k-Gap Interval Graphs and Parameterized Complexity

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A *multiple interval* (t-*interval*) is the nonempty union of a finite number of (at most t) disjoint intervals over the real line. A *multiple interval graph* (t-*interval graph*) is the intersection graph of families of multiple intervals (t-intervals). This natural generalization of interval graphs was independently introduced by Trotter and Harary [3], and by Griggs and West [2]. Even for small fixed $t \geq 2$, these classes are much richer than interval graphs: for example, the class of 2-interval graphs includes circular-arc graphs, outerplanar graphs, cubic graphs, and line graphs. Unfortunately, many problems remain NP-hard on 2-interval graphs (for example, 3-Coloring, Dominating Set, Independent Set, Hamiltonian Cycle, and their Recognition) or 3-interval graphs (for example Clique, whose complexity on 2-interval graphs is open). Parameterized by solution size, Independent Set, Dominating Set, and Independent Dominating Set are $W[1]$-hard on 2-interval graphs, even when all intervals have unit length, whereas Clique is FPT [1].

With the objective to generalize interval graphs while maintaining their nice algorithmic properties, we define *k-gap interval graphs* as multiple interval graphs whose number of intervals exceeds the number of multiple intervals by at most k. Parameterizing problems by k becomes then a reasonable way of scaling up the nice properties of interval graphs to more general graphs. As a case study, we focus on the Clique Cover problem on k-gap interval graphs parameterized by k and show that it has a quadratic kernel.

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References

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