leads to correct results.

(F28') is the central claim in Fernandez (2004a) and is obtained by subtracting (F38) from (F37):

$$[\rho - r_D(1-\tau_C)] - (E_L / D)(r_E - \rho) = (VTS / D)(\rho - g)\,, \tag{F37}$$

[26] For the entire formal proof by Fernandez (2004a), see Appendix B.

$$[\rho - r_D(1-\tau_C)] - (E_L / D)(r_E - \rho) = \tau_C \rho \, . \tag{F38}$$

(F37) is derived for perpetuities with g>0. (F38) is obtained by substituting $VTS = \tau_C D$ (F16), the formula for $g = 0$, in (F37).

The transformation from (F37) to (F38) is only valid, however, if the left-hand side of (F37) does not depend on the growth rate g. Therefore, Fernandez (2004a) makes the important assumption that *if (E_L/D) is constant, the left-hand side of equation (F37) does not depend on growth (g) because for any growth rate (E_L/D), ρ, r_D and r_E are constant.* These and other assumptions by Fernandez (2004a) are examined in chapter 5.

4.2 The Value of Tax Shields IS equal to the Present Value of Tax Shields

Cooper and Nyborg (2006b) derive the value of debt tax shields for constant perpetuities and growing perpetuities, the two cases discussed above, by computing the *present value* of tax savings as suggested by the standard literature. Therefore, the VTS[27] is obtained by applying the valuation theories of ME (1980) and MM (1963) for each of the two cases. As opposed to Fernandez (2004a), Cooper and Nyborg (2006b) claim that “The value of tax shields is equal to the present value of tax shields” and that the VTS does depend on both growth characteristics *and* the specific debt policy of the firm.

4.2.1 The Present Value of Tax Shields for Growing Perpetuities

Cooper and Nyborg (2006b) assume – in line with Fernandez (2004a) - the existence of a growing perpetuity that generates stochastic cash flows and that is expected to grow at a constant rate g. Further, no personal taxes and no costs of financial distress are assumed.

Then, the value of the unlevered growing perpetuity is:

$$V_U = \frac{E(FCF)}{\rho - g} \tag{42}$$

and the value of the levered growing perpetuity is:

$$V_L = E_L + D = \frac{E(FCF)}{\rho - g} + VTS \, . \tag{43}$$

[27] For consistency with the previous section, I continue to use VTS (value of tax shields) instead of PVTS (present value of tax shields), which is in line with both Fernandez (2004a) and Cooper and Nyborg (2006b).

If the ME debt policy (see section 3.4) is assumed, the levered firm value for a growing perpetuity can also be derived by applying the FCF method and by using the constant WACC as the discount rate (Cooper and Nyborg 2006b). Given the ME debt policy, the constant WACC can be written as:

$$WACC = \rho - \tau_C r_D D/(E_L + D)\,. \tag{44}$$

Equation (44) can be decomposed into two parts: The first part, ρ, the unlevered cost of capital, represents the required return due to operating risk only, as if no financial leverage had been used. The second part, $-\tau_C r_D D/(E_L + D)$, represents the benefit of debt financing to the firm. Hence, the tax shield of debt is included in the WACC by adjusting the discount rate downward (Harris and Pringle 1985). Then, the value of the levered firm is:

$$V_L = E_L + D = \frac{E(FCF)}{(\rho - \tau_C r_D D/(E_L + D) - g)}\,. \tag{45}$$

As shown by equation (11), the VTS can generally be derived by subtracting the unlevered value (V_U) from the levered firm value (V_L).Consequently, with the ME debt policy assumption, the VTS for a growing perpetuity is given by:

$$VTS^{ME}(g) = V_L - V_U = \frac{E(FCF)}{(\rho - \tau_C r_D D/(E_L + D) - g)} - \frac{E(FCF)}{(\rho - g)} = \frac{\tau_C r_D D}{(\rho - g)}\,. \tag{46}$$

This is the VTS obtained under the ME debt policy assumption if it is directly valued by computing the *present value* of tax savings (Cooper and Nyborg 2006b).

By contrast, now assume that the same firm follows a fixed debt policy. This is similar to the MM (1963) approach, where future debt levels are known with certainty at time t=0. In this case, however, as opposed to assuming that debt levels are constant as in MM (1963), the amount of debt constantly grows at the same rate as the firm, g. To facilitate comparison with the ME debt policy, this policy is called the “MM debt policy with growth”.

As shown in section 3.2, if future debt levels are fixed initially, the debt tax shield has the same risk as the debt and thus the debt tax shield must be discounted at the cost of debt, r_D. If the tax shield is evaluated directly by computing the *present value* of tax savings, under the assumption of a MM debt policy with growth, one obtains:[28]

[28] For the formal proof, see Appendix C.

$$VTS^{MM}(g) = \frac{\tau_C r_D D}{(r_D - g)}. \tag{47}$$

The VTS is higher in this case than that for the ME debt policy since debt is non-stochastic and consequently the debt tax shield has lower risk (Cooper and Nyborg 2006b).

4.2.2 The Present Value of Tax Shields for Constant Perpetuities

The VTS for constant perpetuities can be easily inferred from either equation (46) or (47) by setting the constant growth rate equal to zero (g=0). For the constant perpetuity following the ME debt policy, the VTS is (Cooper and Nyborg 2006b):

$$VTS^{ME} = \frac{\tau_C r_D D}{\rho}. \tag{48}$$

In contrast, with the MM debt policy (without growth), VTS is given by:

$$VTS^{MM} = \frac{\tau_C r_D D}{r_D} = \tau_C D. \tag{49}$$

Equations (48) and (49) are already known from chapter 3, where the underlying valuation theories are introduced.

As can be seen from equations (48) and (49), the VTS differs by a ratio of ρ / r_D even if both equations assume a constant perpetuity. The difference arises since the discount rate under a ME debt policy is the unlevered cost of capital, whereas it is the cost of debt under the MM debt policy. Hence, Cooper and Nyborg (2006b) draw the conclusion that the VTS is not only dependent on the growth rate, g, but also on the debt policy of the firm. Under the ME debt policy, debt is stochastic and consequently the tax saving is stochastic too, rising and falling with firm value, which follows a random walk. In contrast, the MM debt policy assumes a non-stochastic amount of debt regardless of the firm's value, and accordingly the tax saving is also non-stochastic (Cooper and Nyborg 2006b).

5 Discussion

In this section I analyze the assumptions made by Fernandez (2004a) to derive the VTS for growing perpetuities, which is the central case in his paper.[29] In doing so, I also examine the assumptions in the case of constant perpetuities.

Cooper and Nyborg (2006b) argue (see the previous chapter) that the value of the debt tax shield is a function of the debt policy of the firm that is assumed. However, if Fernandez (2004a) is right, and his approach for valuing the debt tax shield, which is independent of a specific debt policy, is generally valid, then Fernandez' approach and underlying assumptions should be consistent with the ME and MM debt policy.

5.1 The Miles and Ezzell Framework

In this section, a growing perpetuity following the ME debt policy is assumed. This setup should be consistent with the VTS for growing perpetuities derived by Fernandez (2004a) if his approach is generally valid. However, the VTS that he derives, given by equation (F28'), differs from the value derived by Cooper and Nyborg (2006b) in equation (46) under the ME debt policy assumption. Equation (F28') is equation (46) times a factor of ρ / r_D. In the following, it is shown which assumptions of Fernandez (2004a) are inconsistent with the ME debt policy that in turn lead to results that are not valid in a ME setting.

In order to completely understand the assumptions of the ME debt policy, which implies stochastic tax shields, it is helpful to know the assumed *underlying* free cash flow process of the firm. Arzac and Glosten (2005) give the following expression, which describes the explicit stochastic free cash flow process in a ME setting when $g = 0$:

$$FCF_{t+1} = FCF_t (1 + \varepsilon_{t+1}) , \tag{50}$$

with $E_t [FCF_t (1 + \varepsilon_{t+1})] = FCF_t$, where ε_{t+1} is a stochastically independent random variable such that $E_t (\varepsilon_{t+1}) = 0$. The intuition is that expectations about future free cash flows (FCF_{t+1}) are revised at time t on the basis of new information, in particular, the information included by the realized free cash flow at time t (FCF_t) (Arzac and Glosten 2005).

[29] Fieten at al. (2005) point out that Fernandez (2004a) makes several unstated assumptions to derive his results such as zero personal taxes and the absence of non-debt tax shields. Further, he does not specify the stochastic free cash flow process of the firm and it is also not clear from the outset whether the assumed debt is perpetual or finite but perpetually rolled-over.

This stochastic free cash flow process gives rise to firm value following a random walk. Given the assumption of a constant leverage ratio implied by the ME debt policy, debt is stochastic because it adjusts over time as a function of the value of the firm and is therefore a function of random cash flow realizations. This in turn leads to stochastic tax shields. Hence, the process of the stochastic tax shields is dependent not only on the assumed constant leverage ratio policy but also on the stochastic process of the firm's free cash flows (Arzac and Glosten 2005).

The expression describing the stochastic free cash flow process of a constant *growing* perpetuity in a ME setting is (Arzac and Glosten 2005):

$$FCF_{t+1} = FCF_t(1+g)(1+\varepsilon_{t+1}), \tag{51}$$

with $E_t[FCF_t(1+g)(1+\varepsilon_{t+1})] = FCF_t(1+g)$ for $E_t(\varepsilon_{t+1}) = 0$.

If it is further assumed that the free cash flows are positively correlated with the market portfolio, which implies systematic risk in a CAPM world, the firm's value has systematic risk too. This implies, under the assumption of a constant leverage ratio, that the debt and the debt tax shields have systematic risk too (Arzac and Glosten 2005).

Let us now assume a constant perpetuity, subject to systematic risk, following the ME debt policy, which implies the free cash flow process described by equation (50). The effect of the assumptions on the value of the debt tax shield is illustrated by Figure 2 in section 3.4 by the blue line (ME). All firms following the ME debt policy and with a positive asset beta, which implies systematic risk of the free cash flows, have a marginal gain from leverage per additional dollar of debt that is less than the corporate tax rate, τ_C. Hence, the value of tax shields for these firms must be $VTS < \tau_C D$ (Arzac and Glosten 2005).

However, this result is not consistent with the result by Fernandez (2004a). He finds that the value of tax shields for constant perpetuities is $VTS = \tau_C D$, which he claims is independent of a firm's specific debt policy. This is an important result since Fernandez (2004a) uses $VTS = \tau_C D$ for $g = 0$ as a substitution for VTS in equation (F37) to derive the VTS for *growing* perpetuities (Cooper and Nyborg 2006b).

As shown in section 4.2.2, the expression $VTS = \tau_C D$ is correct only if the MM debt policy is assumed, where future debt levels are fixed initially. Under the ME debt policy, VTS is given by equation (48) for constant perpetuities. The error that Fernandez (2004a) makes, and which leads to the results he derives, can be traced back to the assumption that he makes to derive the taxes paid by the levered constant perpetuity. In particular, he assumes that the taxes paid by the levered constant perpetuity are proportional to the ECF, that is, based upon the proportionality assumption, he assumes that the appropriate discount rate for the expected taxes paid by the levered constant perpetuity ($E(Taxes_{L,t})$) must be the expected return to equity, r_E (Cooper and Nyborg 2006b). This relation does not hold in general, however, as can be illustrated by the following two equations (Cooper and Nyborg 2006b):

$$Taxes_{L,t} = (EBIT_t - r_D D_{t-1})\tau_C \text{ and} \tag{52}$$

$$ECF_t = (EBIT_t - r_D D_{t-1})(1-\tau_C) + \Delta D_t . \tag{53}$$

Equation (53) computes the ECF for constant perpetuities. If a constant perpetuity is assumed, $DEP_t = INV_t = 0$. However, $\Delta D_t = 0$ only if $D_t = D_{t-1} = D$. This is only given if all debt levels are constant over time (MM debt policy). If a ME debt policy were assumed instead, $D_t \neq D_{t-1}$ since debt is stochastic and therefore $\Delta D_t \neq 0$. Consequently, in a ME setup, ECF is not proportional to taxes paid by the levered constant perpetuity and hence taxes have a different risk and discount rate than ECF (Cooper and Nyborg 2006b). Fieten et al. (2005) point out that if debt is assumed to be stochastic and positively correlated with equity value (which is the case in a ME setting); the taxes of the levered constant perpetuity are of lower risk than ECF and consequently have a lower discount rate.

Cooper and Nyborg (2006b) reconcile the case for constant perpetuities in Fernandez (2004a) when the ME debt policy is assumed. They argue that under a ME debt policy, the present value of the taxes paid by the levered constant perpetuity must be:

$$G_L = V_U \tau_C /(1-\tau_C) - (\tau_C r_D D)/\rho . \tag{54}$$

The first part of the equation, $V_U \tau_C /(1-\tau_C)$, is the present value of taxes paid by the *unlevered* firm (see equations (38) and (39) for the derivation) and the second part, $-(\tau_C r_D D)/\rho$, is the value of tax shields for a constant perpetuity derived under the assumption of the ME debt policy. Then, the VTS for a constant perpetuity in a ME setting is:

$$VTS^{ME} = G_U - G_L = V_U \tau_C /(1-\tau_C) - [V_U \tau_C /(1-\tau_C) - (\tau_C r_D D)/\rho] = (\tau_C r_D D)/\rho , \tag{55}$$

which is equal to equation (48), the *present value* of tax shields. Further, it can be shown that if Fernandez had used the VTS from equation (55) for $g = 0$ to derive the VTS for growing perpetuities (F28'), he would have found the same result as equation (46), the VTS in the ME setting for growing perpetuities (Cooper and Nyborg 2006b).[30]

To summarize, this section has shown that Fernandez (2004a) erroneously uses an assumption that is true in a MM setup but not in a ME setup in deriving his VTS for growing perpetuities. Therefore, Fernandez' (2004a) result lacks generality (Cooper and Nyborg 2006b).

5.2 The Modigliani and Miller with Growth Framework

In a reply to a working paper version of Cooper and Nyborg (2006b), Fernandez (2004b) clarifies some aspects of the assumptions used in Fernandez (2004a) by the following statement:

> "There is also a subtle difference between the Miles-Ezzell (1980) assumption about the capital structure, namely, D = K E, and the assumption that I use in my paper: E{D} = K E{E}, being E{·} the expected value operator, D the value of debt and E the equity value. K is a constant. Miles-Ezzell (1980) assumption requires continuous rebalancing, while my assumption does not."

Cooper and Nyborg's (2006b) interpretation of this statement is that Fernandez (2004b) assumes a debt policy where future debt levels are fixed initially as under a MM debt policy but at the same time follows a constant leverage ratio. Therefore, in contrast to the ME setup, debt levels are independent of the free cash flow process of the firm and the debt policy is consistent with the result that $VTS = \tau_C D$ for $g = 0$, which makes it a valid substitution for VTS in equation (F37) (Cooper and Nyborg 2006b).

However, since the firm is a growing perpetuity and Fernandez (2004a) assumes a constant leverage ratio, debt has to grow at the same expected rate g as the levered firm's value, so that $D_t = LV_{L,t}$ is satisfied at every time t. The debt policy is called 'MM debt policy with growth' since debt is deterministic as under a MM debt policy but grows at rate g (Cooper and Nyborg 2006b).

Note that since all future debt levels are fixed initially and the leverage ratio remains constant over time, $V_{L,t}$ must be known at time 0. Hence, the following free cash flow process is required given the MM debt policy with growth (Arzac and Glosten 2005):

[30] For the derivation of this result (that Cooper and Nyborg 2006b do not show), see Appendix D.

$$FCF_t = FCF(1+g)^t + \varepsilon_t \text{, with} \tag{56}$$

$$E(FCF_t) = E_{t-1}(FCF_t) = FCF(1+g)^t \tag{57}$$

for $E(\varepsilon_t) = E_{t-1}(\varepsilon_t) = 0$ where $E(\cdot)$ is the unconditional expected value and $E_{t-1}(\cdot)$ the expected value conditional on the information available at time t-1 (Cooper and Nyborg 2006b).

The intuition of the free cash flow process is that expectations are constant over time and *not* revised on the basis of new information and hence firm value does not follow a random walk. This is a less realistic scenario than the assumed free cash flow process in the ME setup, however, because firm value is predictable and hence the efficient market hypothesis does not hold. Note that although the *expectations* are not revised and are independent of free cash flow realizations, the free cash flows are still assumed to be stochastic (Arzac and Glosten 2005).

Given this interpretation by Cooper and Nyborg of Fernandez' (2004b) statement, Fernandez' (2004a) assumptions for a growing perpetuity are as follows (Cooper and Nyborg 2006b):

1. Expected free cash flows grow at rate $g > 0$.
2. Future debt levels are known with certainty at time 0 (Interpretation by Cooper and Nyborg (2006b)).
3. $L = D/V$ is a constant and independent of g and time.
4. ρ, r_E and r_D are constants and independent of g and time.
5. The firm has systematic risk greater than zero in a CAPM world and hence: $\rho > r_f$.

These assumptions given, it is important to note that the independence between the variables (L, ρ, r_E and r_D) and g is critical, since this allows Fernandez (2004a) to substitute $VTS = \tau_C D$ for $g = 0$ into equation (F37) to derive the VTS for growing perpetuities (Cooper and Nyborg 2006b).

Cooper and Nyborg (2006b) show that if $g < r_f$ and debt is risk-free, all of the assumptions above are satisfied, except possibly one, namely, r_E being independent of g. Below I show that r_E must be *dependent* on g if all other assumptions above hold simultaneously. Therefore, the VTS for growing perpetuities derived by Fernandez (2004a) is not valid under the assumption of a 'MM debt policy with growth' setup.

The following reasoning uses risk instead of expected return expressions. This is consistent because expected returns are a function of beta in a CAPM world (assuming the risk premium and risk-free rate are constant).[31] According to Ruback (2002), equation (58) shows the different risk positions of a market-value balance sheet with assets and the debt tax shield on the left and debt and equity on the right, where the systematic risk of the levered firm (β_L) is a value-weighted average:

$$\frac{V_U}{V_L}\beta_U + \frac{VTS}{V_L}\beta_{DTS} = \beta_L = \frac{D}{V_L}\beta_D + \frac{E}{V_L}\beta_E\,. \tag{58}$$

We know that ρ and r_D are constants independent of g. This implies that the risk of the unlevered firm (β_U) and debt (β_D) is constant. Furthermore, since $L = D/V$ is also constant and independent of g; V_U/V_L, D/V_L and E/V_L must be constant too. Consequently, the equity beta (β_E) can only be independent of g if β_L is constant and independent of g. It can be seen from equation (58) that if VTS is a function of g and its fraction of total firm value increases over time, β_L is independent of g if $\beta_{DTS} = \beta_U$, which is the case in the ME setup. However, from section 3.2 we know that if debt levels are fixed initially, as in the case of a MM debt policy with growth, the debt tax shields are as risky as the debt. Hence, $\beta_{DTS} = \beta_D$. In our case, debt is assumed to be risk free and therefore we obtain $\beta_{DTS} = \beta_D = 0$. Thus, β_L and therefore β_E decrease over time and are dependent on g since the VTS of a growing perpetuity is dependent on g and its fraction of total value increases over time. Consequently, r_E must also be dependent on g since expected returns are a function of beta.[32]

Since the debt is risk-free in our case the tax savings per period are: $TS = \tau_C r_f D$. In addition, we know from above that the discount rate of the debt tax shield must be r_f because $\beta_{DTS} = 0$. Hence, the VTS for a growing perpetuity following the MM debt policy with growth must be:

$$VTS^{MM}(g) = \frac{\tau_C r_f D}{r_f - g}\,. \tag{59}$$

[31] The approach I use is different from Cooper and Nyborg (2006b), who use expected returns instead of beta. Their approach is misleading since they assume that the discount rate of tax shields is equal to the interest rate used in the numerator to compute debt tax savings, which is not generally valid, as for instance in the ME setup.

[32] Fieten et al. (2005) also identify this inconsistency in the assumptions by arguing that subtracting a fixed debt from a firm value that follows a geometric random walk results in a levered equity of non-constant risk.

This is equivalent to equation (47), the present value of tax shields derived by Cooper and Nyborg (2006b).

Note further that if Fernandez (2004a) would allow for the dependence of r_E on g in the derivation of (F28'), the VTS would give the same value as in equation (47) (Cooper and Nyborg 2006b).[33] I conclude that Fernandez' (2004a, 2004b) assumptions for deriving his VTS for growing perpetuities are neither consistent with the MM debt policy with growth nor the ME debt policy and their required specific cash flow processes.

5.3 The Value of the Tax Shield if the Leverage Ratio is Constant in Book Values

In a series of papers, Fernandez (for instance, 2004b, 2005a, 2005b and 2007b) replies to the inconsistencies mentioned above. In doing so, he derives the VTS by applying a different approach, which allows him to obtain the VTS without specifying the exact values of G_U and G_L (the present values of taxes paid by the unlevered firm and the levered firm, respectively). Although the approach is different, the VTS is the same as in Fernandez (2004a) for growing perpetuities (equation F28') if a certain set of assumptions is used. In this section, this new approach is derived by applying a concept developed by Arzac and Glosten (2005) and I show *under what circumstances the VTS of Fernandez (2004a) is appropriate to use and how it expands the existent theories.*

We know from equation (29) that the VTS is the difference between G_U and G_L, which are the present values of the two following cash flows:

$$Taxes_{U,t} = (FCF_t + \Delta A_t)\tau_C / (1-\tau_C) \text{ and} \tag{60}$$

$$Taxes_{L,t} = (ECF_t + \Delta A_t - \Delta D_t)\tau_C / (1-\tau_C), \tag{61}$$

where $\Delta A_t = -DEP_t + INV_t$, which is the increase in net assets (in book values) in period t.

In order to derive the correct present value of the cash flows, we must know the appropriate discount rate. To avoid arguments about the appropriate discount rate, a pricing kernel is applied (also called stochastic discount factor). It is assumed that the price of an asset that pays a random amount x_t at time t is the sum of the expectation of the product of the random amount (x_t) and the pricing kernel for time t cash flows (M_t) (Arzac and Glosten 2005). Thus:

[33] For the derivation of this result (that Cooper and Nyborg 2006b do not show), see Appendix E.

$$P_x = \sum_{t=1}^{\infty} E[M_t x_t]\,. \tag{62}$$

I can then derive the VTS as the difference between the present values of equation (60) and (61) by applying the pricing kernel, without worrying about the correct discount rate for each cash flow. The general result for the VTS is then:

$$VTS = \sum_{t=1}^{\infty} E[M_t Taxes_{U,t}] - \sum_{t=1}^{\infty} E[M_t Taxes_{L,t}]\,, \tag{63}$$

which is equal to:

$$VTS = \frac{\tau_C}{1-\tau_C}\left[\sum_{t=1}^{\infty} E[M_t FCF_t] - \sum_{t=1}^{\infty} E[M_t ECF_t] + \sum_{t=1}^{\infty} E[M_t \Delta D_t]\right]. \tag{64}$$

Equation (64) can be rewritten as:

$$VTS = \frac{\tau_C}{1-\tau_C}\left[V_U - E + \sum_{t=1}^{\infty} E[M_t \Delta D_t]\right]. \tag{65}$$

Knowing from equation (5) that $V_U - E = D - VTS$, we get:

$$VTS = \frac{\tau_C}{1-\tau_C}\left[D - VTS + \sum_{t=1}^{\infty} E[M_t \Delta D_t]\right]. \tag{66}$$

Simplifying the above leads to:

$$VTS = \tau_C D + \tau_C \sum_{t=1}^{\infty} E[M_t \Delta D_t]\,. \tag{67}$$

Equation (67) is a general result for the VTS and it can be seen that the VTS is only dependent on the nature of the stochastic process of the net change in debt (Arzac and Glosten 2005 and Fernandez 2005a).

It is important to note that the value of the net change in debt, $\sum_{t=1}^{\infty} E[M_t \Delta D_t]$, depends on whether ΔD_t is correlated with the market portfolio and how strong the correlation is. In other words, the value of the net change in debt depends on the systematic risk of ΔD_t in a CAPM world. If ΔD_t is positively correlated with the market, the value of the net change in debt is less than if it is uncorrelated and has no systematic (or priced) risk so that $\sum_{t=1}^{\infty} E_t[M_t \Delta D_t] < \sum_{t=1}^{\infty} E_t[\Delta D_t]/(1+r_f)^t$. Consequently, if a constant perpetuity with $g = 0$

and $E_t[\Delta D_t] = 0$ is assumed and if ΔD_t is positively correlated with the market, the value of the net change in debt is $\sum_{t=1}^{\infty} E_t[M_t \Delta D_t] < 0$. Hence, the VTS of this constant perpetuity inferred from equation (67) must be $VTS < \tau_C D$ (Arzac and Glosten 2005). This is exactly the result I derived in section 5.1 for a constant perpetuity in the ME setting and that is shown in Figure 2 in section 3.4 for a firm following the ME debt policy. The results must be identical, because the assumptions in sections 3.4 and 5.1 implied a net change in debt that is stochastic and has systematic risk, since I assumed a constant leverage ratio (which implies perfect correlation with firm value) and firm value with systematic risk following a random walk.

Given equation (67), Fernandez (2005b) specifies the stochastic cash flow process he assumes to derive the VTS for *constant growing perpetuities*:

$$FCF_{t+1} = FCF_t (1+g)(1+\varepsilon_{t+1}), \tag{68}$$

with $E_t(\varepsilon_{t+1}) = 0$ and $E_t[M_{t,t+1}\varepsilon_{t+1}] < 0$, which implies systematic risk. $M_{t,t+1}$ is the one-period pricing kernel at time t for cash flows at time t+1. Note that the assumed cash flow process by Fernandez (2005b) is equivalent to equation (51), the cash flow process under the ME assumption.

Further, Fernandez (2005b, 2007b) assumes a debt policy where the firm follows a constant leverage ratio *in book values*, instead of a constant leverage ratio in market values as in the ME setting. Thus, he assumes the following relationship, which describes the assumed debt policy that the leverage ratio is constant in book values.

$$D_t = K * Ebv_t, \tag{69}$$

where Ebv is the book value of equity and K a constant. Equation (69) implies that the increase in debt[34] is equal to the increase of book value of equity:

$$\Delta D_t = K * \Delta Ebv_t. \tag{70}$$

In addition, we know that in terms of book values, the increase in net assets is equal to the increase in equity plus the increase in debt:

$$\Delta A_t = \Delta Ebv_t + \Delta D_t. \tag{71}$$

[34] Since a constant growing perpetuity is assumed, the net change in debt (ΔD) must be an increase in debt.

Solving equations (70) and (71) for ΔEbv_t and setting them equal to one another, we get:

$$\Delta A_t - \Delta D_t = \Delta D_t / K \,. \tag{72}$$

Solving for ΔD_t, it can be seen that the increase in debt is proportional to the increase in net assets and hence they must both have the same risk (Fernandez 2005b).

$$\Delta D_t = \Delta A_t /(1 + 1/ K)\,. \tag{73}$$

It is assumed that the increase in net assets follows the stochastic process defined by:

$$\Delta A_{t+1} = \Delta A_t (1 + g)(1 + \phi_{t+1})\,, \tag{74}$$

where ϕ_{t+1} is a random variable with $E_t(\phi_{t+1}) = 0$ but $E_t(M_{t,t+1}\phi_{t+1}) < 0$, which implies systematic or priced risk (Fernandez 2005b).

Then, by assuming that the firm follows a constant leverage ratio in book values, which implies that the risk of ΔD_t is equal to the risk of ΔA_t, Fernandez (2005a, 2005b) derives the following result for the value of the net change in debt for constant growing perpetuities:

$$\sum_{t=1}^{\infty} E[M_t \Delta D_t] = \frac{gD}{\alpha - g}\,, \tag{75}$$

where α is the appropriate discount rate for the expected increase in net assets and under a constant book-value leverage ratio equal to the appropriate discount rate for ΔD_t. Substituting equation (75) in (67), the VTS we obtain is:

$$VTS = \frac{\tau_C \alpha D}{\alpha - g}\,. \tag{76}$$

If it is now further assumed that *the risk of ΔA_t is equal to the risk of the FCF*, which implies that $\phi_{t+1} = \varepsilon_{t+1}$ and hence ΔA_t is proportional to FCF_t, then the appropriate discount rate is $\alpha = \rho$ for the expected increase in net assets (Fernandez 2005b). Then it must follow:

$$\sum_{t=1}^{\infty} E[M_t \Delta D_t] = \frac{gD}{\rho - g} \text{ and} \tag{77}$$

$$VTS = \frac{\tau_C \rho D}{\rho - g}. \tag{78}$$

Equation (78) is equal to equation (F28') of Fernandez (2004a).

Note that equation (78) only holds if the increase in debt is as risky as the free cash flows, which is a special case and depends not only on the assumption of a constant book-value leverage ratio but also on the assumption that the increase in net assets is as risky as the free cash flows. If a constant book-value leverage ratio is assumed and the risk of ΔA_t is different from the risk of the free cash flows, equation (76) holds with a discount rate of α, reflecting the risk of ΔA_t (Fernandez 2005a). Note that α is difficult to apply in practice since the risk of the increase in net assets (ΔA_t) is difficult to measure.

It should be noted further that equation (78) is only valid for constant growing perpetuities. If constant perpetuities with $g = 0$ are assumed, we would obtain $VTS = \tau_C D$. However, this is not correct because equation (68) assumes a stochastic FCF process with systematic risk, which implies that ΔD_t must have systematic risk too, since ΔD_t is as risky as the free cash flows. We know from above, however, that $\sum_{t=1}^{\infty} E_t[M_t \Delta D_t] < 0$ when ΔD_t has systematic risk (positively correlated with the market). Substituting this result in equation (67), it can be seen that $VTS < \tau_C D$ for perpetuities with $g = 0$, under the assumptions made by Fernandez (2005a, 2005b).

I conclude that equation (F28') is only valid for constant growing perpetuities with a stochastic cash flow process subject to systematic risk as in the ME setting and with a constant book-value leverage ratio debt policy. Further, the risk of the increase in net assets must be equal to the risk of the free cash flows. If the last assumption is not given, formula (76) should be used, although it is difficult to apply in practice. Finally, note that the VTS for constant growing perpetuities derived by Fernandez (2005a, 2005b, 2007b) under the assumptions given in this section expands the existent valuation theories <u>but</u> it does not contradict or correct them.

6 Conclusion

The debate in the finance literature about the correct value of debt tax shields (VTS) is ongoing. One thread of the literature suggests that the VTS be computed by discounting the debt tax savings to the present using a discount rate that reflects the risk of the debt tax savings. The difference across the various theories in this literature lies primarily in the assumed debt policy. MM (1963) and Myers (1974) suggest that the debt tax savings be discounted at the cost of debt when future debt levels are fixed initially, whereas ME (1980) and Harris and Pringle (1985) propose to discount the debt tax savings at the unlevered cost of capital when the firm maintains a constant leverage ratio in terms of market values.[35]

In contrast to this literature, Fernandez (2004a) derives the value of tax shields by computing the difference between the present value of taxes for the unlevered company and the present value of taxes for the levered company. Fernandez' (2004a) results challenge extant valuation theories as he claims that the VTS he derives is generally valid and in fact the only way to compute the correct VTS.

Fernandez' (2004a) claim is not consistent with his set of assumptions, however. It has been shown in this paper that Fernandez' VTS for growing perpetuities is not consistent with the assumption of a firm following a ME- or MM-style debt policy. In addition, the VTS for perpetuities with zero growth must be less than the value that Fernandez derives if the firm is following a ME debt policy and its free cash flows have systematic risk. Consequently, Fernandez' VTS is not generally valid.

In addition, I have shown that the following set of assumptions must hold if the VTS for growing perpetuities by Fernandez is to be valid: the expected growth rate of the firm is greater than zero, the firm follows a stochastic free cash flow process subject to systematic risk and the firm applies a debt policy that follows a constant leverage ratio in terms of book values. Further, the increase in net assets must be as risky as the free cash flows of the firm.

In this paper, it has also been shown that Fernandez' approach of computing the VTS as the difference between two present values does not lead to results that are different from those obtained when directly computing the present value of the debt tax savings, as suggested by the standard literature, if the same assumptions are made. Consequently,

[35] Note that Harris and Pringle (1985) discount all debt tax shields at the unlevered cost of capital, whereas ME (1980) discount the first-period debt tax shield at the cost of debt and all subsequent periods' tax shields at the unlevered cost of capital.

Fernandez' VTS for growing perpetuities expands the existent valuation theories by using an additional set of assumptions that includes a new debt policy, but it does not correct or contradict the standard valuation theories of MM (1963), Myers (1974), ME (1980) or Harris and Pringle (1985), for instance.

Taken together, these findings indicate that it is crucial to explicitly specify at the outset the assumptions one is using in valuing a company and its tax shields, be it in theory or in practice. In particular, the debt policy applied by the firm must be specified in order to correctly choose from among the different valuation theories.

This paper's findings also point to the importance of the choice of valuation method used. It is essential to choose the method that will be less prone to error and more likely to provide a correct estimate of the value of the firm (including the VTS). In general, the different valuation methods should give the same result provided that the same assumptions are used, which makes sense since they analyze the same reality under the same hypotheses. However, as I have shown, the capital cash flow method does not lead always to results that are consistent with the other methods, since it includes implicit assumptions about the risk of the debt tax shield in its valuation.

The following table summarizes on the vertical the different valuation theories as well as the assumptions made by the capital cash flow method and on the horizontal the three different valuation methods analyzed in this paper. A ☼ indicates which method is likely to work best under a given set of assumptions, a √ indicates whether a valuation method is consistent with a certain set of assumptions but difficult to apply in practice and an X indicates which methods are inconsistent with the given assumptions.

Table 2
Which Valuation Method Should Be Used Under Which Set of Assumptions?

Assumptions				Methods		
Assumption name	Growth characteristics	Leverage policy	Future tax status	APV	FCF	CCF
MM (1963)	zero growth	constant and permanent amount of debt	always tax-paying	☼	√	X
Extended MM (Myers 1974)	any growth pattern	future debt levels are fixed initially	always tax-paying	☼	√	X
ME (1980) with continuous rebalancing	any growth pattern	constant *market-value* leverage ratio	always tax-paying	√	☼	√
Fernandez (2004a)	constantly growing	constant *book-value* leverage ratio	always tax-paying	☼	√	X
Capital cash flow (Ruback 2002)	any growth pattern	any debt policy	uncertain; it is assumed that the risk of tax savings is equal to risk of free cash flows	√	√	☼

Based upon: Cooper and Nyborg (2007)

The table shows that the APV and FCF methods can be used with every set of assumptions if they are applied correctly and both should give the same results as long as the same assumptions are used. The FCF method works best for firms that have a long-run target capital structure, which is in line with the assumption of a constant leverage ratio in terms of market values.[36] The FCF method is difficult to apply however, and likely to produce errors if the capital structure changes over time, since the WACC must be adjusted each period to reflect the changes in capital structure.

In the case that the future tax status of the firm is uncertain and the company could become non-taxpaying, the CCF method might be reasonably used as an approximation, if

[36] If Fernandez (2004a) is applied and the increase in net assets is as risky as the free cash flows and the firm is assumed to grow at a constant rate, then the WACC stays constant and the FCF method is also simple to apply as shown by Fernandez (2007a).

one is willing to assume that the risk of the tax savings is equal to the risk of the free cash flows of the firm.

For all other sets of assumptions, the APV method is the best choice since it lays out each component of value separately and is thus highly transparent. It is particularly useful when future debt levels are fixed initially and the capital structure changes significantly over time, as it is likely to be the case in an LBO, for example.

To conclude the above discussion, one should ask two fundamental questions before starting to value a company:

1. Which set of assumptions (valuation theory) is closest to the actual situation the company is facing?
2. Which valuation method works best given the chosen set of assumptions?

The difficult task is then to identify the set of assumptions that is closest to the actual situation of a specific company.

Appendix

Appendix A: Beta of Debt Tax Shields equals Beta of Debt (based upon Ruback 2002)

Assumption: Debt is perpetual and fixed as a dollar amount, B, which does not change as the value of the firm changes.
Then the value of the debt tax shield is

$$PVTS_t = \frac{\tau_C \overline{r_D} B}{r_{D,t}}, \tag{A1}$$

where $\overline{r_D}$ is the fixed yield on the debt and $r_{D,t}$ is the cost of debt in period t, which is determined by equation (8).
The value of the debt can change through time since $\overline{r_D}$ is fixed and the cost of debt might change. The value of debt in period t (D_t) is

$$D_t = \frac{\overline{r_D} B}{r_{D,t}}. \tag{A2}$$

By substituting equation (A2) into equation (A1), the value of the debt tax shield at time t can be expressed as

$$PVTS_t = \tau_C D_t. \tag{A3}$$

Using a modified definition of beta, the beta of the debt tax shield, β_{DTS}, equals

$$\beta_{DTS} = \frac{Cov(PVTS_t, R_M)}{V_{DTS,t-1} Var(R_M)}. \tag{A4}$$

By substituting equation (A3) into equation (A4) and simplifying (τ_C disappears since constant)

$$\beta_{DTS} = \frac{Cov(D_t, R_M)}{V_{D,t-1} Var(R_M)} = \beta_{Debt}. \tag{A5}$$

Appendix B: The derivation of the value of tax shields (VTS) for growing perpetuities by Fernandez (2004a)

For comparison, the numbering of the equations in Appendix B is taken from Fernandez (2004a).

Knowing that:

$$E_L + D = V_U + VTS\,. \qquad \text{(F1)}$$

Notice further that the unlevered value of a firm with an expected growth rate of g is derived by

$$V_U = FCF/(\rho - g)\,. \qquad \text{(F29)}$$

By substituting equation (F29) in (F1), we get

$$E_L + D = FCF/(\rho - g) + VTS\,. \qquad \text{(F30)}$$

The relation between the ECF and the FCF is

$$FCF = ECF + r_D D(1-\tau_C) - gD\,. \qquad \text{(F31)}$$

By substituting (F31) in (F30), we get

$$E_L + D = [ECF + r_D D(1-\tau_C) - gD]/(\rho - g) + VTS\,. \qquad \text{(F32)}$$

As for a levered company $ECF = E_L(r_E - g)$, (F32) may be rewritten as

$$E_L + D = [E(r_E - g) + r_D D(1-\tau_C) - gD/(\rho - g) + VTS\,. \qquad \text{(F33)}$$

Multiplying both sides of equation (F33) by $(\rho - g)$, we get

$$(E_L + D)(\rho - g) = [E(r_E - g) + r_D D(1-\tau_C) - gD] + VTS(\rho - g)\,. \qquad \text{(F34)}$$

Eliminating $-g(E_L + D)$ on both sides of equation (F34), we have

$$(E_L + D)\rho = [r_E E + r_D D(1-\tau_C) + VTS(\rho - g)\,. \qquad \text{(F35)}$$

Equation (F35) can be rewritten as

$$D[\rho - r_D(1-\tau_C)] - E_L(r_E - \rho) = VTS(\rho - g)\,. \tag{F36}$$

Dividing both sides of equation (F36) by D (debt value), we get,

$$[\rho - r_D(1-\tau_C)] - (E_L / D)(r_E - \rho) = (VTS / D)(\rho - g)\,. \tag{F37}$$

If (E_L / D) is constant, the left-hand side of equation (F37) does not depend on growth (g) because for any growth rate (E_L / D), ρ, r_D and r_E are constant. We know that for $g = 0$, $VTS = \tau_C D$.

Then, equation (F37) applied to constant perpetuities $(g = 0)$ is

$$[\rho - r_D(1-\tau_C)] - (E_L / D)(r_E - \rho) = \tau_C \rho\,. \tag{F38}$$

Subtracting (F38) from (F37), we get

$0 = (VTS / D)(\rho - g) - \tau_C \rho$. Solving for VTS, we get

$$VTS = \frac{\tau_C \rho D}{(\rho - g)}\,. \tag{F28'}$$

Appendix C: The derivation of the value of tax shields for growing perpetuities under the assumption of a MM debt policy with growth

The proof I show is based upon Bodie, Kane and Marcus (2008) where it is used to derive the constant-growth dividend discount model.

It is shown in section 3.2, that if future debt levels are fixed initially, the debt tax shield has the same risk as the debt. Hence, the debt tax shields must be discounted at the cost of debt.

$$VTS = \frac{\tau_C r_D D}{1+r_D} + \frac{\tau_C r_D D(1+g)}{(1+r_D)^2} + \frac{\tau_C r_D D(1+g)^2}{(1+r_D)^3} + \ldots \tag{A6}$$

Multiplying both sides of equation (A6) by $(1+r_D)/(1+g)$, we get

$$\frac{(1+r_D)}{(1+g)} VTS = \frac{\tau_C r_D D}{(1+g)} + \frac{\tau_C r_D D}{(1+r_D)} + \frac{\tau_C r_D D}{(1+r_D)^2} \ldots \tag{A7}$$

Subtracting equation (A6) from equation (A7), we obtain

$$\frac{(1+r_D)}{(1+g)} VTS - VTS = \frac{\tau_C r_D D}{(1+g)}, \tag{A8}$$

which implies

$$\frac{(r_D - g)}{(1+g)} VTS = \frac{\tau_C r_D D}{(1+g)}. \tag{A9}$$

Solving for VTS, we find that

$$VTS = \frac{\tau_C r_D D}{(r_D - g)}. \tag{A10}$$

Appendix D: The derivation of the value of tax shields for growing perpetuities in a ME setup when $VTS = \tau_C r_D D / \rho$ for $g = 0$

If a ME setting is assumed, Fernandez' (2004a) analysis is correct up to equation (F37). Thereafter, he must use $VTS = \tau_C r_D D / \rho$ for $g = 0$, as explained in section 5.1. The result that must follow for growing perpetuities in a ME setup is shown below.

$$[\rho - r_D(1 - \tau_C)] - (E_L / D)(r_E - \rho) = (VTS / D)(\rho - g) \tag{F37}$$

Assuming a ME setting, $VTS = \tau_C r_D D / \rho$ for $g = 0$. Then, equation (F37) applied to perpetuities ($g = 0$) is

$$[\rho - r_D(1 - \tau_C)] - (E_L / D)(r_E - \rho) = \tau_C r_D \tag{A11}$$

Subtracting (A 11) from (F 37), we get

$0 = (VTS / D)(\rho - g) - \tau_C r_D$, solving for VTS, we find that

$$VTS = \frac{\tau_C r_D D}{\rho - g}. \tag{A12}$$

This is equal to equation (46) that is the present value of tax shields for growing perpetuities in a ME setting as shown in section 4.2.1.

Appendix E: The derivation of the value of tax shields for growing perpetuities when the dependence of r_E on g is allowed for.

The following formula from Copeland, Koller and Murrin (2000, 475) is used when $g > 0$ so that the formula allows for the dependence of r_E on g.

$$r_E(g) = \rho + (\rho - r_D)\frac{D}{E_L}\left[1 - \left(\frac{r_D}{r_D - g}\right)\tau_C\right]. \tag{A13}$$

Hence, if $g = 0$, (A13) simplifies to

$$r_E = \rho + (\rho - r_D)\frac{D}{E_L}[1 - \tau_C]. \tag{A14}$$

Since equation (F37) assumes $g > 0$, I substitute (A13) for r_E in (F37) and I obtain

$$(\rho - r_D(1 - \tau_C) - (\rho - r_D)\left[1 - \left(\frac{r_D}{r_D - g}\right)\tau_C\right] = (VTS / D)(\rho - g). \tag{A15}$$

And since equation (F38) assumes $g = 0$, I substitute (A14) for r_E in (F38) and I obtain

$$(\rho - r_D(1 - \tau_C) - (\rho - r_D)[1 - \tau_C] = \tau_C \rho. \tag{A16}$$

As in Fernandez (2004a), subtracting (A16) from (A15), I get

$$-(\rho - r_D)\left[1 - \left(\frac{r_D}{r_D - g}\right)\tau_C\right] + (\rho - r_D)(1 - \tau_C) = (VTS / D)(\rho - g) - \tau_C \rho. \tag{A17}$$

(A17) can be rewritten as

$$(\rho - r_D)\left(\frac{r_D}{r_D - g}\right)\tau_C - (\rho - r_D)\tau_C = (VTS / D)(\rho - g) - \tau_C \rho. \tag{A18}$$

(A18) can be rewritten as

$$(\rho - r_D)\left[\left(\frac{r_D}{r_D - g}\right) - 1\right]\tau_C = (VTS / D)(\rho - g) - \tau_C \rho. \tag{A19}$$

By substituting $\frac{r_D - g}{r_D - g}$ for 1 in (A19), I get

$$(\rho - r_D)\left(\frac{g}{r_D - g}\right)\tau_C = (VTS / D)(\rho - g) - \tau_C \rho \,. \tag{A20}$$

(A20) can be rewritten as

$$(\rho * g - r_D * g)\tau_C /(r_D - g) = (VTS / D)(\rho - g) - \tau_C \rho \,. \tag{A21}$$

Adding $\tau_C \rho$ on both sides of equation (A21) and rearranging, I get

$$\big((\rho * g - r_D * g + \rho * r_D - \rho * g)/(r_D - g)\big)\tau_C = (VTS / D)(\rho - g) \,. \tag{A22}$$

(A22) can be rewritten as

$$\tau_C r_D (\rho - g)/(r_D - g) = (VTS / D)(\rho - g) \,. \tag{A23}$$

Solving for VTS, I obtain

$$VTS = \frac{\tau_C r_D D}{r_D - g} \,. \tag{A24}$$

Equation (A24) is equivalent to equation (47), which is the value of tax shields, derived by Cooper and Nyborg (2006b), for growing perpetuities if the MM debt policy with growth is assumed. Therefore, I have shown that if the dependence of r_E on g is allowed for in the derivation of Fernandez (2004a), we obtain the same result as Cooper and Nyborg (2006b).

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