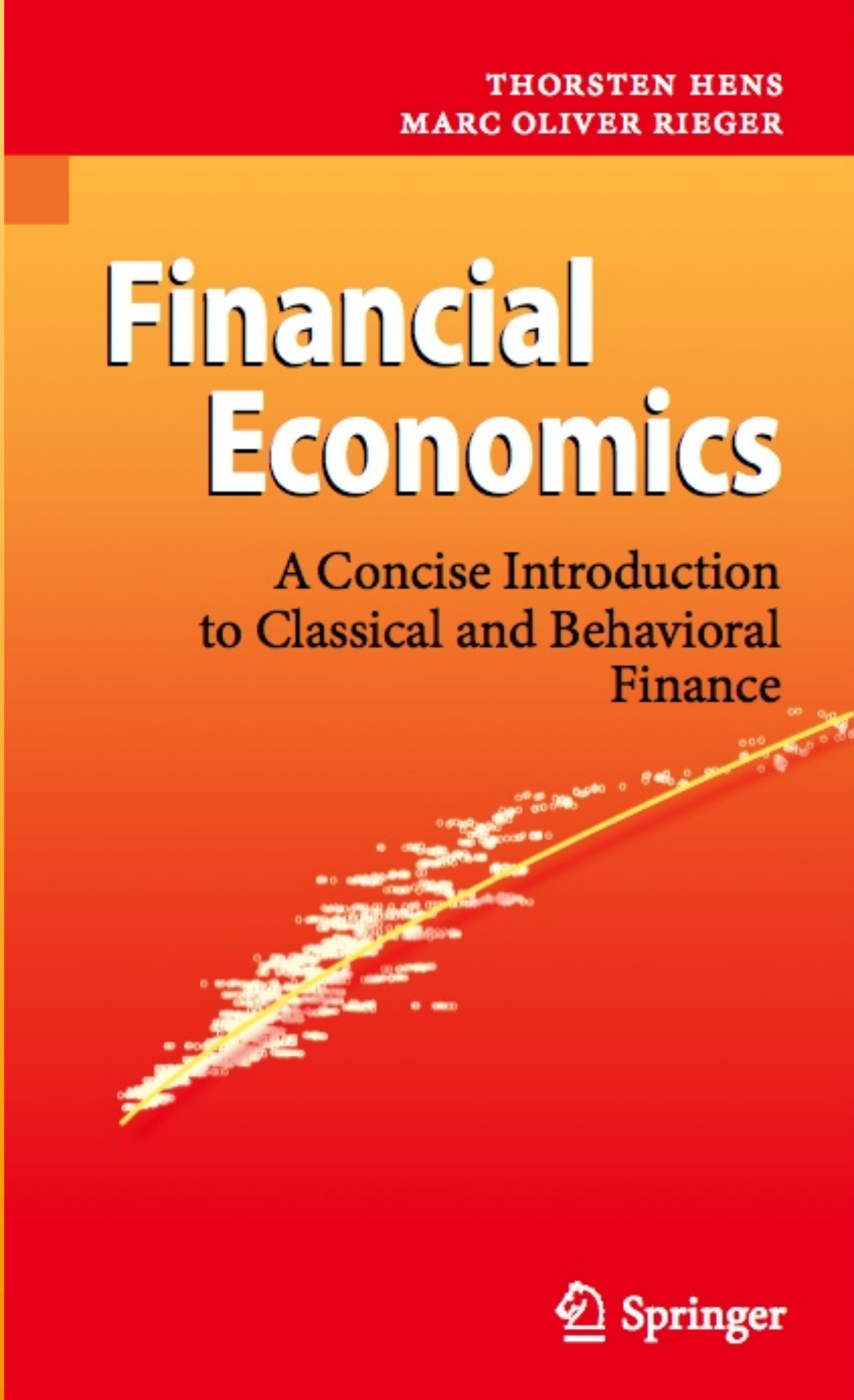

HENS · RIEGER
Financial Economics

Financial economics is a fascinating topic where ideas from economics, mathematics and, most recently, psychology are combined to understand financial markets. This book gives a concise introduction into this field and includes for the first time recent results from behavioral finance that help to understand many puzzles in traditional finance. The book is tailor made for master and PhD students and includes tests and exercises that enable the students to keep track of their progress. Parts of the book can also be used on a bachelor level. Researchers will find it particularly useful as a source for recent results in behavioral finance and decision theory.

The text book to this class is available at www.springer.com

On the book's homepage at www.financial-economics.de there is further material available to this lecture, e.g. corrections and updates.

Financial Economics

A Concise Introduction to Classical and Behavioral Finance
Chapter 6

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Theory of the Firm*

"The great difference between the industry of today as compared to that of yesterday is what might be referred to as the necessity of the scientific approach, the elimination of operation by hunches." Alfred P. Sloan

Theory of the Firm

- We will now extend the financial economy $\mathcal{E}_F$ to cover problems of production and production units, i.e. firms.
- This allows conclusions about the behavior of firms in markets.
- So far we assumed bond payoffs to be exogenous, ignoring the decision-making process of the bonds' issuers.

Basic Model

- The model builds on the two-period model introduced in Sec. 4.1.
- In time $t = 0$ a set of bonds $\mathcal{K} := \{1, \ldots, K\}$ can be traded; in time $t = 1$ they payoff in dependence of the state of the world.
- Formally, there are $S + 1$ states of the world, where $s = 0$ corresponds to $t = 0$ and in $t = 1$ one state $s \in \{1, ..., S\}$ is realized.

Households and Firms

- In this chapter we consider two types of economic agents:
  - A set of households $\mathcal{I} := \{1, \ldots, I\}$ (or agents in the narrower sense).
  - A set of firms $\mathcal{J} := \{1, \ldots, J\}$.
- A household's main interest lies in consumption.
- Let $\mathcal{X}^i \subset \mathbb{R}^{S+1}$ be a set of consumption plans, where $x^i \in \mathcal{X}^i$ describes agent i's consumption for each state s.
- Let for a household i the mapping $U^i : \mathcal{X}^i \to \mathbb{R}$ represent its utility.

Households and Firms

- The firms' genuine task is to produce.
- Each firm $j \in \mathcal{J}$ is characterized by its exogenous production technology $\mathcal{Y}^j \subseteq \mathbb{R}^{S+1}$.
- The net output is denoted by $y^j := (y_0^j, \ldots, y_S^j) \in \mathcal{Y}^j$, where $y_s^j > 0$ is the net output of firm j in state s. $y_s^j < 0$ is interpreted as the net input.
- Let $\mathcal{Y} := \bigcup_{j \in \mathcal{J}} \mathcal{Y}^j$ be the set of all production capabilities and let $Y \in \mathbb{R}^{(S+1) \times J}$ be a production matrix of the entire economy.

Households and Firms

Assumption (Production technology set)

We make the following assumptions on the production technology set $\mathcal{Y}^j$ of a firm:

(i) $\mathcal{Y}^j \subset \mathbb{R}^{S+1}$ *is closed,*

(ii) $\mathcal{Y}^j$ *is convex,*

(iii) $\mathcal{Y}^j \supset \mathbb{R}^{S+1}_{\leq 0}$,

(iv) $\mathcal{Y}^j \cap \mathbb{R}^{S+1}_{\geq 0} = \{0\}$,

(v) *for all* $\omega \in \mathbb{R}^{S+1}_{\geq 0}$ *we have* $(\omega + \mathcal{Y}) \cap \mathbb{R}^{S+1}_{\geq 0}$ *compact.*

Households and Firms

(i) has only technical character.

(ii) implies that the technologies have non-increasing returns to scale.

(iii) means on the one hand that $0 \in \mathbb{R}^{S+1}$ belongs to the technology of every firm; i.e., it has the option of producing nothing.
On the other hand every firm can freely dispose its resources: a given output does not have to be produced with minimal possible input.

(iv) is a kind of no arbitrage condition (NAC) for the production: a positive output in one state can only be produced by investing a strictly positive input in another.

(v) formalizes the limitations of production: given the resources in an economy, production is bounded; which implies that for any price system q cash-flows are bounded.

Financial Market

- Both types of economic agents actively participate in the bond market.
- Let $\mathcal{Z}^i$ be the set of a household's possible portfolios, i.e., its portfolio space.
- An element $z^i := (z^i_1, \ldots, z^i_K) \in \mathcal{Z}^i$ describes its position on the bond market, where z^i_k denotes the position for the k-th bond.
- Let $\mathcal{Z} := \bigcup_{i \in \mathcal{I}} Z^i$ be the set of all portfolio spaces.
- Similarly, we denote by $\mathcal{P}^j \subset \mathbb{R}^K$ a firm's portfolio space, with an element $\xi^j := (\xi^j_1, \ldots, \xi^j_K) \in \mathcal{P}^j$.
- We call a firm's portfolio ξ^j its financial policy to distinguish it from a household's portfolio.
- Let $\mathcal{P} := \bigcup_{j \in \mathcal{J}} P^j$ denote the set of all portfolio spaces.

Financial Market

- In time $t = 0$ the set of bonds $\mathcal{K} := \{1, \ldots, K\}$ are traded at some prices $q \in \mathbb{R}^K$.
- Each bond's payoff can be described by a $S \times K$ matrix with entries A_s^k denoting the payoff of bond k in state s.
- We will distinguish between *non-incorporated* and *incorporated companies*.
- In the context of non-incorporated companies, the agents exogenously hold firm shares, and cannot trade them.
- On the other hand, in an economy with incorporated companies these shares are tradable on a market.
- This distinction leads to two variants of the set up.

Financial Market

- In the benchmark case the financial market A only consists of bonds.
- Firm shares are held by households who do not sell them. Let $\bar{\delta}^i = (\bar{\delta}^i_1, \ldots, \bar{\delta}^i_J) \in \mathbb{R}^J_{\geq 0}$ denote the initial distribution of firm shares of agent i, where $\bar{\delta}^i_j$ is its share of firm j.
- In the case of incorporated companies firm shares are tradeable (in a stock market).
- The market for firm shares is assumed to be open to both, households and firms.
- Let $\mathcal{H}^i \subset \mathbb{R}^J$ be a household's portfolio space on this market.
- An element $h^i = (h^i_1, \ldots, h^i_J)$ means that the i-th agent holds h^i_j shares in the j-th firm.

Financial Market

- As in the benchmark case, we assume that any household is endowed with some portfolio $\bar{\delta}^i$, which can be sold now.
- Since there are no transaction costs, we let the agents sell their stocks $\bar{\delta}^i$, and then demand δ^i on the market.
- The firms now may also buy shares of other firms, so that they are connected by cross ownership.
- We assume they do not hold any such shares at the beginning. Let $\mathcal{T}^j$ be the set of possible firm shares.
- As before, $\mathcal{T}^j \subseteq \mathbb{R}^J$.
- Let $\mathcal{T} := \bigcup_{j \in \mathcal{J}} \mathcal{T}^j$ be the set of firm interdependencies.

Financial Market

- The dependencies can be described from a firm's point of view by vectors $\tau^j = (\tau^j_1, \ldots, \tau^j_J)$, meaning the j-th firm holds τ^j_l shares in the l-th firm.
- In period $t = 0$ firm shares are traded at some prices $p \in \mathbb{R}^J$.
- If traded or not, holding firm shares leads to dividends in both periods.
- Let the matrix $D := (d^1, \ldots, d^J) \in \mathbb{R}^{(S+1)\times J}$ describe the dividend of all firms (in any state).
- Dividends can also be negative which would mean a liability (to subsequent payment).

Example: Dividend policy

- Consider a firm j and only two states $s = \{0, 1\}$.
- For a given production decision $y^j \in \mathcal{Y}^j$ and a given financial policy $\xi^j \in \mathcal{P}^j$ it might fix the following dividends:

$$d^j = y^j + \begin{pmatrix} -q' \\ A \end{pmatrix} \xi^j$$

that is $d_0^j = y_0^j - \sum_{k \in \mathcal{K}} q^k \xi_k^j$ and $d_1^j = y_1^j - \sum_{k \in \mathcal{K}} A_s^k \xi_k^j$.

- Differing financial policies determine different dividends, as can be seen in the following cases.

Example: Dividend policy

- 100% equity finance: $\xi^j = 0$

$$d^j = y^j \quad \rightarrow \quad d_0^j = y_0^j \quad \text{complete equity finance}$$
$$\rightarrow \quad d_1^j = y_1^j \quad \text{full risk}$$

- 100% bonded capital finance: $-q'\xi^j = y_0^j$

$$d^j = y^j + \begin{pmatrix} -q' \\ A \end{pmatrix} \xi^j \quad \rightarrow \quad d_0^j = 0$$
$$\rightarrow \quad d_1^j = y_1^j + A\xi^j$$

- 100% risk coverage: $A\xi^j = -y_1^j$

$$d^j \quad \rightarrow \quad d_0^j = y_0^j - q'\xi^j$$
$$\rightarrow \quad d_1^j = 0.$$

Financial Economy with Production

Definition (Financial Economy with Production $\mathcal{E}_F^P$)

A financial economy with production
$\mathcal{E}_F^P(\mathcal{I}, \mathcal{X}, U, \omega, \mathcal{Z}, \bar{\delta}, \mathcal{J}, \mathcal{Y}, \mathcal{P}, [\mathcal{H}, \mathcal{T}], A)$ *is an economy with*

(i) *I economical agents, their consumption spaces $\mathcal{X}$, utility functions U, initial distributions of goods ω, initial distribution of firm shares $\bar{\delta}$, portfolio spaces $\mathcal{Z}$ (and corresponding firm share portfolios $\mathcal{H}$);*

(ii) *J firms with technologies $\mathcal{Y}^j$, portfolio spaces $\mathcal{P}$ (and corresponding firm share portfolios $\mathcal{T}$);*

(iii) *and a financial market A for bonds (and shares).*

Financial Economy with Production

- We need to restate a few already known concepts in the setting of a financial economy with production.
- Let $\phi_{\mathcal{Y}}, \phi_{\mathcal{P}}$, and $\phi_{\mathbb{T}}$ be the corresponding production allocation, financial policy allocation and share allocation (respectively) of the firms.

Definition (Achievability)

An allocation of goods $\phi_{\mathcal{X}}$ and a production allocation $\phi_{\mathcal{Y}}$ are achievable, if

(i) $x^i \in \mathcal{X}^i$ *for all* i,

(ii) $y^j \in \mathcal{Y}^j$ *for all* j,

(iii) $\sum_{i \in \mathcal{I}}(x^i - \omega^i) \leq \sum_{j \in \mathcal{J}} y^j$.

Budget Restriction / Households' Decisions and Firms' Decisions

- At the beginning of the first period, the agents and firms plan all traded and produced quantities for all $S+1$ states.
- This means that at $t=0$, not only the production plans for $t=1$ have to be set, but in particular their financing has to be guaranteed in advance.
- This set-up leads to a list of budget restrictions. We will keep the distinction between incorporated and non-incorporated firms, because it plays an important role in the description of the budget restrictions.

Non-Incorporated Companies

- Since firm shares are not tradeable, households' hold just their initial firm shares $h^i = \bar{\delta}^i$.
- At time $t = 0$, agent i's expenses for consumption x_0^i and investment in bonds z_k^i may not exceed the value of its initial assets ω_0^i and its dividends $\sum_{j=1}^J d_0^j \bar{\delta}_j^i$ from its firm shares, i.e. at $t = 0$,

$$x_0^i + \sum_{k \in \mathcal{K}} q^k z_k^i \leq \omega_0^i + \sum_{j \in \mathcal{J}} d_0^j \bar{\delta}_j^i. \tag{1}$$

- At $t = 1$, in a given state s, the agent gains the payoff of its portfolio and the dividends which can all be used for consumption.

Non-Incorporated Companies

- Hence the budget restriction for $t = 1$ are

$$x_s^i \leq w_s^i + \sum_{k \in \mathcal{K}} A_s^k z_k^i + \sum_{j \in \mathcal{J}} d_s^j \bar{\delta}_j^i. \qquad (2)$$

- We combine the two conditions (1) and (2), where the latter shall hold for all states $s \geq 1$:

$$\mathbb{B}^i(q, \omega^i, \bar{\delta}^i, A, D)$$
$$:= \left\{ \begin{array}{c} (x^i, z^i) \\ \in \mathcal{X}^i \times \mathcal{Z}^i \end{array} \middle| \begin{array}{l} x_0^i + \sum_{k \in \mathcal{K}} q^k z_k^i \leq \omega_0^i + \sum_{j \in \mathcal{J}} d_0^j \bar{\delta}_j^i \\ x_s^i \leq w_s^i + \sum_{k \in \mathcal{K}} A_s^k z_k^i + \sum_{j \in \mathcal{J}} d_s^j \bar{\delta}_j^i, \quad \text{for all } s \geq 1 \end{array} \right\}.$$

Non-Incorporated Companies

- The budget set can also be written in matrix form:

$$\mathbb{B}^i(q,\omega^i,\bar{\delta}^i,A,D)=\left\{\begin{array}{c}(x^i,z^i)\\ \in\mathcal{X}^i\times\mathcal{Z}^i\end{array}\;\middle|\; x^i\le\omega^i+\begin{pmatrix}-q'\\ A\end{pmatrix}z^i+D\bar{\delta}^i.\right\}$$

- Therefore, an economic agent i is confronted with the following utility maximization problem:

$$\max_{\substack{x^i\in\mathcal{X}^i\\ z^i\in\mathcal{Z}^i}} U^i(x) \quad \text{such that} \quad (x^i,z^i)\in\mathbb{B}^i(q,\omega^i,\bar{\delta}^i,A,D).$$

Non-Incorporated Companies

- A firm, on the other hand, should maximize its total profit.
- Since profits also arise at $t = 1$ and in different states, they have to be discounted or weighted.
- In the following, we assign a vector $\pi^j \in \mathbb{R}^{S+1}$ to every firm j, for which the NAC holds ($q_k = \sum_{s=1}^{S} A_s^k \pi_s^j$, for all k; see Th. 4.2).
- In $t = 0$, a firm's dividends are restricted by its net production and its actions on the bond market.
- In $t = 1$ for any state $s \geq 1$, the dividends may not exceed the net output and the payoffs from the bond market.
- Thus, we consider the following maximization problem:

$$\max_{\substack{d^j \in \mathbb{R}^{S+1} \\ y^j \in \mathcal{Y}^j \\ \xi^j \in \mathcal{P}^j}} \pi^{j\prime} d^j \quad \text{such that} \quad d^j \leq y^j + \begin{pmatrix} -q' \\ A \end{pmatrix} \xi^i.$$

Incorporated Companies

- Households and firms be allowed to trade firm shares at some prices p.
- The budget restriction of a household i needs to be changed to account for this possibility.
- At $t = 0$, agent i has a demand for goods x_0^i and for bonds z_k^i and shares δ_j^i.
- Apart from the initial distribution of goods ω_0^i, the sales revenue from the initial distribution and the dividends of its shares limit a household's decisions, i.e.,

$$x_0^i + \sum_{k \in \mathcal{K}} q^k z_k^i + \sum_{j \in \mathcal{J}} p^j \delta_j^i \leq \omega_0^i + \sum_{j \in \mathcal{J}} d_0^j \delta_j^i + \sum_{j \in \mathcal{J}} p^j \bar{\delta}_j^i. \quad (3)$$

Incorporated Companies

- The revenues and expenses at $t = 1$ in a given state s are:

$$x_s^i \leq \omega_s^i + \sum_{k \in \mathcal{K}} A_s^k z_k^i + \sum_{j \in \mathcal{J}} d_s^j \delta_j^i. \tag{4}$$

- We combine the two conditions of the budget restriction, where the latter shall hold for all states $s \geq 1$:

$$\mathbb{B}^i(q, p, \omega^i, \bar{\delta}^i, A, D) = \left\{(x^i, z^i, \delta^i) \in \mathcal{X}^i \times \mathcal{Z}^i \times \mathcal{H}^i \mid (3) \text{ and } (4) \text{ hold}\right\}$$

which can be written more compactly as

$$\mathbb{B}^i(q, p, \omega^i, \bar{\delta}^i, A, D) = \left\{(x^i, z^i, \delta^i) \in \mathcal{X}^i \times \mathcal{Z}^i \times \mathcal{H}^i \mid (5) \text{ holds}\right\}$$

where

$$x^i \leq \omega^i + \begin{pmatrix} -q' \\ A \end{pmatrix} z^i + \begin{pmatrix} (D_0 - p)' \\ D_1 \end{pmatrix} \delta^i + \begin{pmatrix} p'\bar{\delta}^i \\ 0 \end{pmatrix}. \tag{5}$$

Incorporated Companies

- $D_0 \in \mathbb{R}^J$ is the vector of dividends of all J firms at $t = 0$, and $D_1 \in \mathbb{R}^{S \times J}$ is the matrix of dividends of all firms in all states at $t = 1$.
- An economic agent i is therefore confronted with the following utility maximization problem:

$$\max_{\substack{x^i \in \mathcal{X}^i \\ z^i \in \mathcal{Z}^i \\ \delta^i \in \mathcal{H}^i}} U^i(x) \quad \text{such that} \quad (x^i, z^i, \delta^i) \in \mathbb{B}^i(q, p, \omega^i, \bar{\delta}^i, A, D)$$

- Now consider the budget restriction of an incorporated firm.
- For a given production decision y^j and a given financial policy (ξ^j, τ^j), the maximal dividends of firm j are:

$$d^j = y^j + \begin{pmatrix} -q' \\ A \end{pmatrix} \xi^j + \begin{pmatrix} (D_0 - p)' \\ D_1 \end{pmatrix} \tau^j.$$

Incorporated Companies

- The dividends of firm j depend on the dividends of all firms.
- A firm solves the following maximization problem (where the weights π^j, again serve to evaluate different dividend vectors):

$$\max_{\substack{d^j \in \mathbb{R}^{S+1} \\ y^j \in \mathcal{Y}^j \\ \xi^j \in \mathcal{P}^j \\ \tau^j \in \mathcal{T}^j}} \pi^{j\prime} d^j \quad \text{such that} \quad d^j \leq y^j + \begin{pmatrix} -q' \\ A \end{pmatrix} \xi^j + \begin{pmatrix} (D_0 - p)' \\ D_1 \end{pmatrix} \tau^j .$$

- The budget restriction requires that in any state s the dividends do not exceed the net output plus the payoffs from the financial market.

Incorporated Companies

- Let

$$\mathbb{W}^j(q,p,A,D) := \left\{ \begin{matrix} (d^j, y^j, \xi^j, \tau^j) \\ \in \mathbb{R}^{S+1} \times \mathcal{Y}^j \times \mathcal{P}^j \times \mathcal{T}^j \end{matrix} \;\middle|\; (6) \text{ holds} \right\}$$

where

$$d^j \leq y^j + \begin{pmatrix} -q' \\ A \end{pmatrix} \xi^j + \begin{pmatrix} (D_0 - p)' \\ D_1 \end{pmatrix} \tau^j. \qquad (6)$$

- Now we can formulate the firm's decision problem as

$$\max_{\substack{d^j \in \mathbb{R}^{S+1} \\ y^j \in \mathcal{Y}^j \\ \xi^j \in \mathcal{P}^j \\ \tau^j \in \mathcal{T}^j}} \pi^{j'} d^j \quad \text{such that} \quad (d^j, y^j, \xi^j, \tau^j) \in \mathbb{W}^j(q,p,A,D).$$

Modigliani-Miller Theorem

- We consider financial market equilibria and ask to what extent they depend on the firms' financial policies.
- While in the traditional view, the shareholder value of a firm depends on its debt-equity ratio, Modigliani and Miller show that under well defined conditions the funding of a firm is irrelevant.
- Their proof is based on an intuitive arbitrage argument.

Non-Incorporated Companies

- Consider a financial economy with production $\mathcal{E}_F^P$ according to Def. 1.
- An $\mathcal{E}_F^P$ is considered to be in a state of equilbrium, if four conditions are met:
  (i) Consumers maximize utility within their budget constraints.
  (ii) Firms maximize profits given their constraints, without allowing for arbitrage.
  (iii) The allocations must be achievable.
  (iv) Financial markets clear.
- Given the model of non-incorporated companies introduced in the last subsection, this leads to the following definition.

Non-Incorporated Companies

Definition (Financial Market Equilibrium with Endogenous Production)

A FME for a financial economy $\mathcal{E}_F^P$ *with endogenous production are allocations* $(\overset{*}{\phi}_{\mathcal{X}}, \overset{*}{\phi}_{\mathcal{Z}}, \overset{*}{\phi}_{\mathcal{H}}, \overset{*}{\phi}_{\mathcal{Y}}, \overset{*}{\phi}_{\mathcal{P}})$ *and a pricing system* $(\overset{*}{q}, \{\pi^j\}_{j\in\mathcal{J}})$, *such that*

(i) $(\overset{*}{x}^i, \overset{*}{z}^i, \overset{*}{\delta}^i) \in \mathbb{B}^i(\overset{*}{q}, \omega^i, \bar{\delta}^i, A, \overset{*}{D})$ *and* $\overset{*}{x}^i \in \arg\max U^i(x^i)$ *for all* $i \in \mathcal{I}$,

(ii) $(\overset{*}{y}^j, \overset{*}{\xi}^j, \overset{*}{d}^j)$ *satisfies* $\overset{*}{d}^j \leq \overset{*}{y}^j + \begin{pmatrix} -q' \\ A \end{pmatrix} \overset{*}{\xi}^j$ *and* $\overset{*}{d}^j \in \arg\max \pi^{j\prime} d^j$ *for all* $j \in \mathcal{J}$,

(iii) $\sum_{i\in\mathcal{I}} \overset{*}{x}^i \leq \sum_{i\in\mathcal{I}} \omega^i + \sum_{j\in\mathcal{J}} \overset{*}{y}^j$,

(iv) $\sum_{i\in\mathcal{I}} \overset{*}{z}^i + \sum_{j\in\mathcal{J}} \overset{*}{\xi}^j = 0$,

where π^j *satisfies the NAC for all* j, *i.e., the firms see no opportunity for arbitrage.*

Non-Incorporated Companies

- If we do not require condition (ii) to hold, then we speak of a financial market equilibrium with exogenous production decisions.
- In the following we shall examine how a given equilibrium changes under alternative financial policies of the firms. The answer is given by the Modigliani-Miller theorem (MMT).

Non-Incorporated Companies

Theorem (MMT with non-incorporated companies)

Let $(\overset{*}{\phi}_{\mathcal{X}}, \overset{*}{\phi}_{\mathcal{Z}}, \overset{*}{\phi}_{\mathcal{H}}, \overset{*}{\phi}_{\mathcal{Y}}, \overset{*}{\phi}_{\mathcal{P}})$ *and* $(\overset{*}{q}, \{\pi^j\}_{j\in\mathcal{J}})$ *be a FME with endogenous production decision and let* $\hat{\phi}_{\mathcal{P}}$ *be any financial policy allocation of the firms, then* $(\overset{*}{\phi}_{\mathcal{X}}, \hat{\phi}_{\mathcal{Z}}, \overset{*}{\phi}_{\mathcal{H}}, \overset{*}{\phi}_{\mathcal{Y}}, \hat{\phi}_{\mathcal{P}})$ *and* $(\overset{*}{q}, \{\pi^j\}_{j\in\mathcal{J}})$ *is also an FME with exogenous/endogenous production decision, where*

$$\hat{z}^i = \overset{*}{z}^i + \sum_{j\in\mathcal{J}} \left(\overset{*}{\xi}^j - \hat{\xi}^j\right) \bar{\delta}^i_j, \text{ for all } i \in \mathcal{I}.$$

- This shows that the funding of optimal production plans is irrelevant, as the good allocation $\overset{*}{\phi}_{\mathcal{X}}$, the production $\overset{*}{\phi}_{\mathcal{Y}}$, and the price vector $\overset{*}{q}$ remain unchanged. Intuitively, the consumers can undo the change of the firms' financial policy.
- The proof can be found in the text book on page 275f.

Incorporated Companies

- Apart from the activity on the bond market, firms can now also buy each other's stock.
- In this case, variant of MMT holds (see book, page 279).
- Exceptions for when the MMT does not hold can be found in the text book on page 277f.

Firm's Decisions Rules

- In this section we have a closer look at the decision-making process in firms.
- First, we question why the shareholders, who are consumers in the end of the day, would maximize profits.
- Then, we analyze whether shareholders with different preferences would agree on a common production plan.
- Fisher Separation Theorem

A Simple Market with One Firm

- We consider a situation where there is no uncertainty and only one firm.
- A single security with positive payoff A and price q is traded.
- Let p be the firm's market price and d its dividend at $t = 1$, while there are no dividends at $t = 0$.
- Furthermore, the household initially has a positive share $\bar{\delta}^i$ in the firm.
- This considerably simplifies the budget restriction (3), yielding the following maximization problem for the agent:

$$\max_{x^i \in \mathbb{R}^2_{\geq 0}, \delta^i, z^i} U(x^i) \quad \text{such that} \quad \begin{aligned} & x_0^i + qz^i + p\delta^i \leq \omega_0^i + p\bar{\delta}^i \\ \text{and} \quad & x_1^i \leq \omega_1^i + z^i A + \delta^i d. \end{aligned} \tag{7}$$

Firm's Decisions Rules

- Thus at $t = 0$, an household can consume and buy securities and shares in the firm.
- At $t = 1$ it gains the earnings from its initial share, security payoffs and the firm's dividends.
- Suppose the agent would like to consume less at $t = 0$ and more at $t = 1$.
- This results in a utility change.
- If Δx_0 and Δx_1 are very small, we can approximate the utility change by the corresponding first derivatives:

$$t = 0: \quad U(x_0 - \Delta x_0, x_1) - U(x_0, x_1) \approx \frac{\partial U(x)}{\partial x_0} \Delta x_0$$

$$t = 1: \quad U(x_0, x_1 + \Delta x_1) - U(x_0, x_1) \approx \frac{\partial U(x)}{\partial x_1} \Delta x_1.$$

Firm's Decisions Rules

- Setting aside Δx_0 consumption units at $t = 0$, one can use the free resources to buy $\frac{\Delta x_0}{q}$ units of securities.
- This yields a payoff of $\frac{\Delta x_0}{q} A$ at $t = 1$ to the extent of which one may finance additional consumption: $\Delta x_1 \leq \frac{\Delta x_0}{q} A$.
- If the agent has strictly monotonic preferences, we even have $\Delta x_1 = \frac{\Delta x_0}{q} A$.
- At the optimum, this transfer of consumption may not change the net utility, i.e.:

$$\frac{\partial U(x)}{\partial x_0} \Delta x_0 = \frac{\partial U(x)}{\partial x_1} \Delta x_1.$$

Substituting $\Delta x_1 = \frac{\Delta x_0}{q} A$, we get

$$\frac{\partial U(x)}{\partial x_0} \Delta x_0 = \frac{\partial U(x)}{\partial x_1} \frac{\Delta x_0}{q} A \quad \Leftrightarrow \quad MRS_{01} = \frac{A}{q}. \qquad (8)$$

Firm's Decisions Rules

- We know that the price q of a bond with payoff $A = 1$ is determined by $q = \frac{1}{1+r_f}$, where r_f denotes the riskless rate of interest.
- In this case the right-hand side of (8) becomes $MRS_{01} = 1 + r_f$.
- At the optimum an agent will determine his allocations such that his valuations of consumption at $t = 0$ and $t = 1$ agree with the market's.

Firm's Decisions Rules

- Buying shares instead of securities leads to a similar result.
- Forgoing consumption at $t = 0$ allows one to buy $\frac{\Delta x_0}{p}$ shares to the firm, which in turn yield a profit of $\frac{\Delta x_0}{p} d$ consumption units at $t = 1$.
- Again, at the optimum this transfer of consumption may not change the net utility, and we get:

$$\frac{\partial U(x)}{\partial x_0} \Delta x_0 = \frac{\partial U(x)}{\partial x_1} \frac{\Delta x_0}{p} d \quad \Leftrightarrow \quad MRS_{01} = \frac{d}{p}.$$

- Now we have both $MRS_{01} = 1 + r_f$ and $MRS_{01} = \frac{d}{p}$, so $1 + r_f = \frac{d}{p}$, and thus $p = \frac{d}{1+r_f}$.

Firm's Decisions Rules

- We can divide the income at $t = 1$ by $1 + r_f$ in (7) to convert it to a present income, then sum up both budget restrictions. Some rewriting then leads to

$$x_0^i - \omega_0^i + \frac{x_1^i - \omega_1^i}{1+r_f} + z^i \left(q - \frac{A}{1+r_f} \right) \leq \bar{\delta}^i p + \delta^i \left(\frac{d}{1+r_f} - p \right).$$

- The left-hand expression describes the value of the net swapped amounts in $t = 0$ consumption units.
- The right-hand side denotes the value of the firm share in consumption units.

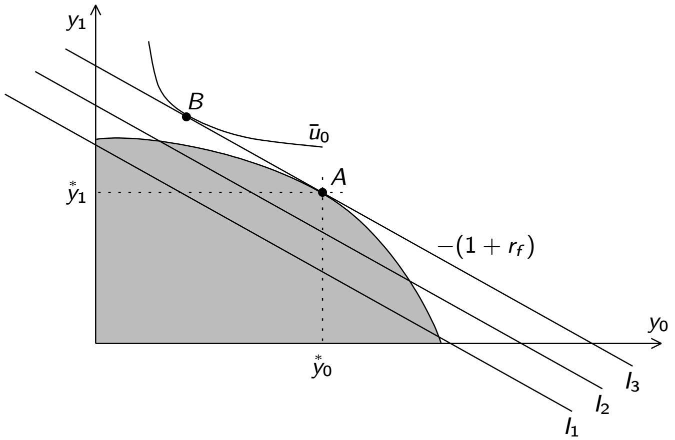
Firm's Decisions Rules

- If we substitute $q = \frac{A}{1+r_f}$ and $p = \frac{d}{1+r_f}$, the bound simplifies to

$$x_0^i - \omega_0^i + \frac{x_1^i - \omega_1^i}{1 + r_f} = \bar{\delta}^i \frac{d}{1 + r_f}.$$

- Hence the possible consumption increases in proportion to the dividend d.
- We have also considered the case $S = 1$, so the demand for a higher dividend can be translated to a demand for a high profit or net present value (NPV).
- Clearly, the shareholders will strive for maximum profit, i.e., carry out the project with the highest cash-flow.

Firm's Decisions Rules



Firm's Decisions Rules

- The product technology set is shown in grey.
- The households, assuming they own and control the firms, will choose a production point that maximizes their available income.
- In the figure this is $(\overset{*}{y}_0, \overset{*}{y}_1)$, which lies on the budget line l_3.
- The result is independent of the respective preferences, except that they are required to be strictly monotonic.
- The households can achieve optimum consumption by trading their production $(\overset{*}{y}_0, \overset{*}{y}_1)$ on the market for securities.

Fisher Separation with Multiple Firms

- With multiple firms the budget restriction looks a bit more general, but is still a special case of (3):

$$\max_{\substack{x^i \in \mathbb{R}^2_{\geq 0} \\ \delta^i \in \mathbb{R}^J \\ z^i \in \mathbb{R}}} U(x^i) \quad \text{such that} \quad x_0^i + qz^i + \sum_{j \in \mathcal{J}} p^j \delta_j^i \leq \omega_0^i + \sum_{j \in \mathcal{J}} p^j \bar{\delta}_j^i$$
$$\text{and} \quad x_1^i \leq \omega_1^i + z^i A + \sum_{j \in \mathcal{J}} d^j \delta_j^i. \tag{9}$$

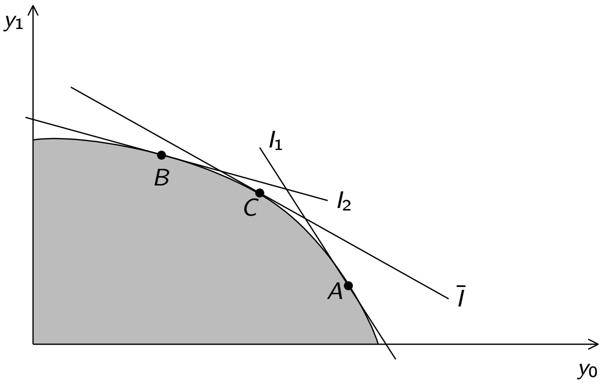
- Similarly to the first part, we obtain the relation of prices of securities and shares. Accordingly, the budget restrictions in (9) can be written concisely:

$$x_0^i - \omega_0^i + \frac{x_1^i - \omega_1^i}{1 + r_f} = \sum_{j \in \mathcal{J}} \bar{\delta}_j^i \frac{d^j}{1 + r_f}.$$

Fisher Separation with Multiple Firms

- Introducing more firms has not changed the result: a shareholder with strictly monotonic utility function again wants the firm to maximize its profit.
- The two types of shareholders are illustrated as the slope of I_1 and I_2, showing very high and very low rates of interest, respectively. Accordingly, the former would choose production point A, while the latter would pick B.
- We compute an average rate of interest, weighted by the corresponding shares, which is shown as the slope of $\bar{I}$.

Fisher Separation with Multiple Firms



The Theorem of Drèze

- In Section 6.2 the sole requirement for the firm's decision rules was that it weights its future profits by the elements of a vector $\pi^j \in \mathbb{R}^{S+1}$.
- If the markets are complete (rank $A = S$), this uniquely determines the firm's target function.
- In the case of incomplete markets, it does not have to be unique.
- Basically one can work with any vector that does not offer an arbitrage opportunity, i.e.,

$$\pi^{j\prime} \begin{pmatrix} -q' \\ A \end{pmatrix} = 0 \quad \text{and} \quad \pi^{j\prime} \begin{pmatrix} (Y_0 - p)' \\ Y_1 \end{pmatrix} = 0.$$

- In the case of complete markets, every shareholder would agree with the uniquely determined criterion π^N.

The Theorem of Drèze

- This is apparent from the first-order conditions of the utility maximization problem: the normalized utility gradients of all consumers matches the normalized target function vector π^N.
- Let firms be barred from running a financial policy, i.e., $\xi^j = 0$.
- It immediately follows that $D = Y$.
- The Modigliani-Miller theorem showed that in our model, nobody will want to object to this limitation.
- Furthermore, we will limit ourselves to the non-incorporated companies model. So we have the following underlying maximization problems:

$$\max_{\substack{x^i \in \mathcal{X}^i \\ z^i \in \mathcal{Z}^i}} U^i(x) \quad \text{such that} \quad (x^i, z^i) \in \mathbb{B}^i(q, \omega^i, \bar{\delta}^i, A, Y) \tag{10}$$

$$\max_{y^j \in \mathcal{Y}^j} \pi^{j\prime} y^j \quad \text{such that} \quad d^j \leq y^j.$$

The Theorem of Drèze

- We now look into firm j's general meeting of shareholders GM^j.
- We also assume that the production policies of all other firms are available.
- What can GM^j improve, given the decisions $\{\bar{z}^i\}_{i\in\mathcal{I}}, \{\bar{y}^l\}_{l\neq j}$?
- We will assume that GM^j only takes into consideration the consumers that own shares of firm j.
- Hence, let

$$\mathcal{I}_j := \{i \in \mathcal{I} \mid \bar{\delta}^i_j > 0\}, \quad \text{for all } j.$$

- How does a consumer i value changes of the production policy $\bar{y}^j$?
- The obvious criterion is the increase in consumer's utility with the new production policy $\hat{y}^j$:

$$U^i(\bar{x}^i + (\hat{y}^j - \bar{y}^j)\bar{\delta}^i_j) U^i(\bar{x}^i).$$

The Theorem of Drèze

Remark

The x^i implied by a given $(\bar{z}, \bar{Y})$ is

$$\bar{x}^i := \omega^i + \begin{pmatrix} -q' \\ A \end{pmatrix} \bar{z}^i + \bar{Y}\bar{\delta}^i.$$

We now give the following unanimity criterion:

Definition (Pareto-efficient with respect to GM^j)

The production policy $\bar{y}^j$ of firm j is Pareto-efficient with respect to GM^j, if $\bar{y}^j \in \mathcal{Y}^j$, and there is no $\hat{y}^j \in \mathcal{Y}^j$ with

$$\begin{aligned} U^i(\bar{x}^i + (\hat{y}^j - \bar{y}^j)\bar{\delta}^i_j) &\geq U^i(\bar{x}^i) \quad \text{for all } i \in \mathcal{I}_j, \text{ and} \\ U^i(\bar{x}^i + (\hat{y}^j - \bar{y}^j)\bar{\delta}^i_j) &> U^i(\bar{x}^i) \quad \text{for at least one } i \in \mathcal{I}_j. \end{aligned}$$

The Theorem of Drèze

- According to this criterion, an existing production policy is only discarded if all shareholders agree.
- In general many production policies may fulfill the criterion, so that a manager may not know which one to choose.
- We therefore allow shareholders to change each other's minds by means of transfer payments: let ρ^i_j be the net side payment made by shareholder i in the voting process.
- Then he values the decision of GM^j according to the criterion:

$$U^i(\bar{x}^i + (\hat{y}^j - \bar{y}^j)\bar{\delta}^i_j - \rho^i_j \mathrm{e}_1) U^i(\bar{x}^i).$$

The Theorem of Drèze

Definition (Pareto-efficiency with side payments with respect to GM^j)

The production policy $\bar{y}^j$ of firm j is Pareto-efficient with side payments w.r.t. GM^j, if $\bar{y}^j \in \mathcal{Y}^j$ and there are no $\hat{y}^j \in \mathcal{Y}^j$ and $\{\rho^i_j\}_{i\in\mathcal{I}_j}$ with $\sum_{i\in\mathcal{I}_j} \rho^i_j \geq 0$, such that
$U^i(\bar{x}^i + (\hat{y}^j - \bar{y}^j)\bar{\delta}^i_j - \rho^i_j e_1) \geq U^i(\bar{x}^i)$ for all $i \in \mathcal{I}_j$, and
$U^i(\bar{x}^i + (\hat{y}^j - \bar{y}^j)\bar{\delta}^i_j - \rho^i_j e_1) > U^i(\bar{x}^i)$ for at least one $i \in \mathcal{I}_j$.

The Theorem of Drèze

- The decision-finding mechanism described in this definition is rather difficult to implement: one would have to iterate through all production policies and transfer payments to find a better production policy.
- In the following we will describe an equivalent, direct mechanism.
- To that end, let $\pi^{N,i}(x^i) \in \mathbb{R}^{S+1}_{\gg 0}$ be his MRS between consumption in state s and present consumption:

$$\pi^{N,i}_s(x^i) := \frac{\partial_{x^i_s} U^i(x^i)}{\partial_{x^i_0} U^i(x^i)}, \quad s \in \mathcal{S}.$$

The Theorem of Drèze

- Now imagine the consumers just tell the manager their vectors $\pi^{N,i}(x^i)$.
- He in turn chooses the production policy such that it maximizes the function $\pi^{j'} y^j$, where $\pi^j := \sum_i \delta^i_j \pi^{N,i}$ is treated as given by the firm.
- Therefore, the discounting vector of firm j is the mean discounting vector of the consumers, weighted by their shares.
- In particular, this decision rule leads to the usual profit maximization rule if the markets are complete.
- The equivalence of the two mechanisms is shown by the Theorem of Drèze.

The Theorem of Drèze

Theorem (Theorem of Drèze)

The production policy $y^j \in \mathcal{Y}^j$ is Pareto-efficient with side payments with respect to GM^j if and only if for a given $\pi^{N,j}$,
$y^j \in \arg\max_{y^j \in \mathcal{Y}^j} \pi^{N,j\prime} y^j$*, where* $\pi^{N,j} = \sum_i \bar{\delta}^i_j \pi^{N,i}$.

The proof can be found in the text book on page 284f.

References

References I