Title: **Approximate Real Function Maximization and Query Complexity**

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Proposal

Let $\mathrm{MAX}, \mathrm{INT} : C[0,1] \to \mathbb{R}$ be functionals for maximization and integration, respectively. I. e.,

$$\mathrm{MAX}(f) = \max_{0 \le x \le 1} f(x), \qquad \mathrm{INT}(f) = \int_0^1 f(t)dt.$$

It has been shown [KF82] that for the restricted class of polynomial-time computable real functions, both functionals can be approximated up to an error 2^{-n} on polynomial-space in n.

In above's framework of computation, each polynomial-time computable real function $f : [0,1] \to \mathbb{R}$ is essentially represented as a type-2 function [Wei97, Wei00], mapping an rational sequence to a rational sequence, i. e., $f \in (\mathbb{N} \to \mathbb{Q}) \to (\mathbb{N} \to \mathbb{Q})$. This enables us to discuss the computational complexity of a real-valued function on classical Turing machines with oracle access. Here, a TM M computing f is presented with an oracle $\phi \in (\mathbb{N} \to \mathbb{Q})$ for $x \in [0,1]$ and a precision $n \in \mathbb{N}$, such that M produces the output $M^\phi(n) \in \mathbb{Q}$ with $|M^\phi(n) - f(x)| \le 2^{-n}$.

When it comes to the computation of the functionals MAX and INT, the encoding of an input for a Turing machine becomes even more critical: Instead of representing an $x \in [0,1]$ by a fast converging sequence $\phi \in (\mathbb{N} \to \mathbb{Q})$, we now have to encode a continuous real-valued function $f : [0,1] \to \mathbb{R}$. The first goal of this thesis is to devise, compare and explore different ways of accessing an unknown continuous real function via black-box queries that model the actual computation of f as supported by certified numerics like interval arithmetic [KLRK98] or the iRRAM [Mü01].

Building on this, the second goal of this Master's Thesis is to determine the query complexity of the MAX-functional[1] with respect to different protocols. In combination with a complexity analysis of the protocols themselves, this should yield more explicit quantitative and parameterized both upper and lower bounds (compared to those presented in the beginning) on the complexity of MAX (and INT).

Remark: This work will not contain any implementation of MAX (or INT) on, for example, the iRRAM.

[1] The examination of the INT-functional is an optional extension.

Literature

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