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A Cautionary Note on Niu and Zeng (2018)

Ji Cao

Marc Oliver Rieger

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Ji Cao* Marc Oliver Rieger†

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We revisit the article by Niu and Zeng (FRL, 2018) on "Corporate financing with loss aversion and disagreement". We find two problematic points in the mathematical derivation of their fundamental results and suggest ways to remedy these problems.

Keywords: Corporate Finance, Capital Structure, Loss Aversion, Prospect Theory

JEL classification: C61, G32

*School of Development Studies, Yunnan University, Cuihu North Road 2, 650091 Kunming, China. Email: ji.cao@ynu.edu.cn

†Department IV, University of Trier, Universitätsring 15, 54296 Trier, Germany. Email: mrieger@uni-trier.de

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We revisit the article by Niu and Zeng (FRL, 2018) on "Corporate financing with loss aversion and disagreement". We find two problematic points in the mathematical derivation of their fundamental results and suggest ways to remedy these problems.

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1 Introduction

One of the main contributions of Niu and Zeng (2018) is to build an equilibrium model that should be useful to analyze the effect of loss aversion to a firm's decision regarding capital structure.

Unfortunately, there are two fundamental problems in the theoretical derivation of their article (Section 3) that we would like to explain in the following. The first one is due to a simplification regarding the definition of the (loss averse) utility function that can lead to wrong results in some situations. We will explain this problem in more detail in Section 2 and give examples when this error is relevant. The second problem is how the paper solves the central utility maximization problem. We will show that this is done in a rather peculiar, non-standard way that in our opinion is inappropriate to yield the desired results. Reasons for this are given in Section 3 where we also explain how a more appropriate approach should look like.

This comment only discusses the part concerning loss averion and capital structure in Niu and Zeng (2018). We do not discuss the remainder of their paper that deals with the relation between disagreement and capital structure.

2 Expected utility calculation

The loss aversion utility function, as given in the paper by Niu and Zeng (2018), is

$$V(X) = \begin{cases} X^{\alpha}, & X \geq 0, \\ -\lambda(-X)^{\beta}, & X < 0, \end{cases} \tag{1}$$

where X is the change of wealth of the agents.

According to the paper, the wealth change for the new investor is $X_1 = wIr + (1-w)I\frac{P_1-P_0}{P_0}$, and for the old shareholder $X_2 = E_0(P_1 - P_0)$[1].

Then, the expected utility of the new investor, as given in the paper, is

$$\mathrm{E}(V(X_1)) = \theta X_1^{\alpha} + (1-\theta)(-\lambda)(-X_1)^{\beta}. \tag{2}$$

And the expected utility of the old shareholder, as given in the paper, is

$$\mathrm{E}(V(X_2)) = \theta X_2^{\alpha} + (1-\theta)(-\lambda)(-X_2)^{\beta}. \tag{3}$$

While at first glance, these equations make sense, they are not always correct: The utility function (1) distinguishes between gains ans losses (following, e.g., the classical prospect theory model by Tversky and Kahneman (1992)). Thus, (2) and (3) assume implicitly that the wealth changes of the agents are distributed as

$$\begin{cases} X_1 \geq 0, & \text{with probability } \theta, \\ X_1 < 0, & \text{with probability } 1-\theta, \end{cases} \tag{4}$$

for the new investor. And for the old share holder,

$$\begin{cases} X_2 \geq 0, & \text{with probability } \theta, \\ X_2 < 0, & \text{with probability } 1-\theta. \end{cases} \tag{5}$$

[1]We mention here that the wealth change of the old shareholder should in our opinion better be modelled as $X_2 = E_0(P_1 - P_2) - wIr\frac{E_0}{E_0+E_1}$. – The paper ignores the repayment of the bond.

There are at least two problems with (4) and (5): First, the state dependence of X_1 and X_2 is insufficiently treated, since the values of X_1 and X_2 (and hence their signs!) depend on other random variables. For example, standing at period $t = 0$, let us assume the stock price at $t = 1$ has the following simple distribution

$$P_1 = \begin{cases} p_G, & \text{with probability } \theta, \\ p_B, & \text{with probability } 1-\theta, \end{cases} \tag{6}$$

with $p_B < P_0 \leq p_G$, then

$$X_1 = \begin{cases} wIr + (1-w)I\frac{p_G - P_0}{P_0}, & \text{with probability } \theta, \\ wIr + (1-w)I\frac{p_B - P_0}{P_0}, & \text{with probability } 1-\theta, \end{cases} \tag{7}$$

for the new investor. And for the old share holder,

$$X_2 = \begin{cases} E_0(p_G - P_0) - wIr\frac{E_0}{E_0+E_1}, & \text{with probability } \theta, \\ E_0(p_B - P_0) - wIr\frac{E_0}{E_0+E_1}, & \text{with probability } 1-\theta. \end{cases} \tag{8}$$

We see, the gains and losses are asymmetric and state-dependent, unlike those given in (4) and (5).

Second, (4) and (5) make the implicit ad hoc assumption that for both the new investor and the old shareholder, their wealth changes have the same probability θ to lie in the gain domain, and the same probability $1-\theta$ to lie in the loss domain. This assumption is only true in special cases, but not in general.

To underline this point, we give a simple example where the assumption is violated:

If $wIr + (1-w)I\frac{p_B - P_0}{P_0} \geq 0$ and $E_0(p_G - P_0) - wIr\frac{E_0}{E_0+E_1} < 0$, then, the wealth change X_1 is *always* non-negative in (7), and X_2 is *always* negative in (8). Wealth changes of the new investor and of the old shareholder do not have the same probabilities to lie in the gain domain, neither in the loss domain.

As another simple case where the assumption is not valid, consider the case that no new equity (SEO) is issued, i.e. $w = 1$. In this case, the new investor has no loss at period $t = 1$. Her terminal wealth is $W_1 = W_0 + Ir > 0$. In other words, her wealth change ($X_1 = Ir$) has a probability of 1 to lie in the gain domain, and a probability of 0 to lie in the loss domain.

While for the old shareholder, her wealth change $X_2 = E_0(P_1 - P_0) - Ir$ depends on the change of the stock price, but not on w. Thus, there is a probability that the old shareholder's wealth change is positive, and also a non-zero probability that her wealth change is negative.

We see in this case very easily that the two agents' wealth changes have different probabilities to be in the gain and loss domains. Although w is endogenous in the model and the extreme case that $w = 1$ may not be optimal, we can not *a priori* exclude this possibility by assuming that the two agents' wealth changes have the same probabilities.

In sum, we are concerned that the paper oversimplifies the expected utilities of the agents. Instead of arbitrarily assuming certain distribution for the wealth changes of the agents, a better approach would be to posit the original source of risk in the model – the outcome of the new project. One should start from assuming the distribution of the return of the new project, then derive accordingly the distribution of the agents' wealth change. An example how that can be done is the online appendix of Malmendier et al. (2011).

3 Utility optimization problem

Let us now discuss the second fundamental issue in the derivations of Niu and Zeng (2018), namely their utility optimization procedure. The paper solves the optimization problem of the two agents by taking the derivatives of their expected utilities with respect to the stock price P_1, respectively, i.e., $\frac{d\mathrm{E}(V(X_1))}{dP_1} = 0$, $\frac{d\mathrm{E}(V(X_2))}{dP_1} = 0$. This seems to us surprising:

The decision parameters for the old shareholder (represented by the manager) are w and r, the proportion of the fund to be raised by bond financing and the return of the bond. The market price of the stock at period $t = 1$,

P_1, is unknown at $t = 0$, when the old shareholder makes the decision, thus is out of her direct control. P_1 should not be the parameter of interest in the optimization problem.

For the new investor, her decision is whether or not to take part in this financing activity initiated by the firm. If the new investor's expected utility of buying the firm's new stocks or bonds exceeds the expected utility of not buying, then she will do so, vice versa. P_1 is again out of her direct control, thus should not be the parameter of interest in the optimization problem.

Again, a correct framework of such discussions could follow the online appendix of Malmendier et al. (2011).

References

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