From Few Components to an Eulerian Graph by Adding Arcs

Manuel Sorge, René van Bevern, Rolf Niedermeier, and Mathias Weller

Institut für Softwaretechnik und Theoretische Informatik, TU Berlin

We report on our findings regarding algorithmics of the Eulerian Extension (EE) problem. A directed graph is called *Eulerian*, if it contains a tour that traverses every arc in the graph exactly once. In the problem of EE, a directed multigraph G and a weight function is given and it is asked whether G can be made Eulerian by adding arcs whose total weight does not exceed a given threshold.

EE is motivated by applications in vehicle routing [2] and flowshop scheduling [3]. However, EE is NP-hard [1, 3] and thus we have analyzed the problem complexity not only with respect to input length, but also with respect to other properties of the input—called "parameters". We tried to confine the combinatorial explosion occurring in all known algorithms for NP-hard problems to these parameters.

Dorn et al. [1] have proven that EE can be solved in $O(4^k n^4)$ time, where k denotes the parameter "number of arcs that have to be added". We have analyzed EE with respect to the parameters "number c of connected components in the input graph" and "sum b over $\text{indeg}(v) - \text{outdeg}(v)$ for all vertices v in the input graph where this value is positive". Both parameters are upper bounded by k.

Using structural observations about the arc sets that have to be added, we have proven that there is an algorithm for EE with $O(4^{c \log(bc^2)} n^4 + n^2 m)$ running time. We restated EE in a matching context. The matching formulation of EE might be an important tool to solve the question of whether EE can be solved within running time whose super-polynomial part depends only on c. Furthermore, we considered polynomial-time preprocessing routines for EE and showed that such routines cannot yield instances whose size depends polynomially only on either of the parameters b, c, k unless coNP $\subseteq$ NP/poly.

Bibliography

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