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Abstract

We investigate the economic impact of stochastic endogenous extreme events and insurance in a growth model. Our analytical results and computational experiments show that i) transparency of the insurance sector is the decisive requisite for abatement activities, implying substantial policy opportunities; ii) we can fully characterize and quantify the impact of uncertainty on the social planner's decisions; iii) a decentralized economy will under-invest in abatement without adequate policy interventions; iv) precautionary beliefs on the frequency of extreme events lead to more sustainability; v) technical change does not change the ordering of the paths but leads to a more sustainable future; v) a social security system which prices insurance fairly is preferable to an insurance industry which provides insurance with an overhead.

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1 Introduction

During the past decades, both the frequency and the strength of natural disasters have substantially increased. The number of natural catastrophes quadrupled between 1970 and 2008, so did the economic losses associated with these events. Estimates suggest that in 2008 alone approximately 238,000 people died from natural catastrophes with a total cost to society of USD 225 billion. These days, an event that qualifies in its extent as a natural catastrophe[1] happens on our planet on average every single day (Swiss Re [14], [15], Hoyois et al. [7]).

Clearly, the threat of natural disasters is non-negligible for investment decisions and for climate policy. In this article we look at the interplay between natural disasters, global climate change, decision-taking under uncertainty and the role of insurance. This comprehensive view is new to the literature. We build a stochastic growth model with endogenous climate change, endogenous natural disasters and a global insurance sector. The social planner in this model can allocate national income between consumption, savings, insurance or greenhouse gas abatement. After analyzing the main characteristics of this model we fully calibrate it against representative data from seminal Integrated Assessment Models. Computational experiments are used to quantify the results of the analytical model and to evaluate the role of insurance.

One of our innovations to the literature is to model natural disasters as extreme events through a Poisson process, which is the natural approach towards understanding the role of uncertain extreme events.[2] Normally, uncertainty in climate change models is analyzed via Brownian motions or sensitivity analysis, which is not useful to understand the economic impact of extreme events (see e.g. Goodess et al [5]). We show that the existence of extreme events modeled through a Poisson process leads to substantial changes from the results obtained in seminal climate policy assessment models (e.g. Nordhaus [13]). Overall, poisson processes have seen surprisingly few applications in economic theory. The works by Waelde [18] and Sennewald and Waelde [16] were among the first continuous-time applications.

The other innovation to the literature is the inclusion of an insurance sector. Disaster insurance as provided by the major players in the re-insurance industry is the main market based tool to mitigate the consequences of extreme events. We therefore introduce an insurance sector into the model and analyze the implications of different pricing strategies on the take-out of insurance policies and consumption, saving and abatement decisions. In particular we model how full and partial information on the pricing mechanism affects the abatement decision of the social planner. Since we assume that the social planner can affect the state of the environment by appropriately deciding between consumption, saving and abatement, and since the state of the environment affects the frequency and strength of extreme events, the planner can affect the costs of insurance. It turns out that the insurance sector can implement a type of climate policy by signaling the consequences of climate change via insurance costs. This again is new to the literature, where climate policy is solely based on government intervention.

[1] According to EM-DAT an event qualifies as a natural catastrophe if ten or more people are reported killed; or hundred people reported affected; or a declaration of a state of emergency or a call for international assistance happened.

[2] An exception here is Gjerde et al. [3] and Keller et al. [8], who analyze the impact of a single extreme event.

One key assumption underlying our analysis is that climate change is endogenous to the decisions of the planner. We believe there exists ample evidence supporting this point of view that we do not need to motivate this assumption further. See for example the comprehensive studies by McCarthy et al. [10] and Stern et al. [17]. The evidence for extreme events being endogenous in terms of number and strength to changes in the climate however is less certain. We studied the climatology literature and the overall conclusion seems to be that in some regions extreme events increase in strength and numbers whereas in other regions they do not (Walsh and Ryan [20], Walsh [19], Henderson-Sellers [6]). Also, in those regions where the strength of hurricanes increases the overall number of hurricanes seems to decrease (Webster et al. [21], Elsner et al. [2]). Furthermore, climate change is going to lead to strong regional changes in the extreme weather conditions, with significant changes in the hot and cold days as well as wetter or drier areas (McCarthy et al. [10], Stern et al. [17]). Given this evidence, we pursue the following path. In the theoretical part we try to be as general as possible and assess the effects of endogenous changes in the number and strength of extreme events. In the computational part we only assess changes in the strength of events. Our damages are here calibrated to be in accordance with the standard integrated assessment literature with the main difference being that they come at uncertain points in time.

The article is structured as follows. Section 2 introduces and solves the theoretical model. Section 3 gives the integrated assessment model with computational experiments. Finally, Section 4 concludes.

2 The Theoretical Model

The setup of the model follows a Ramsey-type growth problem, where an infinitely-lived social planner maximizes his uncertain stream of felicities $u(c(t))$ subject to consumption $c(t)$, abatement $a(t)$, insurance cover $\gamma(t)$ and capital stock $k(t)$. Our model extends the work by Gollier [4] by allowing for abatement, an endogenous number $\lambda(t)$ and size $\psi(t)$ of extreme events and a climate change sector $T(t)$. Capital increases with production and is reduced by constant depreciation ($\delta_k > 0$), consumption, abatement, the size of the insurance premium $P(t)$, the percent of uninsured capital ($\gamma(t) \in [0, 1]$), and uncertain extreme events. The extreme event is modeled as a Poisson process. When $q(t) = 0$ no shock occurs, whereas if $q(t) = 1$, then an extreme event reduces capital stock by $\gamma(t)\psi(T(t))k(t)$. The expected number of extreme events in any point in time is an endogenous, increasing function of temperature and given by $\lambda(T(t)) > 0$. The share of capital destroyed by an extreme event is given by the function $\psi(T(t))$, which too is increasing in temperature. When there is full insurance, thus $\gamma(t) = 0$, then the planner has full insurance coverage and will not be (financially) affected by the extreme event. Finally, temperature is increased by productive activity and decreases due to a natural regeneration. In our model, abatement can reduce emissions, but it cannot affect temperature directly. One should therefore interpret abatement as an investment in greener technology.

The social planner thus solves the following problem:

$$V(k_0, T_0) = \max_{\{c(t),\gamma(t),a(t)\}} E_{t_0}\Big\{ \int_{t_0}^{\infty} u(c(t), T(t))e^{-\rho(t-t_0)}dt \Big\} \tag{1}$$

subject to

$$
\begin{aligned}
dk(t) &= \{f(k(t)) - c(t) - a(t) - (1-\gamma(t))P(t) - \delta_k k(t)\}dt - \gamma(t_-)\psi(T(t_-))k(t_-)dq(t), && (2)\\
dT(t) &= \{g(k(t), a(t)) - \delta_T T(t)\}dt, && (3)\\
&\text{and} \quad k(0), T(0) \quad \text{given.}
\end{aligned}
$$

The interpretation of $T(t)$ as an argument in the utility function is that it represents a short-hand notation for $u(c(t), \lambda(T(t)))$ and means that the social planner's utility is decreasing in the expected number of accidents. If $T(t)$ is interpreted as the stock of CO_2, then another interpretation is simply that it represents the amenity value for the environment or other non-market damages. We assume that capital follows a cádlág process, such that $k(t)$ is continuous from the right having left limits. The left limits are given by $\lim_{s\uparrow t} k(s) = k(t_-)$. Intuitively, if an extreme event occurs, then the size of the jump depends on the amount of capital just before the jump. In the subsequent part we skip this extra piece of notation but hope the reader keeps this in mind. We assume the following functional forms.
Assumption 1: *The utility function $u : \mathbb{R}^2_+ \to \mathbb{R}_+$ is at least twice continuously differentiable, concave in both arguments, with $u_1(c,T) > 0$, $u_2(c,T) < 0$, and $\lim_{c\to 0} u'(c) = \infty$.*
Assumption 2: *The production function $f : \mathbb{R}_+ \to \mathbb{R}_+$ follows $f(k) \geq 0$, $f(0) = 0$, $f'(k) > 0$, $f''(k) < 0$, with $\lim_{k\to 0} f'(k) = \infty$, and $\lim_{k\to\infty} f'(k) = 0$.*
Assumption 3: *The share of capital stock destroyed is $\psi(T) > 0$, $\psi'(T) > 0$.* The assumptions on $\psi(T)$ come from the previous section.
Assumption 4: *The extreme event is modeled via a Poisson process. The endogenous, expected number of extreme events follows $\lambda(T) > 0$, $\lambda'(T) > 0$.*
Assumption 5: *Temperature increases concavely with capital $g_1(k,a) > 0$, $g_{11}(k,a) < 0$, and decreases concavely with abatement activity $g_2(k,a) < 0$, $g_{22}(k,a) < 0$.* These last assumptions on temperature accumulation are standard in the integrated assessment literature.

The Bellman equation of this control problem is given by

$$
\rho V(k_t, T_t) = \max_{\{c_t, a_t, \gamma_t\}} \left\{ u(c_t) + \frac{1}{dt} E_t dV(k_t, T_t) \right\},
$$

and $V(k_t, T_t)$ refers to the optimized utility functional. This equation suggests that the return of having k_t and T_t, denoted in indirect utility terms, should at any point in time be equal to the instantaneous felicity as well as the expected change in the future indirect utility stream. Making use of the Change of Variable formula, which is the equivalent of Itô's Lemma but holds for Poisson processes, see Davis [1], and taking the expectation we arrive at the following Hamiltonian- Bellman-Jacobian equation.

$$
\begin{aligned}
\rho V(k_t, T_t) &= \max_{\{c_t, a_t, \gamma_t\}} \Big\{ u(c_t) + \big(f(k_t) - c_t - a_t - (1-\gamma_t)P_t - \delta_k k_t\big) V_k(k_t, T_t) && (4)\\
&\quad + \big(g(k_t, a_t) - \delta_T T_t\big) V_T + \lambda(T_t)\big[V\big(k_t - \gamma_t \psi(T_t) k_t, T_t\big) - V(k_t, T_t)\big] \Big\}.
\end{aligned}
$$

The part of the Bellman equation with the squared brackets includes the adjustment through extreme event. The term $V\big(k_t - \gamma_t\psi(T_t)k_t, T_t\big) - V(k_t, T_t)$ describes the total cost of the extreme event, since it refers to the difference in indirect utility after a jump occurred minus the indirect utility without a jump. This difference is multiplied by $\lambda(T)$ to transform it in expected value terms.

The dynamic system after optimization is completely characterized by the following system of equations:

$$\begin{aligned}
-\frac{u''(c)}{u'(c)}dc &= \Big\{f'(k) - \delta_k + \frac{g_k}{g_a} - \rho + \frac{u_{cT}}{u_c}\big(g(k,a) - \delta_T T\big) && (5)\\
&\quad +\lambda(T)\Big(\frac{u'(\tilde{c})}{u'(c)}\big(1-\gamma\psi(T)\big) - 1\Big)\Big\}dt - \Big\{\frac{u'(\tilde{c})}{u'(c)} - 1\Big\}dq(t),\\
\frac{g_{aa}}{g_a}da &= \Big\{g_a\frac{u_T}{u_c} - f'(k) + \delta_k - \delta_T - \frac{g_k}{g_a} - \frac{g_{ak}}{g_a}\Big[f(k) - c - a - (1-\gamma)P - \delta_k k\Big] && (6)\\
&\quad +\lambda'(T)\Big[\frac{\tilde{V}-V}{V_T}\Big] + \lambda(T)\Big[\frac{\tilde{V}_T}{V_T} - \frac{\tilde{V}_k}{V_T}\gamma\psi'(T)k - \frac{u'(\tilde{c})}{u'(c)}\big(1-\gamma\psi(T)\big)\Big]\Big\}dt\\
&\quad +\Big\{\frac{u'(\tilde{c})}{u'(c)} - \frac{\tilde{V}_T}{V_T} - \frac{\tilde{g}_a}{g_a} + 1\Big\}dq(t),\\
dk &= \{f(k) - c - a - (1-\gamma)P - \delta_k k\}dt - \gamma\psi(T)k\,dq(t), && (7)\\
dT &= \{g(k,a) - \delta_T T\}dt, && (8)\\
\frac{u'(\tilde{c})}{u'(c)} &= \frac{P}{\lambda(T)\psi(T)k}, && (9)
\end{aligned}$$

where $\tilde{x}$ refers to a variable after a jump occurred, and time subscripts are submitted for simplicity. We can easily interpret this system describing the way preferences and technical possibilities work together. Starting with equation (9) we know this defines the optimal level of insurance cover for the social planner. If the premium is fair and therefore equal to the expected damages, then we obtain $\tilde{c} = c$ and no jump will occur since the social planner will fully insure. This is a standard result in the insurance literature, attributable to Mossin [11]. If there is an overhead on the premium, then $u'(\tilde{c}) > u'(c)$ which by concavity of the utility function implies consumption after an extreme event is lower than before an extreme event. The larger the overhead on the premium the bigger the jump of consumption.

Equation (5) refers to the optimal consumption choice of the social planner. The part in the first curly brackets explains the way the social planner chooses during periods in which no extreme event occurs. This term is a standard Ramsey-Keynes component, where the social planner will choose to increase consumption if the benefits of producing more outweigh the costs of capital depreciation, time preference, and it also involves a valuation of how changes in temperature affect future marginal utilities. We obtain a new result about precautionary savings, which is summarized in the following proposition.

Proposition 1 *Given the control system* (1) *to* (3) *we find that the anticipation of extreme events may either lead to precautionary savings or precautionary consumption. Given an indemnity contract with a fixed overhead $\phi > 1$ we find positive precautionary savings if $\gamma\psi > (\phi-1)/\phi$.*

To understand this result we have to investigate the term $\lambda(T)\big(\frac{u'(\tilde{c})}{u'(c)}\big(1-\gamma\psi(T)\big)-1\big)$, which is new to the literature. If we assume that the currently exogenously given insurance premium sufficiently exceeds the expected damage, then we know that if an extreme event occurs, consumption will decrease and therefore $u'(\tilde{c}) > u'(c)$. Hence, if the jump is large enough, then this term will have a positive effect on consumption growth. We call this precautionary consumption. Since the social planner knows that in the future his consumption might be reduced through an extreme event, he prefers to increase his current consumption in order to fall back to some average level later. However, this term need not always be positive. If insurance cover is low and the percent of capital stock which gets destroyed large, then the overall term might turn negative, leading to a reduction in consumption in favor of either abatement activity, insurance or precautionary capital accumulation. It therefore becomes clear that the theoretical model leaves us with an insurance puzzle: Whether the social planner increases or decreases his future consumption will depend on the relative strength of the precautionary consumption versus the precautionary savings effect. The last term in the curly brackets refers to the adjustment in case an extreme event occurs. Since the ratio $u'(\tilde{c})/u'(c) \geq 1$ we know that the jump in case an extreme event occurs will be negative. Indeed, the size of the jump can be found for the case of an interior solution in $\gamma \in (0, 1)$, constant-relative risk aversion (CRRA) utility and a standard insurance contract with an overhead on an otherwise fair premium. In that case we know from equation (9) that $u'(\tilde{c})/u'(c) = \phi$, where $\phi > 1$ gives the overhead. Thus the jump in consumption will be determined by the level of consumption before the jump, the size of the overhead and the intertemporal elasticity of substitution (IES). It will be given by $\tilde{c} = \phi^{-1/\sigma}c$, where $\sigma > 0$ is the IES. We can easily calculate that $\frac{d\tilde{c}}{d\sigma} > 0$ (taking given the effect of σ on c), implying a stronger consumption smoothing the larger the intertemporal elasticity of substitution. This also allows us to obtain a condition for the precautionary consumption versus savings decision. Given the previous assumptions, precautionary consumption is positive if $\gamma\psi < (\phi-1)/\phi$, whereas precautionary savings are positive otherwise. Clearly, the more capital gets destroyed the more incentive will be for precautionary savings. On the converse, the larger the overhead the more likely is precautionary consumption.[3] Surprisingly, one would therefore expect more precautionary consumption if the social planner insures less.

Equation (6) gives optimal abatement. The terms in the first line are the standard ones describing trade-offs between abatement versus capital for their relative effectiveness on temperature, capital accumulation and future costs. The first term in the second line describes the cost of a marginal increase in the expected number of extreme events on the future stream of utilities. The larger the marginal impact of temperature on the expected number of extreme events and the higher the costs of an extreme event in terms of utility foregone, the stronger will the social planner increase abatement. The second term in the second line gives the amount of precautionary abatement. Precautionary abatement is positive if, in case of an extreme event, we expect a lot of capital to be destroyed and if the impact of changes in temperature on the percent of capital which gets destroyed is very large. Precautionary abatement can be negative though, too. This will be the case if precautionary consumption is very big. This trade-off obviously comes from the capital constraint. The third line of equation (6) gives the impact on abatement in case of an extreme event. Abatement growth itself will respond to an extreme event with either an upward or a downward jump. It will jump upwards if consumption falls significantly after a jump, but it will decline if the marginal impact of temperature on indirect utility after a jump is much higher than before a jump.

In general, this system is not analytically tractable but it allows us to look at specific, important cases which would normally not have been observed or taken into account. These cases provide us with tractable benchmarks which we shall also later use in the computational

[3]Since $\gamma\psi < 1$ is bounded by $\psi < 1$ for $\gamma \to 1$ whereas $(\phi - 1)/\phi \to 1$ for increasing ϕ.

experiments.[4]

2.1 The insurance industry

Our intention now is to introduce the insurance industry in this framework. We start with the strong assumption that the insurance company behaves like a risk-neutral firm in a perfectly competitive market. This has two major implications. Firstly, we do not account for a possible default of the insurance sector. Though this is a viable threat for small insurance companies, we do not believe that this is likely to occur for the global insurance sector. If the premia are chosen with a certain foresight that reflects the actual expected number and size of extreme events, then the probability of a global default of the insurance industry is likely to be small. Furthermore, it is standard in the insurance industry that certain portfolios which smaller insurance companies deem too risky are transferred to reinsurance companies that can control these excess risks much better. Secondly, with this assumption we abstract from the possibility that the insurance sector accumulates capital which it can invest in a capital market.[5] The implications of the insurance industry in our model could therefore be described as follows. The social planner will be able to obtain insurance but in his decisions of insurance he will not include the possibility of default in the insurance industry (since the probability of default is zero). If the insurance industry is able to receive more capital during certain periods than it has to pay out, then this capital will not be invested in the capital market but stays perfectly liquid in order to pay for future claims later. With this in mind we can turn to the problem of the insurance sector.

The expected insurance claims at any point in time are $\lambda(T(t))\psi(T(t))k(t)(1-\gamma(t))$, whereas the insurance premia obtained are $(1-\gamma(t))P(t)$. There may exist transaction costs or operational costs, represented by a mark–up $\phi > 0$ on the insurance claims. The expected profits are therefore

$$E(\Pi(t)) = (1-\gamma(t))P(t) - (1+\phi)\lambda(T(t))\psi(T(t))k(t)(1-\gamma(t)).$$

The zero-profit condition then implies $P(t) = (1+\phi)\lambda(T(t))\psi(T(t))k(t)$, where $(1+\phi)$ represents overhead charges on the premium. If the transaction costs or operational costs are negligible, then the premium will be equal to the expected costs from an extreme event for a fair insurance contract.

[4] In general it is not our objective to solve for the dynamics of these systems. However, it can be shown that a steady state exists (in the certainty case). Solving for the dynamics around steady state will however give conditions which do not help us in interpreting the results.

[5] One could argue that the insurance industry, and especially re-insurance companies, are non-negligible players on the world capital market. However, allowing for a positive profit would raise questions like: To whom belongs this capital? Are these companies then risk-neutral or do they reflect the preferences of their owners? This would obviously give rise to similarly strong assumptions. Finally, a representative insurance company would have to withdraw capital from the capital market in equal amount to the losses from an extreme event, which thus would be equivalent to no insurance at all. One could therefore not study a complete general equilibrium setup with a social planner framework. We are willing to give up a small amount of completeness in order to be able to clearly compare to previous results while still keeping a good degree of analytical tractability.

2.2 The case of naive insurance

In this section we assume that there is no overhead[6] on the insurance premium such that $\phi = 0$.

Proposition 2 *Given the optimal control problem equations* (1) *to* (3) *we find that transparency of the insurance industry is decisive for abatement decisions and thus sustainability.*

Given the dynamic system (5) to (9) we easily obtain that any risk-averse social planner will choose full insurance, thus $\gamma(t) = 0$ (see e.g. Mossin [11]). This implies that our dynamic system reduces to

$$-\frac{u''(c)}{u'(c)}\dot{c} = f'(k) - \rho - \delta_k + \frac{g_k}{g_a} + \frac{u_{cT}}{u_c}\dot{T}, \quad (10)$$

$$-\frac{g_{aa}}{g_a}\dot{a} = -g_a\frac{u_T}{u_c} - \delta_k + \delta_T + f'(k) + \frac{g_k}{g_a} + \frac{g_{ak}}{g_a}\dot{k}, \quad (11)$$

$$\dot{k} = f(k) - c - a - \lambda(T)\psi(T)k - \delta_k k, \quad (12)$$

$$\dot{T} = g(k,a) - \delta_T T. \quad (13)$$

Under a fair insurance premium the only effect which the extreme event might have is that the social planner behaves as if he has a capital stock which is reduced by the expected damage of extreme events (his payment to the insurance industry). Not surprisingly, his consumption and abatement decisions are not affected by any precautionary decision process. Most interestingly, since the social planner takes the evolution of the insurance premium as given, he will not take the impact of his decisions on the expected number and size of extreme events into account. We interpret this case as that of a *naive insurance*, and it is here where the role of the insurance industry becomes dominant. If the insurance industry is not transparent enough and does not inform the social planner of his influence on the evolution of the insurance premium, then this should have drastic consequences for climate change. Since the social planner does not control for the effect of temperature on the premium, one would expect that temperature increases drastically in the naive insurance case, which should lead to large reductions in future capital stocks. We shall confirm this in a subsequent section via computational experiments.

Corollary 1 One could very well imagine that this case corresponds to the way a small agent in a decentralized setup would act who believes that his decisions have no influence on the evolution of the premium. Possible changes in the premium would not figure in his abatement decisions. The only reason for undertaking abatement in this case would be the mitigation of non-market damages in the utility.

As we have seen, transparent pricing of insurance contracts may play a significant role for the evolution of climatic and economic variables. Of course, if the insurance industry feels that future environmental and economic conditions are important, then it could improve upon this situation. Apart from setting the size of the premium and thereby influencing the decisions of the social planner, the insurance company can inform the social planner about the evolution of the premium given his production choices. We shall analyze this case now.

[6] We investigate the implication of an overhead in the next section.

2.3 The case of an internalized insurance premium

Imagine that the social planner knows that he will get a fair premium and will therefore take up full insurance. Furthermore assume that the insurance company transparently communicates to the social planner how their economic decisions impact the premium which they have to pay. Then the social planner will incorporate the changing costs of the insurance premium into his decision taking.

Proposition 3 *Under a fair and internalized insurance premium the social planner will fully incorporate the future climate change costs of his decisions.*

The problem then writes as follows:

$$V(k_0, T_0) = \max_{\{c(t), a(t)\}} \int_{t_0}^{\infty} u(c(t), T(t)) e^{-\rho(t-t_0)} dt \tag{14}$$

subject to

$$\dot{k}(t) = f(k(t)) - c(t) - a(t) - \lambda(T(t))\psi(T(t))k(t) - \delta_k k(t), \tag{15}$$
$$\dot{T}(t) = g(k(t), a(t)) - \delta_T T(t). \tag{16}$$

A rather surprising observation is that this is equivalent to a standard integrated assessment model like DICE by Nordhaus [12]. The DICE model is therefore similar to our model if we have a fair premium and thus full insurance plus a social planner who takes the evolution of this premium into account. One could rephrase this slightly. Assuming one wishes to use the DICE model to analyze extreme events without altering its structure. Then we can conclude that the model will produce acceptable results only under full insurance and the internalization of the evolution of the premium.

Writing the Hamiltonian from the above equations leads to

$$\mathcal{H} = u(c, T) + \mu\big[f(k) - c - a - \lambda(T)\psi(T)k - \delta_k\big] + \xi\big[g(k, a) - \delta_T T\big]. \tag{17}$$

We can derive the following system of equations which characterize the dynamics:

$$\dot{c} = c\sigma(c)\left[f'(k) - \lambda(T)\psi(T) - \rho + \frac{g_k}{g_a} - \delta_k + \frac{u_{cT}}{u_c}\dot{T}\right], \tag{18}$$
$$\dot{a} = a\theta(a)\left[f'(k) - \lambda(T)\psi(T) - \delta_k + \delta_T + \frac{g_k}{g_a}\right. \tag{19}$$
$$\left. - \frac{u_T}{u_c} g_a + g_a\big(\lambda'(T)\psi(T) + \lambda(T)\psi'(T)\big)k + \frac{g_{ak}}{g_a}\dot{k}\right], \tag{20}$$
$$\dot{k} = f(k) - c - a - \lambda(T)\psi(T)k - \delta_k k, \tag{21}$$
$$\dot{T} = g(k, a) - \delta_T T, \tag{22}$$

where we define $\sigma(c) = -u_c/(cu_{cc})$ and $\theta(a) = -g_a/(ag_{aa})$. Two new effects can be singled out. Firstly, the direct effect of the premium's size. If $\lambda(T)\psi(T)$ is large, meaning that many events happen and they destroy a significant part of the capital stock, then this term may lead to a decrease in consumption and abatement growth. Both may be optimally reduced since stronger damage leads to a lower global capital stock which does not allow to continue consumption and abatement at the previous levels. The other new term only affects abatement and it relates to the direct impact of abatement on the change in the expected value of an extreme event through a change in temperature. The larger the marginal effect of abatement on temperature or the larger the effect of temperature on the expected costs of the extreme event, the more will be invested by the social planner into reducing temperature. This term does not show up in the accumulation for consumption since consumption affects temperature or the expected costs of extreme events only indirectly. In the computational experiments we shall study the magnitude of this result in a properly calibrated model.

3 Computational Experiments

The analytical model gives first insights into the dynamics of an economy which is exposed to extreme weather events. To show how these events affect consumption, investment and abatement, this section reports results from computational experiments.

Computational experiments require full specification of functions and parameters. To keep the analysis comparable with the literature, these functions and parameters are taken from seminal models, whenever reasonable and available. Moreover, we develop a technique to solve the model numerically. It is a top–down technique based on non–linear optimization, scenario reduction and time–consistent forward iteration. Details are given below.

3.1 Specifications

Instantaneous utility is assumed CRRA,

$$u(c(t), \Delta T(t)) = \frac{(\theta(\Delta T(t))c(t))^{1-\sigma}}{1-\sigma}, \tag{23}$$

with risk aversion $\sigma = 2$. Temperature perturbation $\Delta T(t)$ measures the temperature difference between period t and the base period. Climate change directly affects utility via the consumption–loss factor $\theta(\Delta T(t))$. These non–market damages (Manne et al. [9]) are assumed quadratic in $\Delta T(t)$:

$$\theta(\Delta T(t)) = 1 - \min\left\{1, \frac{\Delta T(t)^2}{\Omega_U}\right\},$$

where Ω_U is calibrated such that doubling atmospheric carbon relative to pre–industrial levels induces a loss of 1.5 per cent in consumption-equivalent utility.

The climate sub–model takes current carbon dioxide emissions $e(t)$ as input from the economic model and translates these into temperature perturbations $\Delta T(t)$. We decided to map this relation with the CLIMNEG World Simulation Model. A similar sub–model is used in Nordhaus' DICE 2007 version, which however fits the IPCC scenarios slightly less well.[7] The

[7] We are particularly grateful to Philippe Marbaix for new calibrations of the parameters of the climate module as well as to Johan Eyckmans for the code. See Eykmans and Finus [?] for details.

climate sub–model is sufficiently non–linear to approximate the results of larger climate models like those used in the IPCC scenarios. Equations and parameters are listed in Appendix I.

Carbon emissions arise in fixed proportions from production of $y(t)$. However, emissions can be abated, for example by substituting solar energy for oil. Abatement activities are summarized through $m(t) \in [0, 1]$ which gives the share of abated emissions in total emissions.

Production of $y(t)$ is Cobb–Douglas type with labor and capital as inputs. Output elasticity of capital is assumed 0.36. Labor is exogenously given and price–inelastically supplied. Production is spent on consumption, investment, insurance and abatement, with abatement costs $a(t) = .3m(t)^2y(t)$. Completely decarbonising the economy ($m(t) = 1$) would take thirty per cent of GDP in addition to current energy spending. Capital accumulates according to

$$K(t+1) = \big(1 - \gamma(t)\Psi(T(t)\big)dq(t))(1 - \delta_K)K(t) + i(t)$$

where

$$\Psi(T(t)) = 0.05 + \min\left\{1, \frac{\Delta T(t)^2}{\Omega_K}\right\}. \tag{24}$$

The share of uninsured in total capital is $\gamma \in [0, 1]$, which is a decision variable for the social planner. The arrival rate of extreme events is fixed at $\lambda = .2$. Parameter Ω_K is calibrated such that the expected value of damages because of extreme events accounts for a loss of 2.9 per cent in total output at double pre–industrial carbon concentration (equivalent to a rise in temperature of 2.5 C).[8] The discount rate on instantaneous utility ρ is assumed 0.03, and capital depreciation is $\delta_k = .05$.

Simulations are carried out with GAMS/CONOPT3. Time is taken as discrete with a one year time–step. The model is solved by non-linear programming with state–contingent decision variables. To cope with the curse of dimensionality, we reduce the sample of feasible scenarios and therefore work with a representative subsample of paths (effective states of the world). Sensitivity analysis shows that samples of 20 randomly drawn scenarios already give sufficiently robust results. The scenarios are sampled such that the social planner observing these scenarios would conclude from statistical inference that he faces a Poisson process with the given arrival rate $\lambda = .2$.[9]

At any point in time, the social planner maximizes expected utility by looking 70 years ahead. He starts in the base year 2005 by deciding about consumption, investments and abatement up to 2070, contingent upon the unfolding sequence of extreme weather events. After reaching and carrying out insurance, abatement and investment decisions, an extreme weather event either does occur or not. In 2006, the social planner revises his original state–contingent policies, looking now forward till 2071. Again by sensitivity analysis we found that a 70 years forward looking period ensures time–consistency.

We run computational experiments to get quantitative answers on the following questions:

[8] This is equivalent to a forty per cent capital damage in case of an extreme event. Since the expected number of events is .2, and the output elasticity .36, this ends up with an expected loss of less than three per cent.

[9] A scenario is a sequence $dq_t, t = 1, ...$ of 0, 1 or 2, indicating the number of extreme events in t. More than two extreme events per year are not taken into account.

What is the implication of uncertainty and insurance for policy decisions? What is the role of transparency in the insurance sector? What happens if the social planner holds false beliefs about the risk of extreme events? What is the role of technical change? Who should provide insurance?

These questions define the scenarios. Based upon the theoretical model we single out three broad scenario types.

- *Fully internalized insurance* occurs for a fair insurance premium, i.e. the premium equals expected damages from extreme events. In this case, risk averse social planner transfer all risks to the insurance company. He takes into account that he endogenously determines the insurance premium through non–abated emissions. This is similar to a deterministic case with fully internalized climate change like in the DICE or MERGE models. We take this scenario as reference. It is called "Perfect Insurance" for simplicity.
- *Stochastic* means that the social planner can not insure at all, hence he is fully exposed to extreme weather events. He anticipate his influence on the future climate.
- *Naive insurance* refers to the scenario where the social planner insures against extreme weather events but – in contrast to the full insurance scenario – considers the insurance premium as exogenous. This scenario may also correspond to a competitive market outcome where agents do not perceive the impact of their decisions on the insurance premia.

In principle, we compare a stochastic economy (scenario Stochastic) with an economy where climate damages unfold smoothly and deterministically (scenario Fully Internalized Insurance). Evidently, the variance of the stochastic paths tends to infinity over time due to the Poisson process. Reporting single paths or a small selection of paths therefore makes no sense as one might have picked up very unlikely paths with either many or very few events. We avoid this problem by plotting the average stochastic path. This corresponds to an averaging over a large number of stochastic paths and is equivalent to taking the conditional expectations at each point in time. Our procedure for averaging is explained in the Appendix.

We analyze two fully stochastic paths which are randomly drawn from the set of all feasible scenarios, the average stochastic path, the case of fair insurance with internalized premium (Perfect Insurance) and without internalized premium (Naive Insurance).

> Figure 2 to Figure 4 about here <

The basic results refer to the Stochastic Average and Perfect Insurance scenarios. Recall that the social planner in the Stochastic Average scenario fully bears the risk of extreme events. In Perfect Insurance, he can – and indeed does – transfer all risk to the insurance company, which charges a premium given by expected damages. Figure 2 shows that this makes a significant difference, uncertainty matters. A risk-exposed social planner decreases consumption to spend more on abatement and capital formation. The precautionary saving motive dominates precautionary consumption. In the long run, however, more abatement and capital formation pays back; carbon stocks are smaller without insurance, hence climate change induced damages are smaller in the long run.

The individual stochastic paths can differ significantly. Two randomly-drawn stochastic paths in Figure 2 to Figure 4 demonstrate this. They clearly grow vastly different. This confirms our previous suggestion that it makes no sense to base policy decisions solely on a randomly-drawn path and supports our arguments for using the average stochastic path.

Realized Welfare (without technical progress)				
FairIns	AvgStoch	Naive	Jump1	Jump2
-567	-587	-743	-535	-584

Finally, we consider the naive insurance scenario. It is equivalent to a non-transparent or non-internalized insurance policy, and it is furthermore equivalent to a competitive economy where a small agent considers himself sufficiently small not to bear any impact on the insurance price. Some minor abatement occurs to mitigate non-market damages. Since the social planner does not take his impact on the insurance premium into account, it leads to large initial increases in investment allowing for high consumption in the subsequent periods, followed by a fast worsening of the climate feedback. In our simulation, these increases in temperature lead to a large damages due to extreme events. After around 25 years this implies massive destructions of capital stock, leading to a non-sustainable evolution of consumption. In terms of realized welfare, the naive insurer does the worst of all. What do these results imply for policy?

3.2 What is the role of transparency in the insurance sector?

One way to understand the role of the insurance industry in environmental and economic policy is to investigate the importance of transparency. A fully transparent insurance policy is equivalent to the perfect insurance scenario, whereas an opaque policy would be depicted by the naive insurance case. As already discussed above, both scenarios differ heavily in terms of realized welfare, climate impact and consumption. It cannot be emphasized too much that the insight into how oneself affects the premium is vital for sustainable consumption and global welfare. This opens up a potential role for regulatory efforts toward stronger information revelation and mandatory insurance. If the insurance industry reveals its pricing mechanism, then the social planner can incorporate this into his decision process.[10] As a welcomed side–effect, this tightens competition in the insurance market such that overheads will decrease and policy uptakes increase. As we show later under the analysis of partial insurance, this is likely to increase global welfare. In addition, regulatory efforts directed toward a reduction of asymmetric information could improve the climate–change–signaling character of insurance premia. For example, insurance companies generally dispose over detailed information about expected extreme events and disaster hot spots. It is however hardly possible to obtain any data from the insurance sector due to the fear of competitive pressure. It is exactly here where policy regulations can provide the greatest benefits.

[10] In a decentralized version one would assume that agents who are aware of their impact on the insurance would act in a cooperative way, through e.g. voting, in order to reduce climate change.

3.3 Are precautionary beliefs welfare-improving?

We have previously argued that information on how the social planner can influence the evolution of the insurance premium is one of the key determinants of a sustainable path of the main economic and environmental variables. Information about the evolution of the insurance premium is based on the expected number (λ) and size (ψ) of extreme events. However, in case the policy maker does not know the true λ, what would be the effect of under-estimating or over-estimating this parameter? In other words, is precaution preferable to a possible under-estimation of events?

It is not reasonable to compare expected welfare for this case, due to the different probabilities attached. But one can ask: Is precaution[11] preferable in terms of realized welfare or in terms of some sustainability measure?

> Figure 8 to Figure 10 about here <

We first consider the results from a discounted utilitarian perspective. Figures 8 to 10 show the optimal paths of consumption, temperature and abatement for the case of under-estimating λ, ($\lambda = 0.1$), for a correct belief ($\lambda = 0.2$), and for over-estimated frequencies of extreme events ($\lambda = 0.3$). If the social planner under-estimates the expected number of extreme events, he will abate less carbon emissions, leading to significant increases in temperature and short–term consumption. However, due to accelerated climate change, strong increases in extreme event induced damages reduce the capital stock and eventually decrease consumption below the level of correct beliefs. The scenario with precautionary beliefs, or an over-expectation of extreme events ($\lambda = 0.3$) behaves exactly the opposite way: high abatement activity, low overall temperature changes, and increasing consumption levels in the future. From a discounted utilitarian perspective, we find that average realized (i.e. ex–post) welfare is higher if the social planner under-estimates the amount of extreme events than if he over-estimates or correctly estimates the amount of extreme events. This is explained through the inertia in the climate system and discounting. Clearly, looking at the realized welfare from a discounted

Average realized welfare with false beliefs

lambda=0.1	lambda=0.2 (correct)	lambda=0.3
-515	-587	-597

utilitarian perspective it might be advisable to place more emphasis on consumption now and simply give fewer weight to the climate system. Indeed, this could be one of the main reasons for questioning the use of the discounted utilitarian criterion in climate analysis altogether. The path which is most inequitable in terms of utility, consumption or environmental quality is the one which gives the highest realized utility. On the other hand, the precautionary beliefs give the lowest realized utility but fare best in terms of realized welfare in the distant future and sustainable consumption. To answer our question, we find that precautionary beliefs maximize future welfare, whereas current welfare is evidently maximized by just doing the opposite. To put it more bluntly: An under-estimation of extreme events is equivalent to

[11] By precaution we here mean that, given a policy maker is faced with a possible distribution of λ's, he would choose a high one.

choosing the Damocles sword, whereas an over-estimation of extreme events is equivalent to choosing a safety net.

3.4 Implication of Technical Progress

We assume that technical change is Harrod-neutral with an annual growth rate in total factor productivity (TFP) of 1.5% from 2005 to 2075, and zero thereafter. As expected, this supports higher consumption, abatement, but also leads to a higher stock of atmospheric carbon and thus temperature. Since income is higher in the future, more money can be spent on abatement in later periods, which thus drives temperature down below the case of no technical change. Increases in TFP allow for abatement levels which lead to almost carbon free production. It is again the full and fair insurance option which implies lower abatement efforts and therefore stronger climate change.

> Figure 5 to Figure 7 about here <

There are no qualitative changes in the order of our previous results, and neither are there significant changes in the distances between the optimal paths.

We can however observe that consumption in this case is non-decreasing on all paths except for naive insurance. Nevertheless, even for naive insurance, consumption does not any longer drop below base period levels. This gives a slight hope that technical progress helps when it comes to e.g. the satisfaction of the basic needs criterion. Finally, again just like in the previous cases, realized welfare under naive insurance and technical progress is by far the worst.

Realized Welfare with technical change				
FairIns	AvgStoch	Jump1	Jump2	Naive
-469	-471	-476	-441	-743

3.5 Who should provide the insurance?

For many regions, the occurrence of extreme events has significant impacts on people's welfare and quality of life. Access to insurance can be seen as one of the major means to make those people less vulnerable. At the same time one must remember that the insurance industry is not driven by considerations of charity but by profit maximization and shareholder value, like any other industry. It is thus important to question as to who should provide insurance in case of disasters which protrude through the whole society.

> Figure 11 to Figure 13 about here <

Many insurance companies work locally, in small units. Under those circumstances one would expect higher overhead charges on the premia. Furthermore, the risk which the smaller insurance companies can not bear are sold to the reinsurance industry, which again implies additional overhead costs. Re-insurance can pool risks at a global level, hence minimizes the risk of default. But these shifting of portfolios between the insurance intermediaries tends to make insurance quite expensive. The re-insurance industry is highly concentrated, therefore it may exercise significant market power. Other designs like some type of social security system where the government aims at full insurance could potentially be more welfare-improving.

We investigate this hypothesis by analyzing the effects of overheads on the insurance premium which lead to unfair premia and therefore to partial insurance. If an insurance policy which provides insurance at a fair premium provides higher welfare than one which is supplied at an unfair premium, then this would indeed rise some questions about market power and regulation. We look at several policies. Perfect Insurance, which implies full insurance at any time serves as a benchmark again. Mark Up 2.5, Mark Up 5, and Mark Up 10 assume an overhead of 2.5%, 5% and 10% respectively; Stochastic Average is equivalent to a sufficiently high mark-up on the premium such that the social planner does not want to insure any more.

We find that fair-priced insurance leads to the highest global welfare. Under a small overhead (here 2.5%), we notice that the social planner insures himself fully after only a few periods.[12] This however implies that he has to pay more than the expected costs of the extreme events, which therefore implies a lower global welfare. Furthermore, we find that overheads which lead to partial insurance still fare better in terms of realized welfare than no insurance at all or small overheads which lead to full insurance.

Realized Welfare under unfair insurance				
FairIns	PartialInsur1025	PartialInsur105	PartialInsur11	AvgStoch
-567	-586	-572	-573	-587

4 Conclusion

This paper investigates the role of uncertainty and the insurance industry in the economics of endogenous extreme events. Our theoretical growth model, where extreme events are endogenously determined but a social planner can insure himself against these events, yields several important and new results. Firstly, we find a role for precautionary savings but also for precautionary consumption. Which of the two prevails over the other critically depends on the percent of capital insured as well as how much of the capital the social planner expects to lose. Secondly, we notice that the DICE model corresponds to our model in case of full insurance and if the social planner is fully aware about the impact of emissions on the insurance premium. This is an important result since this allows us to define a benchmark and thus allows for comparability to other results, and it also suggests the limitations of the DICE or MERGE model in dealing with extreme events or catastrophes. Thirdly, we find a significant role for transparent pricing of insurance contracts. The more transparent the insurance industry, or the more information it leaks as to how it sets its premium, the more sustainable will the economic and environmental system be.

We then develop a fully calibrated Computable General Equilibrium Model based on our

[12] See Gollier [4] for the theoretical argument.

theoretical model in order to quantify the analytical results. This model is calibrated with key data from 2005, and it uses similar functional forms and parameters as the seminal DICE and MERGE models. In addition, we introduce endogenous, Poisson-driven shocks and an insurance sector.

We firstly asked how relevant uncertainty is for the policy maker's decisions. We find that if one constructs an average path from a large set of Poisson-driven uncertain paths, then the difference between the resulting average stochastic path and the full insurance path when the premium is internalized is stems fully from the uncertainty. This therefore allows for a quantification of the impact of uncertainty in terms of many measures like consumption-equivalent variation, like ex-post welfare or a comparison based on various sustainability criteria.

We then take a look at the role of transparency in the insurance sector. We observe drastic differences in the evolution of the premium in case the insurance industry provides full information, in comparison to when the policy maker does not internalize his decisions on the evolution of the premium. If the premium is not internalized, then abatement effort is negligible, climate change will take drastic forms and consumption can even drop below current levels, which would violate any sustainability criterion. We suggest that transparency is the major determinant of the economic and environmental system, opening the possibility for important regulative possibilities.

We find that false beliefs accentuate the problems of the discounted utilitarian criterion in the analysis of climate change. The highest realized welfare is obtained when the social planner under-estimates the amount of disasters, which leads to strong climate change, high current consumption but an unsustainable consumption in the future. On the contrary, an over-estimation of the frequency of disasters leads to high abatement, little climate change and high consumption in the future. One would wonder whether our initial question, namely if precautionary beliefs are welfare-improving, should not be re-phrased into the old question: Whose welfare should we take care of?

In addition, we analyze the implication of technical change. Our main finding is that this does not change the qualitative results of the model. We however find that technical progress avoids decreasing consumption in all scenarios except for naive insurance. In that scenario consumption will drop, but not below the current consumption levels. We therefore expect that technical change helps in avoiding the violation of basic needs criteria.

Our final contribution is in approaching the question of whether insurance of extreme events that affect a potentially significant proportion of the population should be left to the private sector. Our conclusion is that if the private sector demands an overhead due to market power or dis–economies of scale which a social insurance system may not demand due to possible advantages in risk pooling and without the need for the various insurance intermediaries, then a social insurance system should be preferred from a welfarist point of view.

In terms of future research, we believe that the most useful improvements in this field will be in the insurance sector. Allowing for the default of the insurance industry will be a good extension, although it is questionable whether this will lead to qualitative changes in the results. In case of default, the insurance industry will most likely keep a certain cushion of capital which it will call upon in case of multiple consecutive events. We should therefore expect a slightly higher premium in the first periods, which implies an overhead on the premium, and our results from the unfair insurance premium should apply.

More interesting will be if one allows the insurance industry to undertake investments in the capital market. This will however require an unfair premium such that the insurance industry can gather capital which it can invest in the capital market. It will also imply stochastic capital markets and therefore uncertain returns to capital, implying further uncertainties in the social planner's budget constraint. And finally, one will have to deal with the allocation of the excess profits of the insurance industry. Most importantly, one would have to move away from the social planner framework to a multi-agent one which will be difficult from an analytical point of view and will not allow the comparison with the standard results in the literature.

The last remark obviously directly suggests that one of the most important extensions of this work will be a regional approach like the RICE model but with a global insurance industry. One can then study how risk gets transferred from more risky to less risky regions and from higher income regions to lower income ones. We are currently extending our work in this direction.

Finally, we have hinted at the possibility that a social insurance system might be welfare-improving in comparison to a private insurance industry. For example, one could look at this from a paternalistic point of view, or a redistributive one, and finally from the problems of market failure. Indeed, one could think about a social insurance system which might be able to internalize the externality of today's emissions imposed on future generations in the insurance premia now, which would somewhat adhere to the polluter pays principle.

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5 Appendix

5.1 Climate Submodel

The climate sub-model is a version of the CLIMNEG World Simulation model. Carbon emissions $e(t)$ accumulate in the atmosphere according to

$$M_A(t+1) = (1+\tau_{11})M_A(t) + \tau_{21}M_U(t) + e(t), \tag{25}$$

where M_A and M_U denote atmospheric carbon concentrations in the atmosphere and upper ocean, respectively. For parameters τ_{11} and τ_{21}, see Table 1. Upper ocean carbon concentrations depend upon concentrations in both the lower ocean M_L and the atmosphere M_A,

$$M_U(t+1) = (1+\tau_{22})M_U(t) + \tau_{12}M_A(t) + \tau_{32}M_L(t). \tag{26}$$

Lower ocean concentrations evolve according to

$$M_L(t+1) = (1+\tau_{33})M_L(t) + \tau_{23}M_U(t).$$

Forcing f_o is given by

$$f_o(t) = 4.03\frac{\log(M_A(0)/590)}{\log(2)}.$$

Ocean temperature T_O and atmospheric (surface) temperature T follow

$$T(t+1) = .2562\,T(t) + \tau_4 f_o(t) + \tau_5 T_O(t) \tag{27}$$

and

$$T_O(t+1) = \tau_6 T(t). \tag{28}$$

Table 1: Climate System Parameters

Parameter	Value
τ_{11}	-0.033384
τ_{21}	0.027607
τ_{23}	0.011496
τ_{33}	-0.000422
τ_4	1.7
τ_5	0.794
τ_6	0.03609

5.2 The Average Stochastic Path

It should be clear that if one wishes to integrate a Poisson-driven sample path in CGE models, then any interpretation of one given sample path could be rather misleading since the variance of a random variable driven by a Poisson distribution tends to infinity over time. As we interpret the sample paths in the graphs from the CGE experiment based on their relative position to other paths, the probability of obtaining that one path shown, and thus its interpretative power, is very low. One could try to plot a large sample of Poisson paths, which is however extremely time-intensive and not very instructive. We therefore suggest to use an average Poisson path. Our idea is the following. The random variable $q(t)$ is defined over a suitable probability space $(\Omega, \mathcal{F}, P)$, where Ω is a set of possible events ω, $\mathcal{F}$ is a σ-algebra of subsets of Ω, and P is a probability measure. We assume that $q(t)$ is driven by a Poisson process and gives the number of events in the interval $[s, t]$. Just like before, we assume its parameter is given by $\lambda > 0$, which is its mean. However, we also know that its mean is given by $E(q) = \int_\Omega q(\omega)dP(\omega)$, where $\omega \in \Omega$. The expected amount of jumps in a short interval of time is therefore given by λ itself. If we therefore solve the optimization problem under uncertainty but in each period of time we draw the expected value of a jump, then we obtain a path which represents the average path out of all possible sample paths in that point in time. We shall call this path the average Poisson path. The interpretative power of the average Poisson path is obviously much higher than that of a randomly-drawn sample path. Figure 1 shows a sample of twelve stochastic scenarios (with jumps) and the resulting average stochastic scenario.

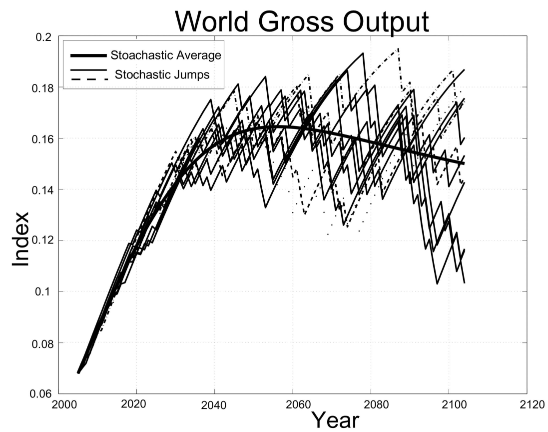

Figure 1: World Output

5.3 Figures

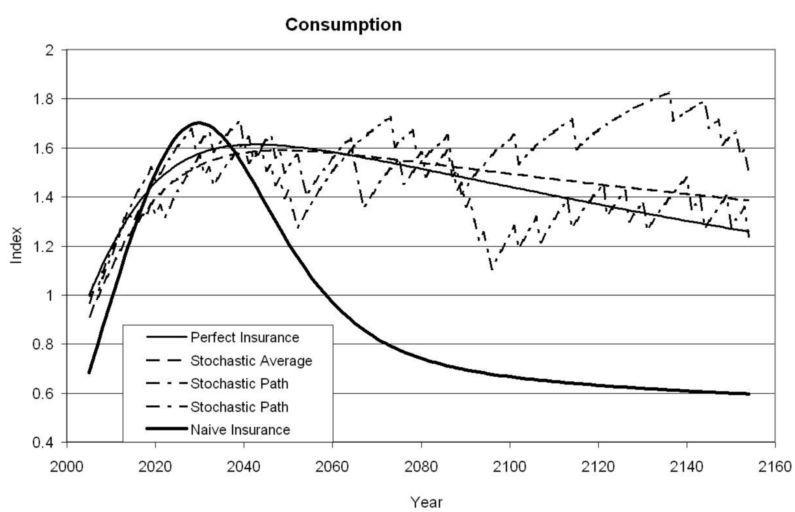

Figure 2: Consumption without technical change

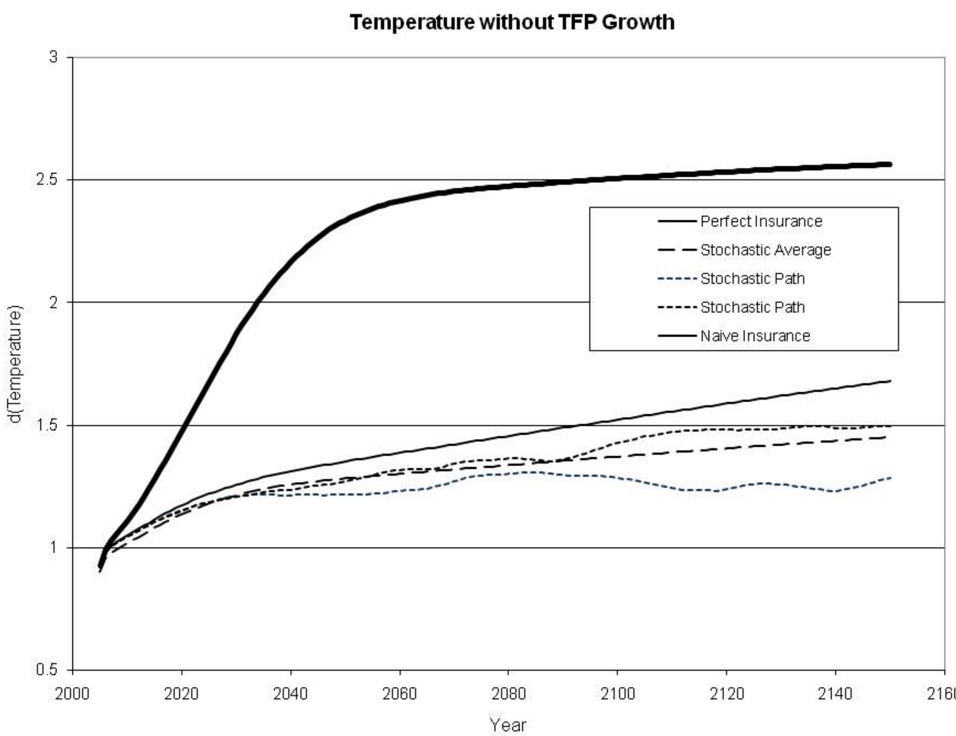

Figure 3: Temperature without technical change

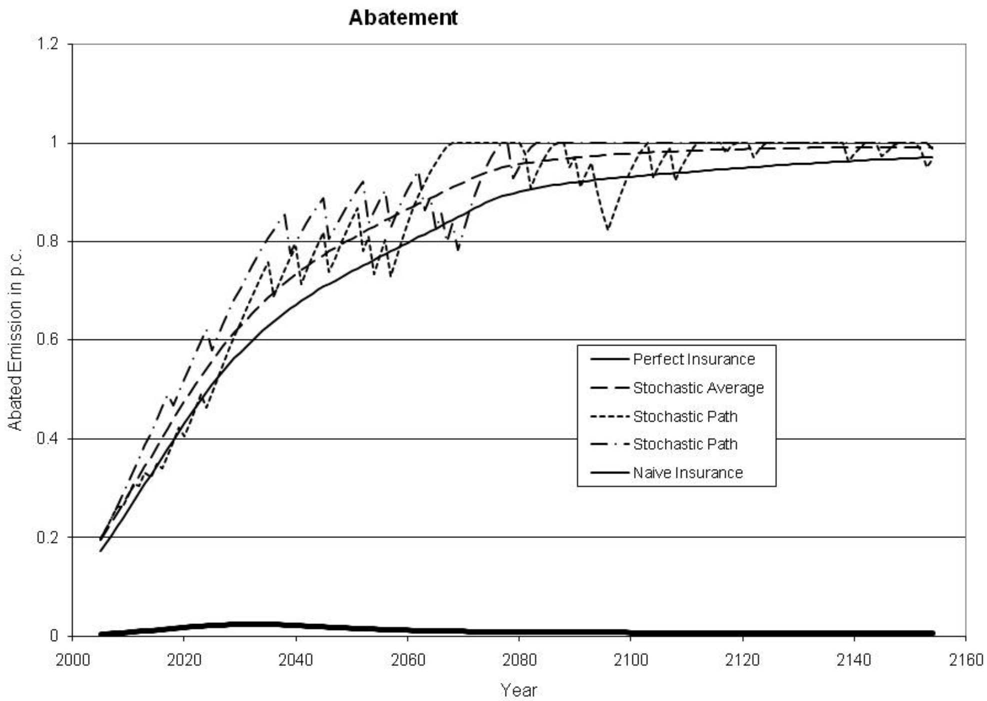

Figure 4: Abatement without technical change

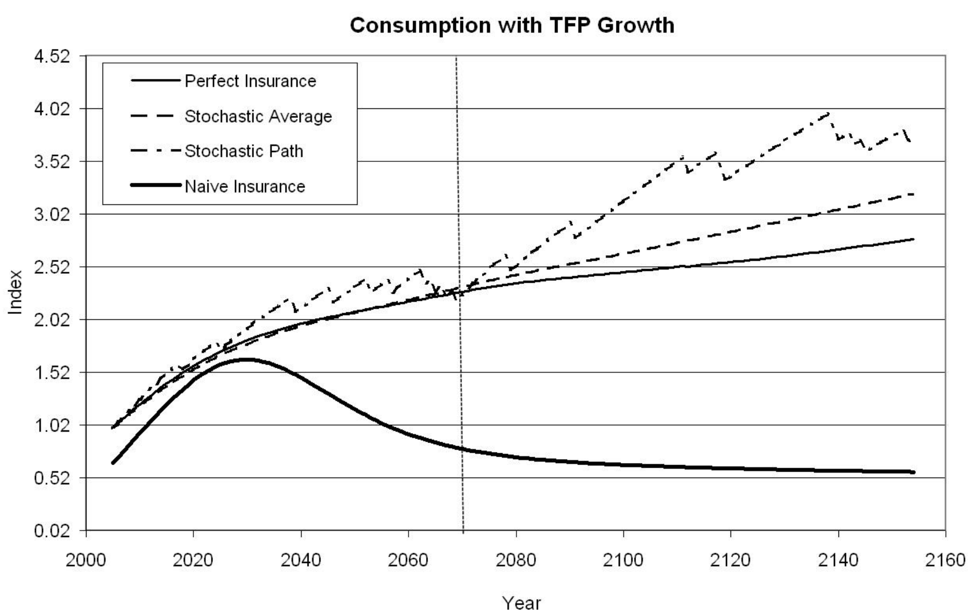

Figure 5: Consumption with technical change

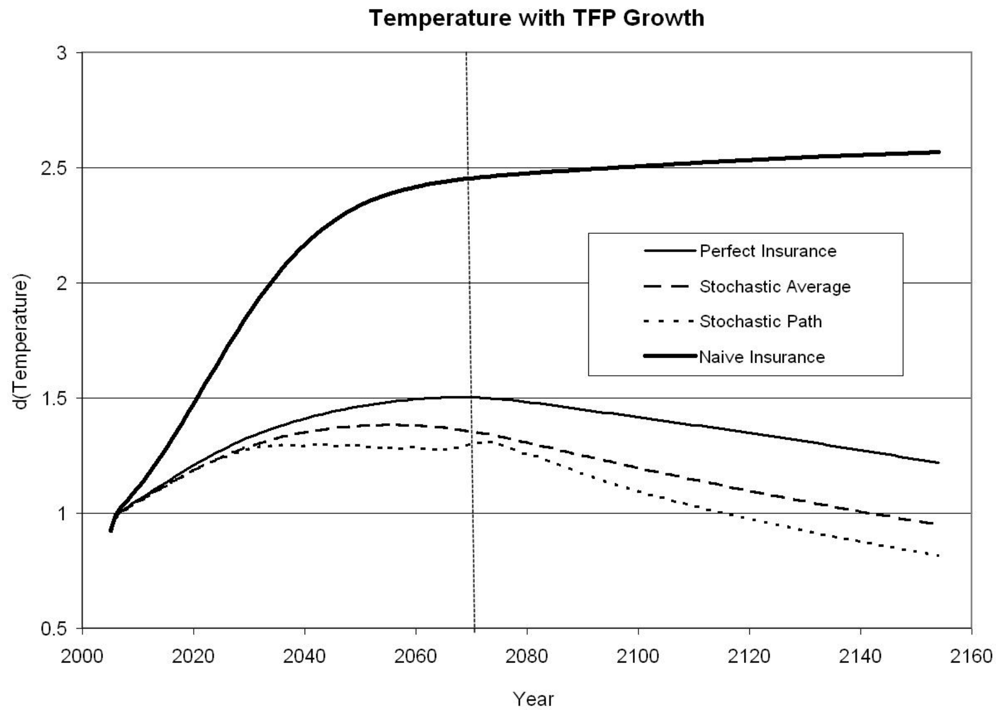

Figure 6: Temperature with technical change

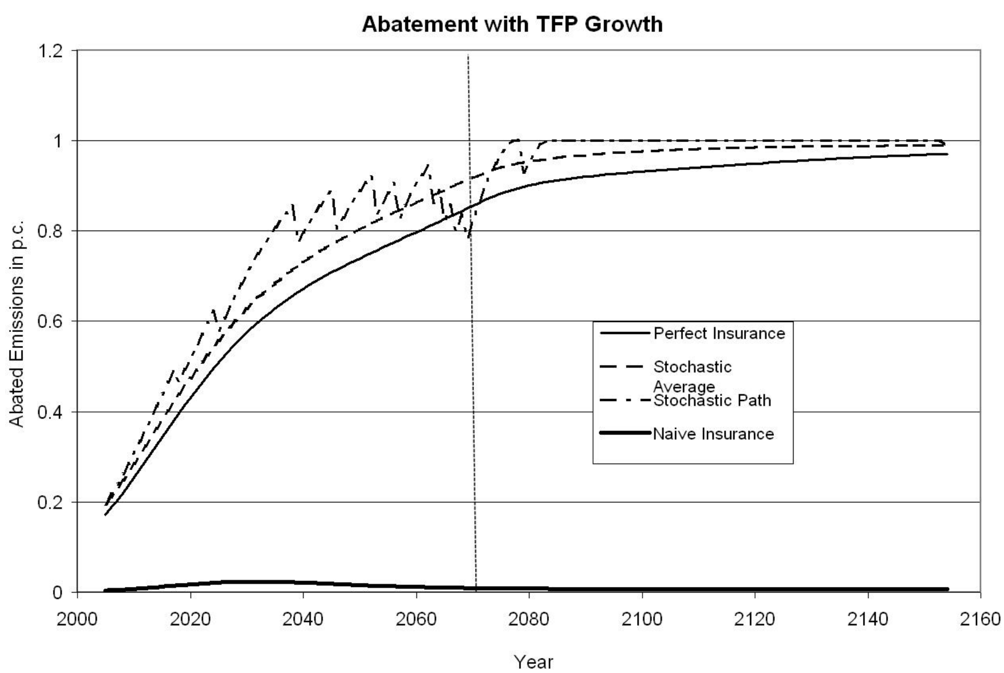

Figure 7: Abatement with technical change

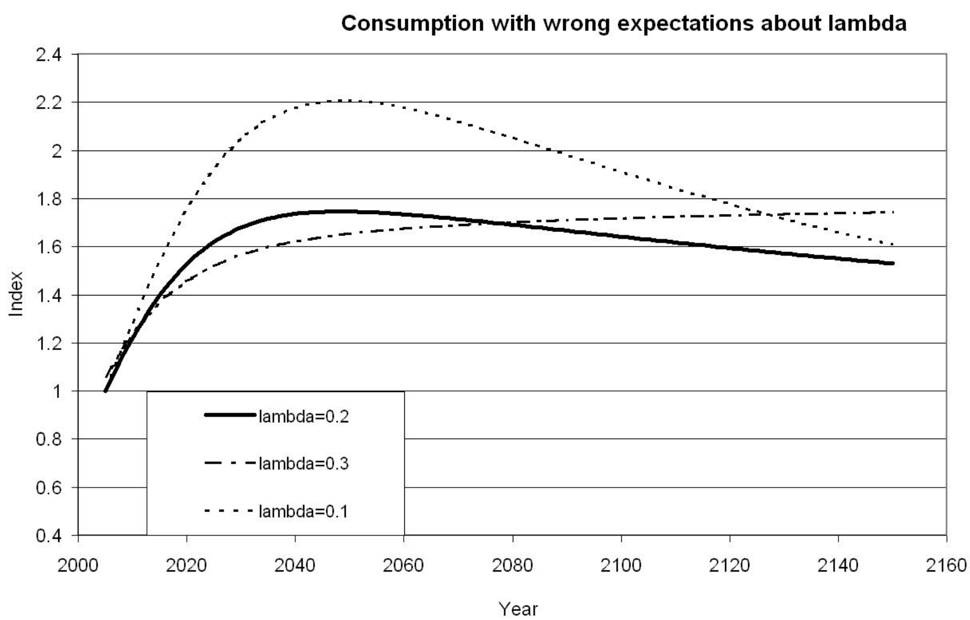

Figure 8: Consumption under false beliefs

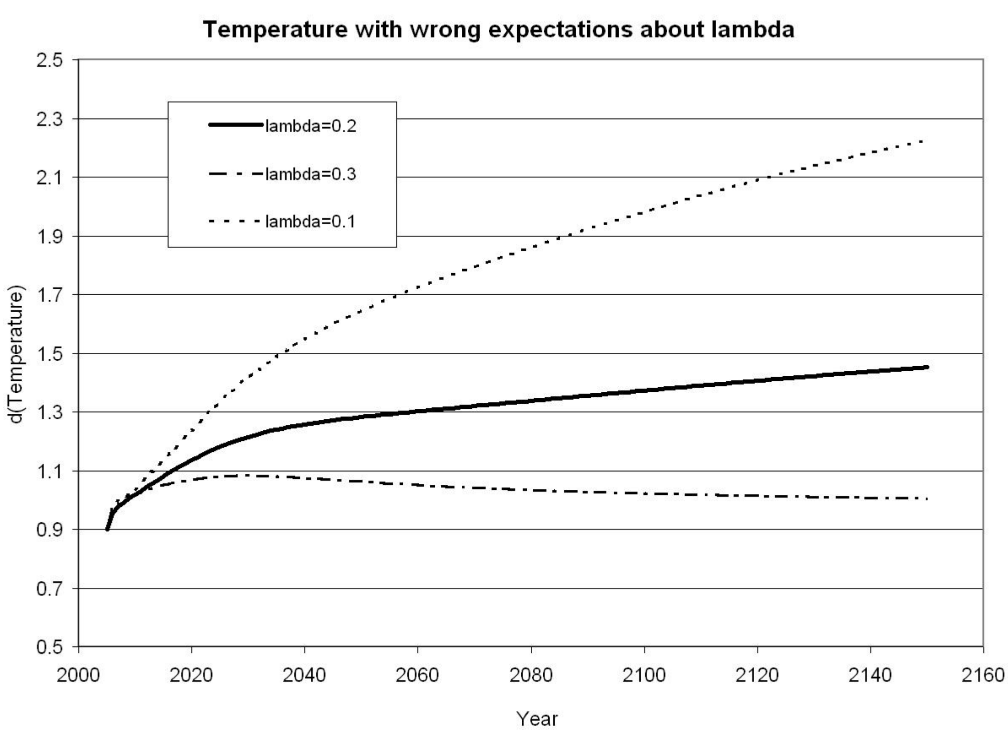

Figure 9: Temperature under false beliefs

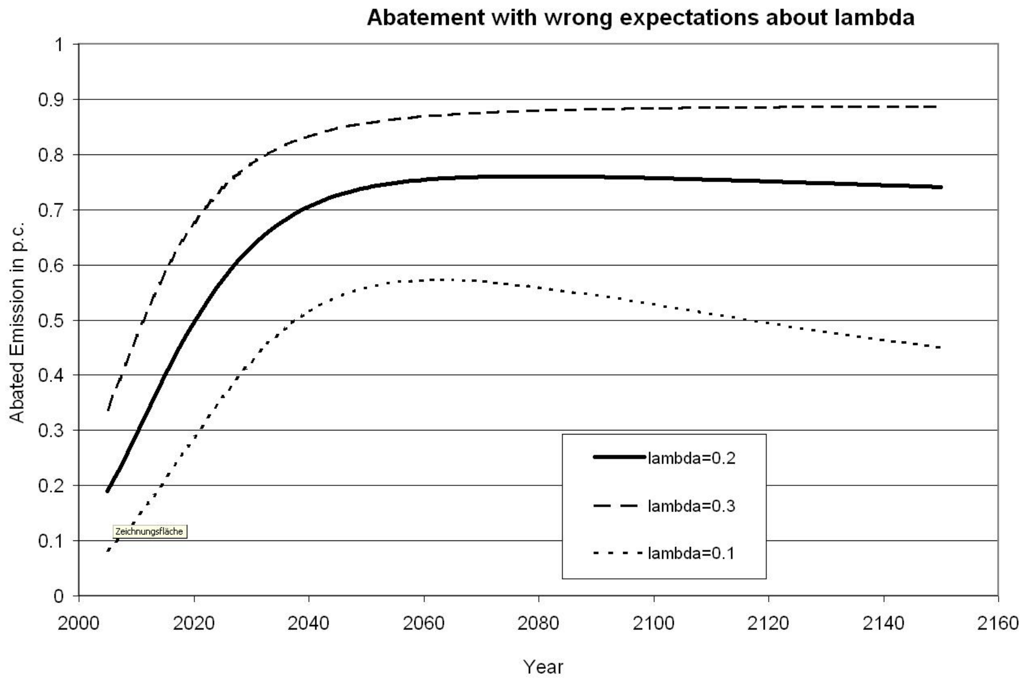

Figure 10: Abatement under false beliefs

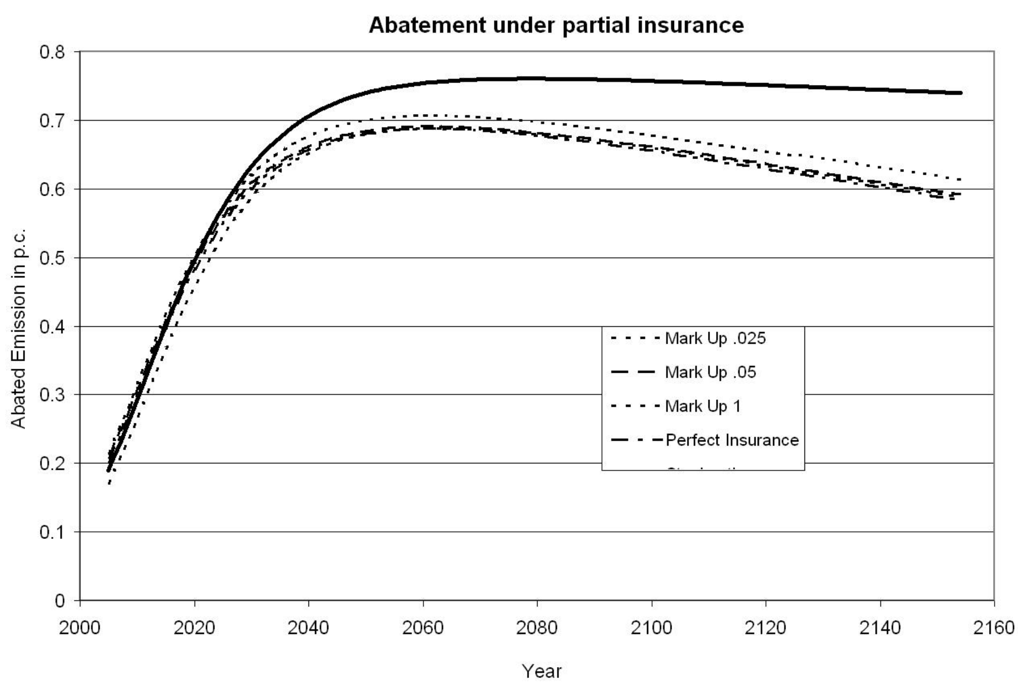

Figure 11: Abatement under partial insurance

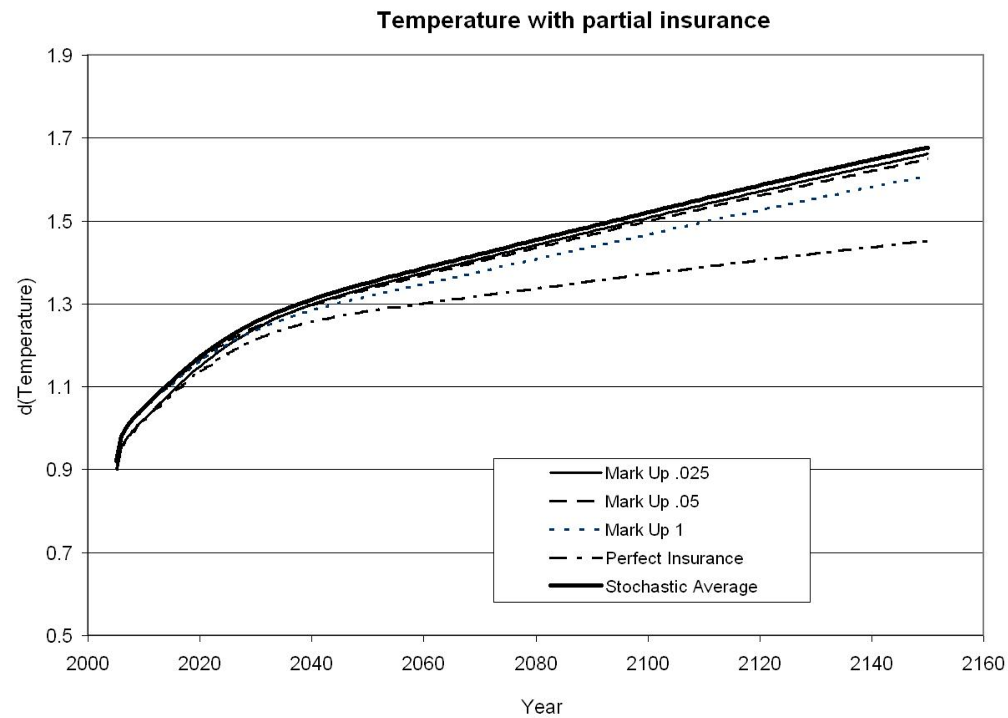

Figure 12: Temperature under partial insurance

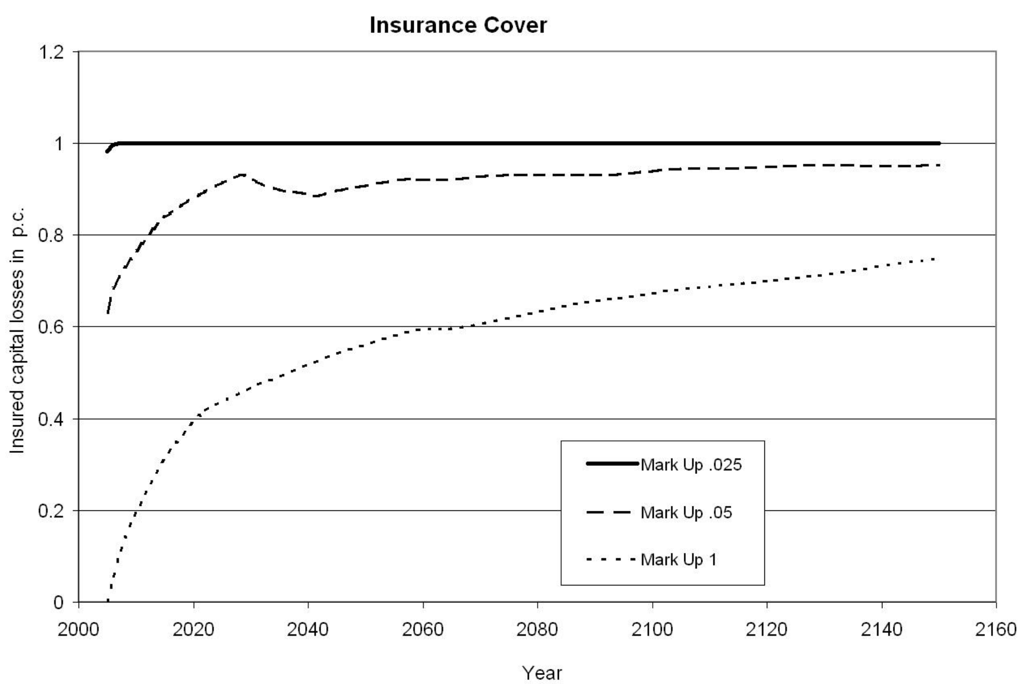

Figure 13: Insurance Cover under partial insurance