

Definition: A mixed-strategy profile $\sigma^* = (\sigma_1^*, \dots, \sigma_n^*)$ is a mixed-strategy Nash equilibrium if for each player, $i = 1, \dots, n$, and every $\sigma_i \in \Delta S_i$ we have $\pi_i(\sigma^*) \geq \pi_i(\sigma_{-i}^*, \sigma_i)$; that is choosing σ_i^* is at least as good as choosing any other σ_i given that the other players choose σ_{-i}^* .

How can we solve for a mixed-strategy Nash equilibrium? The answer is provided by the fundamental theorem of mixed strategy Nash equilibrium.

Fundamental theorem of mixed-strategy Nash equilibrium

A mixed strategy profile σ is a Nash equilibrium if and only if for any player $i = 1, \dots, n$ with pure-strategy set S_i if the following conditions are met:

- If $s_i, s_i' \in S_i$ occur with positive probability in σ_i , then the expected payoffs to s_i and s_i' are equal when played against σ_{-i} .
- If s occurs with positive probability in σ_i and s' occurs with zero probability in σ_i , then the expected payoff to s_i' is less than or equal to the expected payoff to s played against σ_{-i} .