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Markets with Technological Progress: Pricing, Quality, and Novelty

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Markets with Technological Progress: Pricing, Quality, and Novelty☆

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Abstract

New and old products differ in two respects: quality and newness. Whereas a higher quality of a new product always benefits consumers, the newness itself benefits some consumers, but not others, and for some, it is even a disadvantage. We capture these features in a Hotelling model of OverLapping Innovators (HOLI model), entailing a sequence of static Hotelling games of horizontal product differentiation (newness), that we extend by vertical product differentiation (quality). In this model, the firms compete on quality and price. Using advanced dynamic hedonic regression methods, we empirically investigate the pricing policy of firms in the German laser printer market. We show that their pricing corresponds to our model with the seller of the new product acting as the Stackelberg follower.

Keywords: Hotelling, vertical product differentiation, hedonic regression, Stackelberg, laser printer

JEL classification: L11, L63, C23

1. Introduction

Many markets are characterized by regular product innovations and improvements. As a rule, when new products of higher quality enter the market, they do not immediately replace all older ones. Therefore, at any point in time, product variants of different

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quality and vintage coexist and only the products with the worst price-performance ratio exit the market.

However, higher quality is not the only difference between new and old products. A second distinctive feature of new products is their novelty. Most consumers would still differentiate between new and old products, even if they were of the same quality. Products that have been introduced long ago, may be considered as no longer up-to-date or simply "boring". Consumers may fear that buying these products will make them seem dull individuals to others, whereas consumers buying a newly introduced product may expect to be regarded as modern and interesting. Other consumers, however, may prefer established products, if they are doubtful about the quality and ease of use of the new product. These consumers continue to buy the time-tested products as long as they are available.

In summary, whereas a higher quality of a new product always benefits consumers, the product's newness benefits some consumers, but not others, and for some, it is even a drawback. Therefore, the analysis of markets characterized by continuous technical improvements should account for both quality and novelty.

In the industrial organization literature, competition between products of different quality is generally modeled as vertical product differentiation, whereas differentiated consumer tastes are commonly captured by horizontal product differentiation in the tradition of Hotelling's (1929) spatial model. This suggests that markets characterized by technical progress should be studied by means of models that combine horizontal differentiation (capturing different preferences for novelty) with vertical differentiation (capturing product quality differences). In the present paper, we specify a simple *Vertically Extended Hotelling (VEH)* model. It combines horizontal and vertical product differentiation and can easily be embedded into a dynamic context.

In our VEH model, all consumers appreciate the difference in quality (the vertical characteristic) in the same way. However, they differ in their preferences for novelty (the horizontal characteristic). The incumbent product is an established commodity of basic quality and zero novelty that competes against the entirely novel entrant product. The seller of the entrant product can choose a quality that is superior to the basic quality of the incumbent product. The cost of the entrant product increases with the magnitude of the quality improvement. In this setup, the duopolists compete on price and quality.

Almost all studies of Hotelling's spatial model utilize the Nash equilibrium concept, and our VEH model is no exception. However, competition between an incumbent and an entrant product is not necessarily symmetric. Therefore, we also consider two additional scenarios. In the first, we capture the asymmetry by a Stackelberg game in which the seller of the incumbent product is the Stackelberg leader and competes against the seller of the entrant product (the Stackelberg follower), whereas in the second scenario, we consider the reverse situation.

Generally, two opposing pricing strategies appear sensible. When consumers appreciate the novelty of new products, firms should introduce new products into the market at prices above those of the older products of comparable quality, and during the later stages of the product's life cycle, price it below newer products of comparable quality. In the marketing literature, this strategy is often referred to as *skimming* (e.g., Noble and Gruca, 1999). However, if consumers are doubtful about new products, the sellers of new products can counter such initial scepticism by an aggressive pricing policy during the early stages of the product life cycle. Once a sufficiently large customer base has been established, in the later stages of the product life cycle, the price is set above those of newly entering rival products of comparable quality. In line with the marketing literature, we denote this strategy as *penetration*.

Does our VEH model predict skimming or penetration? It will be shown that the prediction depends on the underlying equilibrium concept. If the VEH model makes the Nash assumption or the Stackelberg assumption with the seller of the entrant product being the Stackelberg leader, firms decide to skim. In contrast, if the seller of the incumbent product is the Stackelberg leader, the model predicts penetration.

Which of the three alternative assumptions is the most plausible one? As theoretical considerations remain debatable, an empirical examination is necessary. Fortunately, our theoretical findings suggest a specific empirical approach. Using advanced dynamic hedonic regression methods, we can investigate whether skimming or penetration prevails in the market under consideration. If penetration prevails, the VEH model with the seller of the novel product acting as Stackelberg follower is the best tool for analysing this market. If skimming prevails, the VEH model with either the

Nash assumption or the seller of the established product acting as Stackelberg follower would fit better.

Therefore, in addition to the analytical contribution, this paper also provides an empirical case study of the German laser printer market. This is a mature market with technological progress and rapid changes in the availability of product variants. Laser printer prices tend to decline substantially over their life cycle, which *prima facie* looks like skimming. However, the observed raw prices are not the relevant ones, as model verification requires quality-adjusted prices of laser printers. Using the Markov Chain Monte Carlo (MCMC) method, we derive such quality-adjusted prices. Only if novel printer models are sold at a larger quality-adjusted price than established printer models, can we conclude that skimming prevails.

Markets characterized by frequent product turnover and technical progress, are dynamic in nature, whereas our VEH model is static. Therefore, we extend the VEH model to a dynamic context. For this purpose, we assume that the duopoly game is played repeatedly with today's entrant product being the incumbent product of the next period, while today's incumbent product leaves the market and is replaced by a new entrant product. This generates an infinite-horizon model with overlapping products (firms), in the same manner as common overlapping generation models. We refer to this as the *Hotelling model of OverLapping Innovators (HOLI model)*. It is complex enough to account for the key aspects of markets with product turnover and technical progress. At the same time, it is sufficiently simple to generate, for each of the three scenarios that we consider (the Nash scenario and the two Stackelberg scenarios), a unique equilibrium path.

To the best of our knowledge, the HOLI model is the first to merge horizontal and vertical product differentiation into a dynamic framework capturing continuing technical progress and product turnover. Since the HOLI model is a sequence of static VEH models, the literature most closely related to our approach is that which explores static models combining horizontal and vertical product differentiation. However, in this body of literature, the nexus between novelty and quality is never considered. Furthermore, all of these studies are confined to Nash equilibria and few can be classified as VEH models.

The paper is organized as follows. Section 2 introduces the VEH model, and in Section 3, we derive the subgame-perfect equilibrium for the Nash scenario and the two Stackelberg scenarios. Section 4 introduces the HOLI model. The related literature is discussed in Section 5. Section 6 presents the empirical case study of the laser printer market and relates it to the predictions of the HOLI model. Section 7 concludes.

2. The VEH Model

We consider a VEH model featuring an *incumbent product* (Product I) and an *entrant product* (Product E) that compete on price and quality. Product I is offered at price P_I and Product E at price P_E.

2.1. Consumer Rent

The consumers are of mass 1 and uniformly distributed along the interval $[0, 1]$. The consumer's location is equivalent to her taste parameter $x \in [0, 1]$. Each consumer can buy either one unit of Product I or one unit of Product E or no unit at all. The established Product I exactly matches the taste of the consumer located at $x = 0$ and the novel Product E exactly matches the taste of the consumer located at $x = 1$ (horizontal product differentiation). More specifically, the consumer rents derived from the products I and E are defined by

$$\begin{aligned} U_I(P_I) &= Q - tx - P_I \\ U_E(P_E) &= Q + \Delta - t(1 - x) - P_E \ , \end{aligned}$$

where Q is consumer x's willingness to pay for a product that conforms precisely to her own taste and has the same quality as Product I. The difference in quality between products E and I is indicated by Δ (vertical product differentiation). The parameter $t > 0$ measures the intensity of preferences, that is, the sensitivity of consumer rent with respect to the distance between the consumer's location x and the product's location ($x_I = 0$ and $x_E = 1$). The larger the t, the greater the extent to which consumers dislike a given distance between their own and the product's location.

Without loss of generality, the consumer rents can be expressed in units of t:

$$u_I(p_I) = q - x - p_I \tag{1}$$

$$u_E(p_E) = q + \delta - (1 - x) - p_E \tag{2}$$

where $u_I(p_I) = U_I(P_I)/t$, $u_E(p_E) = U_E(P_E)/t$, $q = Q/t$, $\delta = \Delta/t$, $p_I = P_I/t$, and $p_E = P_E/t$.

2.2. Consumer Demand

The consumer who is indifferent between products I and E is denoted as the marginal consumer $\bar{x}$. Equations (1) and (2) imply that the marginal consumer is located at

$$\bar{x} = \frac{1}{2}(p_E - p_I + 1 - \delta) \ . \tag{3}$$

We assume that the basic quality q is sufficiently large to ensure that at equilibrium, the consumer rent (1) of the indifferent consumer $\bar{x}$ is nonnegative:

$$p_E + p_I \leq 2q + \delta - 1 \ . \tag{4}$$

As a consequence, each consumer will buy either of the two products. We refer to inequality (4) as the *maturity condition*.

We denote the demand for products I and E with D_I and D_E. The demand functions of products I and E directly follow from the marginal consumer $\bar{x}$:

$$D_I = \frac{1}{2}(p_E - p_I + 1 - \delta) \tag{5}$$

$$D_E = \frac{1}{2}(p_I - p_E + 1 + \delta) \ . \tag{6}$$

The demand functions (5) and (6) depend on the quality differential δ and on both prices p_E and p_I. A price increase of the incumbent product will increase the customer base of the entrant product and *vice versa*.

2.3. Cost and Profits

It is assumed that the cost of producing the basic quality level q is always zero. For given prices p_I and p_E, the demand functions (5) and (6) imply a positive relationship between the quality differential δ chosen by firm E, and Product E's customer base.

If the cost of Product E were invariant with respect to the choice of δ, firm E would always choose an infinitely large quality differential δ. It is more realistic that the cost c of Product E increases with its quality differential δ. We model this relationship by the following simple cost function:

$$c = q\delta^2 \qquad (q > 0) \; .$$

This specification implies that the marginal cost of quality is increasing. Furthermore, the larger the basic quality q, the more costly a given quality differential δ.

Then, the profit functions are

$$\pi_I = p_I \left(\frac{1}{2} (p_E - p_I + 1 - \delta) \right) \qquad (7)$$

$$\pi_E = p_E \left(\frac{1}{2} (p_I - p_E + 1 + \delta) \right) - q\delta^2 \; . \qquad (8)$$

2.4. Three Different Scenarios

It takes time for new products of higher quality to achieve marketability. Therefore, we model our duopoly game as a two-stage game in which the first stage is firm E's choice of quality differential δ. The quality of the incumbent product is given. Otherwise it would be a new product and not an incumbent product.

The second stage of our duopoly game is the price game. When modelling markets in which product variants of different quality and vintage coexist, the appropriate equilibrium concept for this second stage is not obvious. Therefore, we consider three different equilibrium concepts.

Stackelberg scenario I→E: First, firm I chooses p_I, anticipating firm E's optimal reaction p_E. Afterwards, firm E chooses p_E. In other words, firm I is the Stackelberg leader, whereas firm E is the Stackelberg follower. It emerges that in this stage of the game, the Stackelberg follower has a strategic advantage.

Stackelberg scenario E→I: This is the reverse situation of the Stackelberg scenario I→E. Here, firm E is the Stackelberg leader, whereas firm I is the Stackelberg follower.

Nash scenario: Neither firm anticipates the other firm's price. Instead, each firm merely reacts to the other firm's price. The equilibrium is defined by those prices p_I and p_E at which neither firm wishes to change its own price, given that of the other firm.

3. Equilibrium Analysis

The equilibrium is derived by backward induction. We begin with the second stage (price game) and take the quality differential δ as given.

3.1. Stackelberg Scenario I→E

The marginal profit of firm *E* is

$$\frac{\partial \pi_E}{\partial p_E} = \underbrace{\frac{1}{2}(p_I - p_E + 1 + \delta)}_{\text{inframarginal}} - \underbrace{\frac{1}{2}p_E}_{\text{marginal}} \ . \tag{9}$$

The first component in (9) reflects the revenue gains from the existing customer base (*inframarginal effects*). The second component reflects the revenue losses caused by the contraction of the customer base (*marginal effects*). Setting (9) equal to zero yields firm *E*'s reaction function

$$p_E(p_I) = \frac{1}{2}(p_I + 1 + \delta) \ . \tag{10}$$

Noting that $\mathrm{d}p_E/\mathrm{d}p_I = 1/2$, the marginal profit of firm *I* (Stackelberg leader) can be written as

$$\frac{\partial \pi_I}{\partial p_I} = \underbrace{\frac{1}{2}(p_E - p_I + 1 - \delta)}_{\text{inframarginal}} - \underbrace{\frac{1}{2}p_I}_{\text{marginal}} + \underbrace{\frac{1}{4}p_I}_{\text{complementary}} \ . \tag{11}$$

As before, the first two components are the inframarginal and marginal effects. The last component, $(1/4)p_I$, is the positive effect of firm *E*'s price reaction (as a response to a unit price increase by the firm *I*) on the customer base of firm *I* (*complementary effect*).

Inserting (10) into (11) and setting the resulting expression equal to zero yields firm *I*'s optimal price

$$p_I^* = \frac{1}{2}(3 - \delta) \ . \tag{12}$$

Inserting this price into (10) yields firm E's optimal price

$$p_E^* = \frac{1}{4}(5+\delta) \ . \tag{13}$$

Inserting prices (12) and (13) into the profit functions (7) and (8) yields

$$\pi_I^* = \frac{1}{16}\delta^2 - \frac{3}{8}\delta + \frac{9}{16} \tag{14}$$

$$\pi_E^* = \frac{5}{16}\delta + \frac{1}{32}\delta^2 - q\delta^2 + \frac{25}{32} \ . \tag{15}$$

Next, we turn to the first stage of the game. Maximizing expression (15) with respect to δ gives firm E's profit maximizing quality differential

$$\delta^* = \frac{5}{32q-1} \ . \tag{16}$$

Inserting δ^* into (12) and (13) yields

$$p_I^* = \frac{48q-4}{32q-1} \tag{17}$$

$$p_E^* = \frac{40q}{32q-1} \ . \tag{18}$$

Solutions (16) to (18) characterize the equilibrium of the Stackelberg scenario I→E. This equilibrium is consistent with the maturity condition (4), if and only if

$$q \geq 1.8204 \ . \tag{19}$$

Does the Stackelberg equilibrium (16) to (18) represent skimming or penetration? In order to answer this question, the quality-adjusted equilibrium prices $\hat{p}_I^* = p_I^*/q$ and $\hat{p}_E^* = p_E^*/(q+\delta^*)$ are required. Penetration prevails if $\hat{p}_I^*/\hat{p}_E^* > 1$, and skimming if $\hat{p}_I^*/\hat{p}_E^* < 1$. The ratio of quality-adjusted equilibrium prices is

$$\frac{\hat{p}_I^*}{\hat{p}_E^*} = \frac{p_I^*}{p_E^*}\frac{q+\delta^*}{q} = \left(\frac{12q-1}{10q}\right)\left(1+\frac{5}{(32q-1)\,q}\right) .$$

For all q-values satisfying the maturity assumption (19), this ratio yields $\hat{p}_I^*/\hat{p}_E^* > 1.186$. Therefore, our analysis can be summarized as follows.

Proposition 1. *In the Stackelberg scenario I→E, penetration prevails* ($\hat{p}_I^* > \hat{p}_E^*$).

The upper solid line in Figure 1 plots the values of the price ratio $\hat{p}_I^*/\hat{p}_E^*$ as a function of q. The other two solid lines and the points can be ignored for the moment. For $q \to \infty$, the ratio approaches $6/5$ from below.

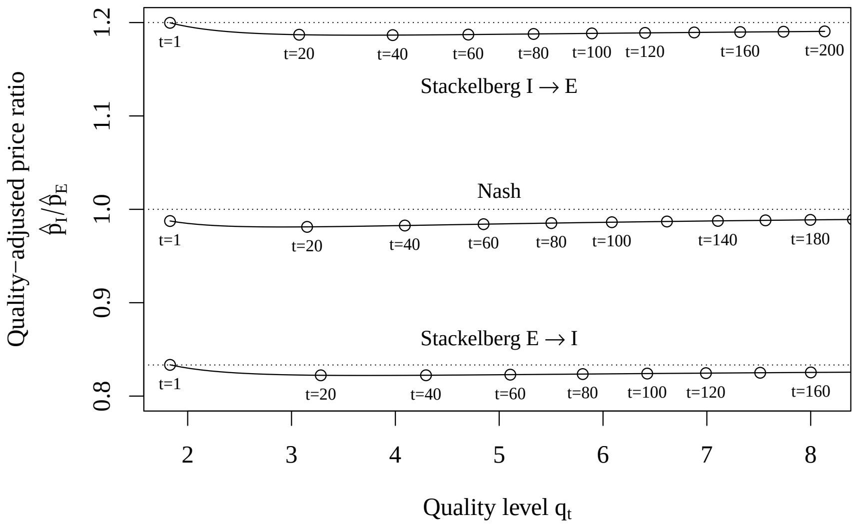

Figure 1: Quality-adjusted equilibrium prices and evolution of quality levels q_t over time in the three scenarios of the VEH model.

3.2. Stackelberg Scenario E→I

In the second scenario, firm I is the Stackelberg follower. Maximizing (7) with respect to p_I yields the reaction function of firm I:

$$p_I(p_E) = \frac{1}{2}(p_E + 1 - \delta) \ . \tag{20}$$

In this scenario, the positive complementary effect devolves to firm E. The derivation of the equilibrium is perfectly symmetric to the derivation in the Stackelberg scenario I→E. For the second stage of the game, this yields the optimal prices

$$\begin{aligned} p_I^{**} &= \frac{1}{2}(3 + \delta) \\ p_E^{**} &= \frac{1}{4}(5 - \delta) \ . \end{aligned}$$

Inserting these prices into the profit functions (7) and (8) yields

$$\pi_I^{**} = \frac{1}{32}\delta^2 - \frac{5}{16}\delta + \frac{25}{32} \tag{21}$$

$$\pi_E^{**} = \frac{3}{8}\delta + \frac{1}{16}\delta^2 - q\delta^2 + \frac{9}{16} \ . \tag{22}$$

Maximizing expression (22) with respect to δ gives the following equilibrium:

$$\begin{aligned} \delta^{**} &= \frac{3}{16q-1} \\ p_I^{**} &= \frac{20q-2}{16q-1} \\ p_E^{**} &= \frac{24q}{16q-1} \;. \end{aligned} \tag{23}$$

This equilibrium is consistent with the maturity condition (4), if and only if

$$q \geq 1.8353 \;. \tag{24}$$

The ratio of quality-adjusted prices is

$$\frac{\hat{p}_I^{**}}{\hat{p}_E^{**}} = \frac{p_I^{**}}{p_E^{**}} \frac{q+\delta^{**}}{q} = \left(\frac{10q-1}{12q}\right)\left(1+\frac{3}{16q^2-q}\right) \;.$$

For all q-values satisfying the maturity assumption (24), this ratio gives $\hat{p}_I^{**}/\hat{p}_E^{**} < 5/6$.

Proposition 2. *In the Stackelberg scenario E→I, skimming prevails* ($\hat{p}_I^{**} < \hat{p}_E^{**}$).

The lower solid line in Figure 1 plots the values of the price ratio $\hat{p}_I^{**}/\hat{p}_E^{**}$ as a function of q. For $q \to \infty$, the ratio approaches $5/6$ from below.

3.3. Nash Scenario

Both firms choose their prices simultaneously. The reaction functions (10) and (20) yield the following second stage equilibrium prices:

$$p_I^{***} = \frac{1}{3}(3-\delta) \tag{25}$$

$$p_E^{***} = \frac{1}{3}(3+\delta) \;. \tag{26}$$

Inserting these two prices into the profit functions (7) and (8) yields

$$\pi_I^{***} = \frac{1}{18}\delta^2 - \frac{1}{3}\delta + \frac{1}{2} \tag{27}$$

$$\pi_E^{***} = \frac{1}{3}\delta + \frac{1}{18}\delta^2 - q\delta^2 + \frac{1}{2} \;. \tag{28}$$

Maximizing expression (28) with respect to δ gives firm E's optimal quality differential

$$\delta^{***} = \frac{3}{18q-1} \;. \tag{29}$$

Inserting δ^{***} into (25) and (26) yields

$$p_I^{***} = \frac{18q-2}{18q-1}$$
$$p_E^{***} = \frac{18q}{18q-1} .$$

Inserting these prices and (29) into the maturity condition (4) yields

$$q \geq 1.4398 . \quad (30)$$

The ratio of quality-adjusted prices is

$$\frac{\hat{p}_I^{***}}{\hat{p}_E^{***}} = \frac{p_I^{***}}{p_E^{***}} \frac{q+\delta^{***}}{q} = \left(1-\frac{1}{9q}\right)\left(1+\frac{3}{(18q-1)\,q}\right) .$$

For all q-values satisfying the maturity assumption (30), this ratio gives $\hat{p}_I^{***}/\hat{p}_E^{***} \leq 1$.

Proposition 3. *In the Nash scenario, skimming prevails* ($\hat{p}_I^{***} \leq \hat{p}_E^{***}$).

The solid line in the middle of Figure 1 plots the values of the price ratio $\hat{p}_I^{***}/\hat{p}_E^{***}$ as a function of q. For $q \to \infty$, the ratio approaches 1 from below.

3.4. Further Results

Inspection of the equilibrium prices of the three scenarios reveals that

$$\hat{p}_I^{***} < \hat{p}_I^{**} < \hat{p}_I^{*} \qquad \text{and} \qquad \hat{p}_E^{***} < \hat{p}_E^{*} < \hat{p}_E^{**} . \quad (31)$$

For both firms, the prices in the two Stackelberg scenarios are larger than in the Nash scenario. These differences are due to the complementary effect in the profit maximization of the Stackelberg leader. In the Nash scenario, there is no such price-increasing effect.

In the second stage of our VEH model, the Stackelberg follower has a strategic advantage. Inserting (16) into (14) and (15), (23) into (21) and (22), and (29) into (27) and (28) yields the following findings.

Proposition 4. *The larger prices of the Stackelberg scenarios translate into larger profits for both firms:*

$$\pi_I^{***} < \pi_I^{*} < \pi_I^{**} \qquad \textit{and} \qquad \pi_E^{***} < \pi_E^{**} < \pi_E^{*} .$$

Firm I earns a lower profit than firm E, unless firm I is the Stackelberg follower (Stackelberg scenario E→I):

$$\frac{\pi_I^*}{\pi_E^*} < \frac{\pi_I^{***}}{\pi_E^{***}} < 1 < \frac{\pi_I^{**}}{\pi_E^{**}} \,. \tag{32}$$

Which part of these findings is caused by firm E's ability to choose the quality differential δ? To answer this question, it is necessary to consider the equilibrium that would arise without a quality differential ($\delta = 0$). In that case, the two products are of identical quality and the two-stage game simplifies to a one-stage game, namely the simplest variant of Hotelling's spatial model: Both firms have fixed locations at the end-points of the interval representing consumer tastes ($x_I = 0$ and $x_E = 1$). In the Stackelberg scenario I→E, this setup would lead to $\pi_I^*/\pi_E^* = 18/25$, whereas the Stackelberg scenario E→I would yield $\pi_I^{**}/\pi_E^{**} > 25/18$. Of course, in the Nash scenario, this symmetric setup would lead to $\pi_I^{***}/\pi_E^{***} = 1$.

It can be seen from firm E's profit function (15) that in the Stackelberg scenario I→E, firm E can always choose a sufficiently small value of δ such that its profit increases relative to the situation $\delta = 0$. At the same time, firm I's profit function (14) reveals that firm E's decision to raise δ above 0 lowers firm I's profit. The same argument applies to the Stackelberg scenario E→I and to the Nash scenario. Accordingly, in all three scenarios, firm E's additional choice variable δ increases its profits and reduces the profits of firm I relative to the case with $\delta = 0$. Furthermore, it can easily be verified that the profit reduction of firm I is larger than the profit increase of firm E. In other words, total profits fall when firm E has the option of offering a better product than firm I.

Figure 1 shows that

$$\frac{\hat{p}_I^{**}}{\hat{p}_E^{**}} < \frac{\hat{p}_I^{***}}{\hat{p}_E^{***}} < 1 < \frac{\hat{p}_I^{*}}{\hat{p}_E^{*}} \,. \tag{33}$$

This implies that in the Stackelberg scenario I→E (firm E is Stackelberg follower) penetration ($\hat{p}_I > \hat{p}_E$) prevails, wheras in the Nash scenario and even more so in the Stackelberg scenario E→I (firm I is Stackelberg follower) skimming ($\hat{p}_I < \hat{p}_E$) prevails.

In order to model a market characterized by continuous technical improvements, it would be extremely useful to know which of the three scenarios of our VEH model is the most plausible. Though our theoretical findings do not provide us with a direct answer, they suggest an empirical approach for determining the most plausible scenario. Applying hedonic regression analysis, it is possible to examine whether new products are offered at lower or higher prices than existing products of comparable quality. In other words, hedonic regression can be used to determine whether skimming or penetration prevails. If skimming prevails, then the Stackelberg scenario E→I or the Nash scenario would provide the best fit. Unfortunately, it would remain unclear which of the two scenarios were more appropriate. However, if penetration prevailed, the Stackelberg scenario I→E would be the most suitable and no ambiguity would arise. Therefore, Section 6 presents a hedonic regression analysis of the laser printer market.

4. The HOLI Model

When a market is characterized by frequent product turnover and ongoing technical progress, the products usually exhibit an ever-improving cost-performance ratio. Such markets are dynamic in nature, whereas our VEH model is static. Therefore, we embed the VEH model into a dynamic framework that can account for such features as an improving cost-performance ratio. We call the resulting model the Hotelling model of OverLapping Innovaters (HOLI model).

The HOLI model is a sequence of VEH models. The two-stage game of the VEH model is played repeatedly. Before the start of the present period, the last period's incumbent product leaves the market and is replaced by a new entrant product, while the last period's entrant product is the present incumbent product of quality q. The present entrant product is new to the market. It is of quality $q + \delta$, where δ is the quality differential chosen by the present firm E.

At the end of the period, the present Product I exits the market, and the present Product E becomes the Product I of the next period. The quality of this product remains unchanged at $q' := q + \delta$. It will then compete with a new entrant product which will have the quality $q' + \delta'$, where δ' is the quality differential chosen by the new firm E. Therefore, the supply conditions during the new period are exactly the same as in the previous period, the only difference being the improved basic quality of the products I and E (q' instead of q).

On the demand side, we assume that the consumers live forever and have constant preferences over time. In other words, their preference for novelty, that is, their location x does not change as they consume Product I or Product E. For example, consider a consumer located close to $x = 1$, who during the last period, consumed Product E. During the current period, this product is offered as Product I. In spite of being familiar with this product, the consumer's strong preference for novelty induces her to switch to the new Product E.

Under these assumptions, the outcome of the present period has no influence on the starting conditions of the next period. Each period can be analyzed separately from all other periods. Consequently, the equilibria derived for the VEH model can be transfered directly to the HOLI model. Of course, more complex extensions of the static VEH model to a dynamic framework are conceivable. However, we consider the simple structure of the HOLI model as one of its major advantages.

For each scenario and each given quality level q, we already know the equilibrium quality differential δ. Therefore, we also know the pace of technical progress for each scenario. Comparing the results (16), (23), and (29) yields the following proposition.

Proposition 5. *For each given quality level q, the technical progress, δ, is faster in the Stackelberg scenario E→I (incumbent is Stackelberg follower) than the Nash scenario which in turn has a faster technical progress than in the Stackelberg scenario I→E (entrant is Stackelberg follower):*

$$\delta^* < \delta^{***} < \delta^{**} .$$

Figure 1 illustrates the evolution of the basic quality level over time, with t indicating the period. The points highlight the sequence of basic quality levels, q_t, arising from the entrant's optimal quality differentials, δ_t, taking the q-values from the maturity condition as the starting point (q_0).

5. Related Literature

In markets with frequent product turnover and technical progress, products are characterised by two properties: novelty and quality. It is tempting to re-interpret quality as a second horizontal characteristic and to utilize Launhardt's (1885) two-dimensional extension of Hotelling's spatial model or the multiple-dimension extension by Irmen and Thisse (1998). However, the re-interpretation of quality as a second horizontal characteristic is inappropriate. Suppose that consumers have to choose between two equally novel products offered at the same price. If quality were a horizontal characteristic, some consumers would prefer the low-quality product. However, this would contradict the very meaning of quality. Therefore, quality must be captured by vertical product differentiation.

Building upon Launhardt's (1885) work, Dos Santos Ferreira and Thisse (1996) extend the Hotelling spatial model not by quality differences, but by differences in product versatility. Therefore, it would be misleading to classify their model as a VEH one.

A model of markets characterized by continuing technical progress and product turnover should satisfy two basic requirements: it should account for the evolutionary process of such markets and it should capture the nexus between novelty and quality by combining horizontal and vertical product differentiation. To the best of our knowledge, the HOLI model is the first to satisfy these two requirements. The HOLI model is a sequence of static VEH models. Since other VEH models have been proposed in the literature, we review these VEH models and some other work combining horizontal and vertical product differentiation. We emphasize, however, that none of these studies consider the Stackelberg equilibrium concept or the nexus between novelty and quality.

The first attempt to combine horizontal and vertical product differentiation can be found in Ireland (1987), although he does not use Hotelling's spatial model. The first VEH model is proposed by Neven and Thisse (1990). In their duopoly model, consumers are differentiated with respect to both the horizontal characteristic and quality valuation. The two competing firms decide on the horizontal characteristic, the vertical characteristic, and the price of their respective products. However, the firms are restricted in their quality choice to some specific interval, and the level of quality does not affect the firm's cost. The analysis is confined to the Nash equilibrium concept and no interior equilibria can be found in this VEH model.

In view of the sparse body of literature, Lambertini (2006) concludes that horizontal and vertical product differentiation are usually studied in isolation. However, since 2006, several studies have been published that combine horizontal and vertical product differentiation. Deltas and Zacharias (2006) consider the sub-game perfect Nash equilibrium of a two-period game in which every consumer buys one unit of a durable product, but must choose between period 1 in which only the low quality product is available, and period 2 in which there is a choice between the old low-quality product and a better-quality new one. The subgame of the second period resembles our VEH model, although the customer base is truncated to those consumers who abstained from purchasing during the first period. Furthermore, the quality differential δ is exogenous.

Some important studies are concerned with the link between imperfect competition and incomplete information about product quality; e.g. Daughety and Reinganum (2007, 2008), Gabszewicz and Resende (2012). However, the underlying models cannot be classified as VEH models. A notable exception is Levin et al. (2009), who propose a VEH model that is similar to ours. However, in their model, quality is an exogenous random variable and the firms can decide whether they want to disclose the quality of their respective product.

Our HOLI model can be interpreted as a repeated duopoly game with equilibrium price paths that depend on the applied equilibrium concept (Stackelberg versus Nash). None of the models listed in this literature review can be viewed as a repeated game. Furthermore, they are not concerned with the product characteristic of novelty and they exclusively utilize the Nash equilibrium concept. This is the standard equilibrium concept applied in the context of Hotelling's spatial model. One notable exception to this rule is Anderson (1987), who applies the Stackelberg equilibrium concept to Hotelling's original spatial model and shows that Stackelberg equilibria may exist where Nash equilibria do not.

6. Case Study: Laser Printer Market

The HOLI model predicts that penetration ($\hat{p}_I > \hat{p}_E$) prevails in the Stackelberg scenario I→E, wheras skimming ($\hat{p}_I < \hat{p}_E$) prevails in the Nash scenario and even more so in the Stackelberg scenario E→I. Unfortunately, it is unclear whether the Stackelberg scenario I→E, the Stackelberg scenario E→I, or the Nash scenario is the most appropriate one for describing the behaviour of real-world firms. Ultimately, this is an empirical question.

Therefore, we now proceed to a hedonic regression analysis of a market that has been characterized by continuous technical progress, namely the laser printer market. In contrast to products such as smartphones, laser printers are not status products. When given the choice between an old and a new printer model of the same quality and price, it is unlikely that all consumers would buy the new model purely for reasons of social prestige. Therefore, it is more reasonable to assume that the continuum of consumers can be approximated by a uniform distribution over the interval $[0, 1]$, than by a cluster near $x = 1$ (high preference for the new product). Furthermore, laser printers are mature products. Although newly introduced printers generally have better quality characteristics than older ones, the quality increments are small in relation to overall quality. In terms of the model, the ratio δ/q is small.

For various reasons, the laser printer market is a suitable field for empirical investigation. Price data is readily available and can easily be obtained from online retailers. In addition, most quality aspects of a printer can be defined and measured in terms of its technical specifications, e.g. the number of pages printed per minute, or whether duplex printing is possible. Most non-measurable quality characteristics can be subsumed reasonably well in fixed brand effects. Furthermore, the market is concentrated and dominated by a small number of large firms. Finally, the life cycle of laser printers is relatively short: the median lifetime is 33 months in our data set. It is therefore possible to observe many market entries and exits within a few years of market observation.

To investigate empirically whether the quality-adjusted prices of newly entering products are higher or lower than that of the incumbent products, one needs a method to take into account the different quality characteristics of laser printers. For this purpose, the hedonic regression approach is a powerful tool. In the following analysis, we briefly present the data set. We then describe the dynamic hedonic regression model that is used to determine quality-adjusted prices, the estimation method, and the empirical results.

6.1. *Data*

We collected monthly online data generated in the German market for black-and-white laser printers over the 48 months from January 2003 to December 2006.[1] The following continuously measurable attributes were recorded: print speed (pages/min), processor speed (MHz), standard memory (MByte), extended memory (MByte), memory that can still be added (MByte), printing resolution (dpi), paper capacity of the multi-purpose tray (pages), standard paper capacity of the main paper tray (pages), supplementary paper capacity (pages), optional paper capacity (pages), and maintenance cost per page (Euro cent).

In addition, there are dummy variables for: interfaces with and without network connectivity, maximum paper size A3, equipped with network connectivity, optional upgrade with network connectivity, printer language PCL5, printer language PCL5 or PCL6, GDI-printer (Graphical Device Interface), equipped with Postscript 2, equipped with Postscript 3, optional upgrade with Postscript, built-in duplex, upgraded with duplex, optional upgrade with duplex.

Since non-measurable quality aspects can be subsumed by brand dummies, we also included dummies for: Brother, Canon, Epson, Hewlett-Packard, Kyocera, Lexmark, Minolta, Oki, and Samsung. No other brands were included in the sample.

In order to handle lifetime effects, and to avoid bias due to left-truncated observations, the entry month of all printers was determined, even if they had entered the market before the observation period. The number of months on the market (presence) is added to the set of attributes. The total number of printer attributes is then $K = 35$.

Prices of the same product in a given month may differ from retailer to retailer. To bypass problems caused by such differing prices, an average price was computed from the individual prices offered by internet vendors.

The number of printers available in each month varies between 176 and 272. The number of different printer models is 597 and the total number of printer-month observations is 10,853. The data cover well above 95 percent of the German market. Table 1 reports descriptive summary statistics of the variables.

[1] A detailed description of the data set can be found in Auer and Trede (2012).

The price data of our sample reveal that the prices of almost all laser printers fall over the duration of their market presence, measured in months since market entry. In other words, the price ratio of a printer, relative to its entry price, falls the longer the product is on the market. The thick line in Figure 2 indicates the average of these price ratios for each month for which the product is available on the market. The broken lines are pointwise 95% confidence intervals. Typically, the price falls rapidly. Six months after market entry, the price has already declined by about 5% on average. After two years, printers are about 15% cheaper than at the time of market entry.

Figure 2 suggests that entrant products are more expensive than incumbent products. This seems to allude to skimming. However, this conjecture is premature. It disregards the fact that new printers generally have better quality characteristics than older printers. What matters are the quality-adjusted prices and not the observed raw prices.

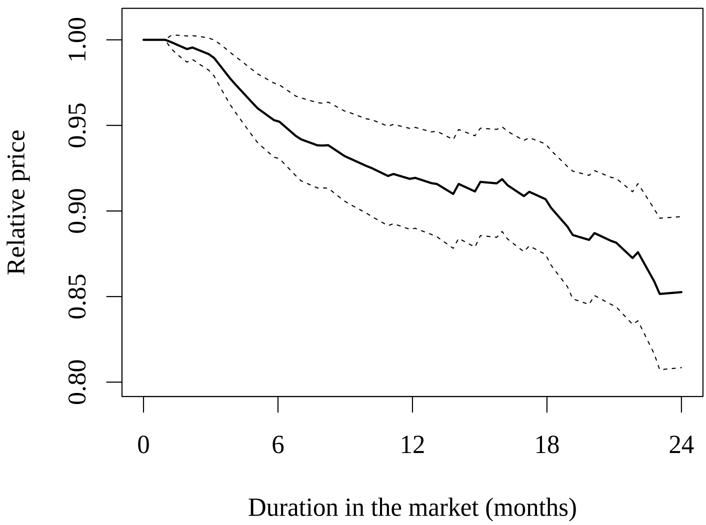

Figure 2: Price of laser printers in relation to their entry price as a function of their duration on the market (in months). The thick line is the average and the thin lines are pointwise 95% confidence intervals.

	Means				Standard deviations			
	2003	2004	2005	2006	2003	2004	2005	2006
# of observations	2390	2507	2862	3103				
Price	1634	1541	1399	1263	1334	1326	1283	1212
Print speed	26	29	30	31	11	10	10	10
Processor speed	221	266	296	330	104	104	128	138
Standard memory	28	39	47	54	21	28	40	47
Added memory	10	11	19	22	32	40	61	55
Opt memory ext	183	239	256	295	122	147	162	174
Print resolution	952	1041	1084	1102	293	269	249	223
Multipurp tr cap	569	597	627	614	638	634	654	639
Main tray cap	140	131	104	96	180	179	119	109
Add paper cap	86	92	113	135	352	360	399	430
Opt paper cap ext	1204	1137	1133	1084	1147	1071	1126	1173
Presence	16	17	19	19	11	13	13	13
Cost (×10)	0.15	0.13	0.14	0.15	0.07	0.07	0.08	0.08
A3 paper size	0.31	0.29	0.27	0.23	0.46	0.45	0.44	0.42
Network connect	0.42	0.49	0.49	0.55	0.49	0.50	0.50	0.50
Opt net connect	0.46	0.41	0.43	0.41	0.50	0.49	0.49	0.49
Interface (net)	0.24	0.35	0.38	0.45	0.45	0.52	0.55	0.55
Interface (no net)	0.46	0.39	0.41	0.38	0.57	0.54	0.55	0.53
PCL 5	0.03	0.04	0.03	0.03	0.18	0.18	0.17	0.16
PCL 5 and 6	0.40	0.37	0.38	0.33	0.49	0.48	0.48	0.47
GDI	0.05	0.04	0.05	0.05	0.21	0.20	0.22	0.22
PostScript 2	0.18	0.07	0.04	0.02	0.38	0.26	0.19	0.15
PostScript 3	0.59	0.76	0.77	0.79	0.49	0.43	0.42	0.41
Opt PostScript	0.12	0.06	0.06	0.04	0.32	0.23	0.23	0.20
Duplex	0.20	0.23	0.23	0.30	0.40	0.42	0.42	0.46
Opt Duplex	0.11	0.12	0.10	0.09	0.31	0.32	0.30	0.28
Added Duplex	0.39	0.40	0.35	0.32	0.49	0.49	0.48	0.47

Table 1: Descriptive statistics of variables, monthly observations have been averaged over years.

6.2. Dynamic Hedonic Regression

A well-established method for adjusting for quality differences is hedonic regression. Hedonic regressions have a long and lasting tradition in the economic literature (Waugh, 1928; Court, 1939; Chow, 1967; Triplett, 1969; Berndt and Rappaport, 2001; Pakes, 2003). A disadvantage of the traditional hedonic approach is its static nature. Only cross-sectional price variations over different product variants are explained by the basic hedonic regression. However, some dynamic techniques also exist: adjacent year regression (Berndt and Rappaport, 2001), continuously changing coefficients (Auer, 2007), the NTP-method (Nelson, Tanguay and Patterson, 1994), linear splines, and semiparametric approaches (see Auer, 2007, for a discussion of all these techniques). Below, we apply the dynamic hedonic regression approach of Auer and Trede (2012). A major advantage of this approach is its capability to deal rigorously with many market entries and exits.

The number of products observed in period t is N_t which may change over time due to market entries and exits. Let $\mathbf{y}_t = (y_{t1}, \ldots, y_{tN_t})'$ denote the vector of log-prices of the N_t products belonging to some product category in period t. The number of product attributes K is constant. The K attributes are organized in an $(N_t \times K)$-matrix $\mathbf{Q}_t$. The hedonic regression model at time t is

$$\mathbf{y}_t = \mathbf{Q}_t \boldsymbol{\beta}_t + \mathbf{u}_t \, , \qquad \mathbf{u}_t \sim N(\mathbf{0}, \sigma_t^2 \mathbf{I}_{N_t}) \, . \tag{34}$$

The vector $\boldsymbol{\beta}_t$ could simply be estimated for each period $t = 1, \ldots, T$ by running T separate OLS regressions. However, Arguea and Hsiao (1993) demonstrate that this approach can suffer from large standard errors and erratic changes in the estimated attribute prices from one period to the next. Auer and Trede (2012) show that the estimation can be improved by adding the assumption that the coefficients follow a random walk process:

$$\boldsymbol{\beta}_t = \boldsymbol{\beta}_{t-1} + \mathbf{q}_t, \quad \mathbf{q}_t \sim N(\mathbf{0}, \mathbf{W}) \, , \tag{35}$$

for $t = 1, \ldots, T$, where $\mathbf{W}$ is a symmetric, positive definite $(K \times K)$-matrix, and $\mathbf{q}_t$ is a random K-vector. As usual, we assume that the disturbance vectors $\mathbf{u}_t$ and $\mathbf{q}_t$ are independent. The start vector $\boldsymbol{\beta}_0$ is a random variable with distribution

$$\boldsymbol{\beta}_0 \sim N(\mathbf{m}, \mathbf{D}) \, . \tag{36}$$

Equations (34), (35) and (36) constitute a state space model (or dynamic linear model; see, for example, West and Harrison, 1997). Equation (35) is the transition equation, and (34) is the measurement equation, while $\boldsymbol{\beta}_1, \ldots, \boldsymbol{\beta}_T$ are the state vectors. Maximum likelihood estimation of the model's coefficients $\mathbf{m}, \mathbf{D}, \mathbf{W}$ and $\sigma_1^2, \ldots, \sigma_T^2$ has to rely on numerical methods and is notoriously unstable. Auer and Trede (2012) suggest estimating all coefficients of interest – $\mathbf{W}$, $\sigma_1^2, \ldots, \sigma_T^2$ and the state vectors $\boldsymbol{\beta}_0, \ldots, \boldsymbol{\beta}_T$ – simultaneously by the Markov Chain Monte Carlo (MCMC) method, and setting an uninformative prior distribution for the initial state $\boldsymbol{\beta}_0$ parameterized by $\mathbf{m}$ and $\mathbf{D}$.

Being a Bayesian method, MCMC treats both the state variables $\boldsymbol{\beta}_1, \ldots, \boldsymbol{\beta}_T$ and the other coefficients $\boldsymbol{\psi} = (\mathbf{W}, \sigma_1^2, \ldots, \sigma_T^2)$ as random vectors. Their prior distribution is assumed to be uninformative for all coefficients.

The joint posterior distribution of $\boldsymbol{\beta}_1, \ldots, \boldsymbol{\beta}_T$ and $\boldsymbol{\psi}$, given the observed data, that is, the observed prices $\mathbf{y}_t$ and the observed product attributes $\mathbf{Q}_t$ for $t = 1, \ldots, T$, can be computed by Gibbs-sampling (see Auer and Trede, 2012, for details). After a burn-in period of R_0 drawings, the subsequent R drawings $\boldsymbol{\beta}_1^{*r}, \ldots, \boldsymbol{\beta}_T^{*r}$, $r = 1, \ldots, R$ are stored and averaged to obtain point estimators of expectations of the posterior distribution of the state variables, $E(\boldsymbol{\beta}_1|\mathbf{Y}_T), \ldots, E(\boldsymbol{\beta}_T|\mathbf{Y}_T)$, given the observed data $\mathbf{Y}_T = (\mathbf{y}_1, \mathbf{Q}_1, \ldots, \mathbf{y}_T, \mathbf{Q}_T)$. The estimator of $E(\boldsymbol{\beta}_t|\mathbf{Y}_T)$ is

$$E\,\widehat{(\boldsymbol{\beta}_t|\mathbf{Y}_T)} = \frac{1}{R}\sum_{r=1}^{R} \boldsymbol{\beta}_t^{*r} \ .$$

and similarly for the other parameters.

In order to quantify the uncertainty of the point estimators, we determine pointwise $(1-\alpha)$ confidence bands for the time path $\beta_{1k}, \ldots, \beta_{Tk}$ of the k-th component of the attribute price vector. The R random draws $\beta_{tk}^{*1}, \ldots, \beta_{tk}^{*R}$ of the Gibbs sampler are ascendingly ordered separately for each time period $t = 1, \ldots, T$. Denote the order statistics as $\beta_{tk}^{*(1)} \leq \ldots \leq \beta_{tk}^{*(R)}$, then

$$\left[\beta_{tk}^{*(\alpha R/2)}, \beta_{tk}^{*((1-\alpha)R/2)}\right], \quad t = 1, \ldots, T \ ,$$

is a pointwise $(1-\alpha)$ confidence band for the k-th attribute price.

6.3. Results

We estimate the dynamic hedonic regression (34) for the laser printer data. The printers are described by their quality characteristics, as well as by the number of months they have been on the market (*presence*). The coefficient belonging to the covariate *presence* is the coefficient of interest. A positive coefficient would indicate that the quality-adjusted price of an older printer is greater than that of a newly introduced printer or, in the terminology of the HOLI model, that the entrant product's quality-adjusted price, $\hat{p}_E$, is lower than the incumbent product's, $\hat{p}_I$. Hence, by running a dynamic hedonic regression, we can determine whether there is skimming or penetration on the market.

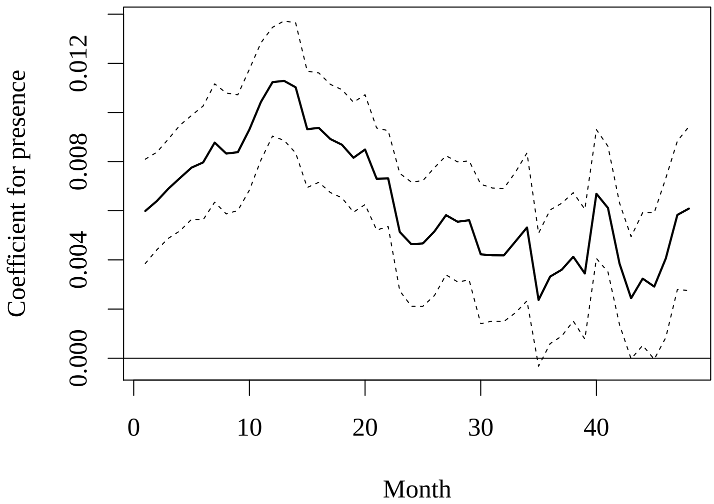

Figure 3: Development of the coefficient of the variable *presence*.

The solid line in Figure 3 represents the development of the coefficient $\hat{\beta}_{t,presence}$ for $t = 1, \ldots, T$ (where $T = 48$ months), the broken lines are pointwise 95% confidence intervals. The coefficient is significantly positive in almost all periods, indicating that an older printer is more expensive than a new one with the same quality characteristics. The size of the effect is also economically significant. The coefficient's value ranges from 0.002 to 0.011 with an average value of about 0.0064, implying that, on average, the quality-adjusted price of a printer rises by 0.64% per month, or about 8% per year.

In other words, the price of a printer entering the market is $1/(1.08) - 1 = 7.4\%$ lower than an incumbent printer of the same quality with a market presence of one year. This is market penetration and therefore conforms to the prediction of the HOLI model for the Stackelberg scenario I→E.

7. Concluding Remarks

The study of markets characterized by technical progress usually relies on rather complex analytical tools. In this paper, we introduced a much simpler alternative that we refer to as the Hotelling model of OverLapping Innovators (HOLI model). This model transforms an essentially dynamic market process into an overlapping sequence of static market situations. The model can be seen as a combination of two basic components.

The first component is Hotelling's (1929) spatial model, extended by vertical product differentiation. Though developed in the context of industrial organization, this Vertically Extended Hotelling (VEH) model is applicable to decision problems in various fields within and beyond that of economics (e.g., political science, medical science). In this paper, we were concerned with pricing in markets with regular product turnover and technical progress. Therefore, our VEH model combines different preferences for novelty (horizontal differentiation) with quality differences (vertical differentiation). We considered different equilibrium concepts leading to different interiour solutions.

The second component is the consistent application of our VEH model in a dynamic context. For this purpose, we assumed that last period's entrant product is the incumbent product of the present period. This yields an infinite-horizon Hotelling model with OverLapping Innovaters (HOLI).

The HOLI model allows to analyze markets in which a product starts its life cycle as an entrant product, becomes the incumbent product and then exits the market. In such markets, two opposing pricing strategies appear sensible and rational: introducing the entering product at a premium price and selling the exiting product at a discount (skimming) or doing the reverse (penetration). Our HOLI model reveals that the pricing strategy depends on the underlying equilibrium framework. Penetration only occurs when the seller of the entrant product acts as the Stackelberg follower.

The theoretical part of this paper has been supplemented by an empirical study of the German market for laser printers. In a hedonic regression analysis based on MCMC estimation techniques, it could be shown that, on average, the quality-adjusted prices of established laser printers exceed those of novel printers. In other words, the German market for laser printers exhibits penetration. This empirical result conforms to the predictions of the HOLI model with the seller of the novel product pursuing the strategy of the Stackelberg follower.

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