6.1 Static Bayesian Games

Let us consider n players: i = 1, …, n. The normal form representation of a static Baysian game is as follows:

- For each player i = 1,…, n, we have a set S_i of strategies. We call $s = (s_1, \ldots, s_n)$ a strategy profile of the game where $s_i \in S_i$.
- For each player we have a type space T_i. This is the set of possible types of player i. We call $t = (t_1, \ldots, t_n)$ a type vector of the game where $t_i \in T_i$.
- The probability that player i is of type t_i is denoted by $p(t_i)$. Player i observes her own type. But she cannot observe the types of the other players. However, she can compute beliefs by using the Bayesian rule: $p_i(t_{-i}|t_i)$. This is the conditional probability that the other players are of certain types given that player i has observed her own type. Of course, this can also involve the special case, that the types of the players are independent so that the conditional probability is equal to the unconditional probability: $p_i(t_{-i}|t_i) = p_i(t_{-i})$.
- The payoff of player i depends on the chosen strategies and the types of the players: $\pi_i = \pi_i(s,t) = \pi_i(s_1, \ldots, s_n; t_1, \ldots, t_n)$.

The timing of a static Bayesian game is as follows:

1. Nature draws a type vector t = (t_1, …, t_n).
2. Nature reveals type t_i to player i but not to any other players. Each player updates her beliefs by using the Bayesian rule: $p_i(t_{-i}|t_i)$.
3. Players simultaneously choose their strategies. This is the reason why the game is called static.
4. Payoffs are received: $\pi_i = \pi_i(s,t) = \pi_i(s_1, \dots, s_n; t_1, \dots, t_n)$.

Definition: Pure-strategy Bayesian Nash equilibrium

A pure strategy profile s* = $(s_1^*, \dots, s_n^*)$ is a Nash equilibrium if for each player i and for each of i's types t_i the strategy $s_i = s_i^*(t_i)$ solves:

$$\max_{s_i} \sum_{t_{-i} \in T_{-i}} p_i(t_{-i}|t_i)\pi_i\left(s_1^*(t_1), \dots, s_{i-1}^*(t_{i-1}), s_i, s_{i+1}^*(t_{i+1}), .., s_n^*(t_n)\right)$$

given the strategies of all other players s_{-i}^* and given the own belief $p_i(t_{-i}|t_i)$.