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Abstract

This paper presents a theoretical analysis of the simulated impact of uncertainty in a New Keynesian model. In order to incorporate uncertainty, the basic three-equation framework is modified by higher-order approximation resulting in a non-linear (dynamic) IS curve. Using impulse response analyses to examine the behavior of the model after a cost shock, I find interest rates in the version with uncertainty to be lower in contrast to the case under certainty.

JEL Codes: E12, E17, E43, E47, E52.

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1 Introduction

For the last 20 years, the New Keynesian framework has been one of the workhorses of macroeconomic analysis. The framework combines market frictions with optimization behavior by the model's agents, and, in its original construction, assumes perfect foresight. However, the treatment of uncertainty is an important issue since its effect can fundamentally change the prediction of these models.

This paper examines possible effects of uncertainty in a simple New Keynesian model (NKM) augmented with stochastic terms and non-linearity that enters the model through a second-order Taylor approximation regarding the IS curve. In line with the literature (see, among others, the textbooks by Galí 2015 and Walsh 2010), cost shock and demand shock are utilized for the New Keynesian Phillips curve (NKPC) and the forward-looking IS curve, respectively. Schmitt-Grohé and Uribe (2004) use second-order approximation in neoclassical growth models. Bauer and Neuenkirch (2015) were the first to use such a framework in the context of a NKM and found empirical evidence that central banks, indeed, take the resulting uncertainty into account. Moreover, their paper provides strong arguments that linear macroeconomic models found in monetary policy literature are less than optimal (see also Boneva et al. 2016 and Fernández-Villaverde et al. 2011). The main contribution this paper offers is the analysis of how the economy evolves after cost shocks, and the extent to which persistence plays a role.

In order to extend the NKM, we include a quadratic approximation in all derived equations. First, in the analytical part, demand and supply side (including monopolistic competition and price rigidity), where firms use second-order approximation when setting the prices, yields the NKPC. Second, the forward-looking IS curve with uncertainty follows from the households' Euler equation. This method differs fundamentally from standard approaches. Finally, to close the model, a (standard) targeting rule is derived by the central bank's optimization under discretion.

After adding AR(1) processes to the derived equations, conditional expectations and variances can be substituted by solving forward. Next, parameter values are selected for the resulting equilibrium condition (or instrument rule) with the focus on persistence and shock strength. A numerical simulation analyzes differences to the basic model. Finally, to examine the adjustment of macro variables in the medium term, impulse responses are carried out and contrasted with the linear counterpart.[1] In the same vein, but without an explicit derivation of the uncertainty, De Paoli and Zabczyk (2013) compare linear and non-linear models.

The remainder of this paper is organized as follows. Section 2 derives a basic version of the NKM augmented with a quadratic IS curve. Section 3 expands this model with shocks and discusses the resulting equilibrium condition. Section 4 carries out the numerical simulation of both the static equilibrium condition and the dynamic view of an impulse response analysis. Section 5 concludes.

2 New Keynesian Model with Uncertainty

2.1 New Keynesian Phillips Curve

For deriving the NKPC, two optimization problems involving private households and firms are employed, leading to aggregated demand and supply. Furthermore, price rigidity is modeled using the method introduced by Calvo (1983).[2] From the Calvo Pricing section on, the time index t is used because it is needed to make a distinction between the different periods.

Demand and Supply Side

Consumers

On the demand side, the representative consumer can choose from a variety of goods C_ξ which results in an aggregate consumption of C. Usually, the CES function is used to model monopolistic competition,[3] one of the two market frictions incorporated into the NKPC:

$$C = \left(\int_0^1 C_\xi^{\frac{\varepsilon-1}{\varepsilon}} \, d\xi \right)^{\frac{\varepsilon}{\varepsilon-1}}. \tag{1}$$

[1]To keep the framework easily understandable, government, investments, money supply, and labor markets are omitted. Consequently, neither money holdings nor working hours (or leisure time) will enter the households' utility function.

[2]This paper focuses on the standard approach. For non-linear versions of the Phillips curve see the articles by Collard and Juillard (2001), Dolado et al. (2005), and Schaling (2004).

[3]Dixit and Stiglitz (1977) developed this approach. Although they used a discrete sum and no integral, they received the same results.

Here, $\xi \in [0,1]$ can be viewed as a continuum of firms from 0 to 100%. The exponent is a measure for the substitutability between the goods C_ξ, where ε represents the elasticity of substitution.

A Hicksian-like optimization helps to solve for the demand curve by means of the Lagrangian function:

$$\mathscr{L}(C_\xi, \lambda) = \int_0^1 P_\xi \cdot C_\xi \, d\xi - \lambda \left(\left(\int_0^1 C_\xi^{\frac{\varepsilon-1}{\varepsilon}} \, d\xi \right)^{\frac{\varepsilon}{\varepsilon-1}} - C \right). \tag{2}$$

Since firms have pricing power, the representative consumer takes prices P_ξ as given. Minimizing expenditures $\int P_\xi C_\xi$ with the constraint of a certain consumption level C requires the following first-order conditions:[4]

$$\frac{\partial \mathscr{L}}{\partial C_\tau} = P_\tau - \lambda C_\tau^{-\frac{1}{\varepsilon}} \left(\int_0^1 C_\xi^{\frac{\varepsilon-1}{\varepsilon}} \, d\xi \right)^{\frac{1}{\varepsilon-1}} = 0. \tag{3}$$

Differentiating with respect to λ provides the constraint, Eq.(1). Rearranging condition (3) and defining $\lambda \equiv P$ as the aggregated price level yields

$$C_\tau = \left(\frac{P}{P_\tau} \right)^\varepsilon C, \tag{4}$$

the demand for good i.[5] The aggregated price level can be described by substituting this in Eq.(1). Rearranging the formula gives us:

$$P = \left(\int_0^1 P_\xi^{1-\varepsilon} \, d\xi \right)^{\frac{1}{1-\varepsilon}}. \tag{5}$$

The lack of investment and governmental spendings in this model leads to $Y_\tau = C_\tau$. Each firms' production Y_τ will be consumed completely by private households and hence $Y = C$.

Firms

Because any single firm is too small to directly influence other prices or productions, each firm takes the aggregated demand function and the aggregated price

[4] Note that τ denotes a continuum of derivatives.

[5] When the consumption constraint is relaxed by one unit, total consumption expenditures (see Galí 2015, 53) will increase to $(C+1)P = CP + P$, where P is the amount by which the optimum will change. This is exactly the information the Lagrange multiplier λ contains. See Appendix A.1 for the missing steps in this paragraph.

level P as given. It chooses its own price P_τ and faces the typical (real) profit maximization problem

$$\max_{P_\tau, Y_\tau} \left\{ \frac{P_\tau Y_\tau}{P} - K(Y_\tau) \right\} \tag{6}$$

with the cost function $K(.)$. Using Eq.(4), the first-order condition is straightforward and leads to

$$P_\tau^* = \left(\frac{\varepsilon}{\varepsilon - 1} \right) K'(Y_\tau) \cdot P, \tag{7}$$

an important result that states that the optimal price P_τ^* equals the nominal marginal costs and a mark-up bigger than one for all $\varepsilon > 1$.[6] Log-linearizing and using the fact that the long-run marginal costs equal the multiplicative inverse of the firms' mark-up ($K_{ss} = 1 - \varepsilon^{-1}$) yields

$$p_\tau^* - p = \psi y_\tau, \tag{8}$$

where ψ is a parameter for the long-run cost elasticity and, therefore, log deviations of marginal costs from their long-run trend are assumed to be linear.[7] Inserting the log-version of Eq.(4) gives

$$p_\tau^* - p = \left(\frac{\psi}{1 + \psi\varepsilon} \right) y. \tag{9}$$

Making use of $\widehat{y}$, the GDP growth rate around the steady state, as an approximation for y and using $\alpha_\psi \in [0, 1[$ as a summarizing parameter, Eq.(9) yields

$$p_\tau^* - p = \alpha_\psi \widehat{y}, \tag{10}$$

a description of the steady state output growth rate, depending on price level growth and microeconomic behavior. The next section introduces a non-optimal price setting scheme which replicates the actual observed economic patterns.[8]

Calvo Pricing

Nominal rigidities, the second market friction in the basic NKM, are implemented through the assumption that the firms' infrequent price adjustment fol-

[6] See Appendix A.2 for the missing steps.

[7] Note that lower case letters denote the log value of a variable in capital letters minus their long-run log value, e.g. $y = \ln(Y) - \ln(Y_{ss})$. See Appendix A.3 for the missing steps.

[8] See the survey by Taylor (1999), that came to abundant evidence. See also Galí (2015, 7–8) for a literature overview.

lows an exogenous Poisson process.[9] This implies that all firms have a constant probability (ϕ) of being unable to update their price in each period with $\phi \in [0,1[$ (i.e., $\phi = 0$ in the absence of price rigidity). It is crucial that price setters do not know how long the nominal price will remain in place. Only the expected value is known due to probabilities that are all equal and constant for all firms and periods. This implies a probability of ϕ^j for having today's same price in j periods, so the average expected duration between price changes will be $1/(1-\phi)$.

From this point forward, the time index t will be used because more than one period is being considered. Simultaneously, the firm index τ is no longer important since it is sufficient to calculate with a share of firms ϕ (or $1-\phi$). Hence, $p_\tau^* \equiv p_t^*$ and $p \equiv p_t$. When x_t is the price that firms set in period t (provided they are able to do so), the following applies:

$$x_t = \frac{p_t - \phi p_{t-1}}{1-\phi} \quad \Rightarrow \quad E_t x_{t+1} = \frac{E_t p_{t+1} - \phi p_t}{1-\phi}. \tag{11}$$

Because firms act on the probability of not being able to adjust prices in future periods, they attempt to establish a price x_t that is not necessarily the optimal price p_t^*, derived in the previous section. Also, in the presence of price rigidities, $x_t \neq p_t^*$ generally holds.

To reveal the mechanics behind the staggered price setting, it is convenient to verbally treat p_t and x_t as level variables. Strictly speaking, firms set price growth paths in the following optimization problem rather than maximizing a discounted profit as the difference between revenue and costs.[10] In the following, the optimal reset price, determined by the discounted sum of future profits, is derived through a quadratic approximation of the per-period deviation from maximum-possible profit with $\beta \in [0,1[$, the discount factor over an infinite planning horizon. Therefore, firms minimize their loss function, the discounted deviations from p_t^* over all t:

$$\min_{x_t} \left\{ E_t \left[k \sum_{j=0}^{\infty} \beta^j \phi^j \left(x_t - p_{t+j}^* \right)^2 \right] \right\}. \tag{12}$$

[9] Calvo (1983) originally wrote his article in continuous time. However, using discrete periods immensely helps the clearness and is more realistic with regard to how firms actually operate. Moreover, Calvo (1983, 396–397) shows the equivalence of both approaches.

[10] See Walsh (2010, 241–242) for the use of level variables in Calvo pricing.

The parameter $k > 0$ enters the loss function multiplicatively and indicates all exogenous factors that will influence the costs of not setting the optimal price in each period.[11] The first-order condition is

$$\frac{\partial}{\partial x_t} = E_t \left[2k \sum_{j=0}^{\infty} (\beta\phi)^j (x_t - p^*_{t+j}) \right] = 0. \tag{13}$$

After rearranging[12] and expressing x through p with Eq.(11), it follows that

$$p_t - \phi p_{t-1} = \beta\phi(E_t p_{t+1} - \phi p_t) + (1-\phi)(1-\beta\phi)p^*_t \tag{14}$$

only contains parameters and variants of the variable p. Expressing p through π,[13] as well as isolating $(p^*_t - p_t)$ and replacing it with the result in Eq.(10), gives

$$\pi_t = \beta E_t \pi_{t+1} + \frac{\alpha_\psi (1-\phi)(1-\beta\phi)}{\phi} \widehat{y_t}. \tag{15}$$

In a final step, a summarizing parameter $\kappa > 0$ for all parameters, multiplied with $\widehat{y_t}$, will be defined. This yields the NKPC:[14]

$$\pi_t = \beta E_t \pi_{t+1} + \kappa \widehat{y_t}. \tag{16}$$

Both the expected inflation rate $E_t \pi_{t+1}$ and the GDP growth rate around the steady state $\widehat{y_t}$ (or output gap) have a positive impact on π_t since $\beta, \kappa > 0$. Moreover, the slope of the NKPC (κ), depends on all four parameters (β, ψ, ε, and ϕ) of this section.[15]

[11] Note that it can also come up as an additive term or any other positive monotonic transformation and does not alter the results.

[12] See Appendix A.4 for the missing steps.

[13]
$$p_t - p_{t-1} = \ln P_t - \ln P_{ss} - (\ln P_{t-1} - \ln P_{ss}) = \ln\left(\frac{P_t}{P_{t-1}}\right) = \ln(1+\pi_t) \approx \pi_t.$$

[14] In contrast to the IS curve discussed in the next section, the NKPC is still linear. Simulations in MATLAB show that the effect of a non-linear NKPC is rather small. That is why our focus on uncertainty relies on the IS curve.

[15] Depending on the exact model, the slope of the NKPC can have a slightly different meaning, e.g. Walsh (2010, 336) uses a measure for the firm's real marginal costs instead of the output gap.

2.2 The Quadratic IS Curve

The objective is to derive an Euler equation via maximizing utility with a dynamic budget constraint. Initially, it is not necessary to formulate an explicit utility function. On the contrary, the general marginal utility provides a better insight into the intertemporal mechanics. The only specific assumption is not taking money, working hours or any other possible utility-gainer into consideration. The utility function solely relies on consumption, thus, households maximize their intertemporal discounted utility

$$\max_{C_t} \left\{ E_t \left[\sum_{s=t}^{\infty} \beta^{s-t} U(C_s) \right] \right\}. \tag{17}$$

Taking into account an intertemporal budget constraint with prices and the interest rate i_t, the maximization problem leads[16] to the Euler equation

$$U'(C_t) = \beta(1+i_t) \cdot E_t \left[\frac{P_t \cdot U'(C_{t+1})}{P_{t+1}} \right], \tag{18}$$

revealing the intertemporal relationship of the marginal utility out of consumption. Marginal utility in period t equals the counterpart in $t+1$, corrected by discount factor, nominal interest rate, and the ratio of current and expected future price level. Assuming i_t rises, marginal utility in t would also rise relative to period $t+1$. Given the diminishing marginal utility property and, therefore, concavity, consumption will be higher in the future.[17]

One convenient formulation for such a function is $U(C_t) = (1-\sigma)^{-1} \cdot (C_t^{1-\sigma} - 1)$ with $\sigma > 0$ implying $1/\sigma$ as the intertemporal elasticity of substitution (IES). Substituting this in the Euler equation gives

$$Y_t^{-\sigma} = \beta(1+i_t) \cdot E_t \left[\frac{P_t \cdot Y_{t+1}^{-\sigma}}{P_{t+1}} \right], \tag{19}$$

when recalling the market clearing condition $Y = C$. The long-run real interest rate r enters the equation through β since it equals $1/\beta - 1$.[18]

[16] See Appendix A.5 for the missing steps.

[17] Note that present consumption could also increase because of the income effect.

[18] The relation follows from the steady state Euler equation. See Galí (2015, 132) for a more complex definition of the long-term real interest rate.

Quadratic Approximation

Eq.(19) can be prepared for quadratic approximation by inserting $1/(1+r)$ for β, treating t-measurable variables as constants for the conditional expectation, rearranging, and taking logs:

$$\ln\left(\frac{1+r}{1+i_t}\right) = \ln E_t\left[\left(\frac{Y_{t+1}}{Y_t}\right)^{-\sigma}\right] - \ln E_t\left[\frac{P_{t+1}}{P_t}\right]. \tag{20}$$

Ignoring Jensen's inequality is equivalent to first-order Taylor series expansions of both logarithm and exponential function. Furthermore, the right side of Eq.(20) can be written as[19]

$$\gtrapprox E_t\left[\ln\left(\left(\frac{Y_{t+1}}{Y_t}\right)^{-\sigma}\right)\right] - E_t\left[\ln\left(\frac{P_{t+1}}{P_t}\right)\right] \tag{21}$$

and thereby be expressed in growth rates:[20]

$$E_t[-\sigma\ln(1+\widetilde{y_{t+1}})] - E_t[\ln(1+\pi_{t+1})]. \tag{22}$$

Instead of linearizing, the logarithm will be represented by a second-degree polynomial:[21]

$$\approx E_t\left[-\sigma\left(\widetilde{y_{t+1}} - \frac{1}{2}\widetilde{y_{t+1}}^2\right)\right] - E_t\left[\pi_{t+1} - \frac{1}{2}\pi_{t+1}^2\right] \tag{23.1}$$

$$= -\sigma E_t\widetilde{y_{t+1}} + \frac{\sigma}{2}E_t\widetilde{y_{t+1}}^2 - E_t\pi_{t+1} + \frac{1}{2}E_t\pi_{t+1}^2 \tag{23.2}$$

$$= \sigma\widehat{y_t} - \sigma E_t\widehat{y_{t+1}} + \frac{\sigma}{2}E_t\widetilde{y_{t+1}}^2 - E_t\pi_{t+1} + \frac{1}{2}E_t\pi_{t+1}^2. \tag{23.3}$$

Bringing together the linearized form of the left side in Eq.(20) yields the quadratic IS curve:

$$\widehat{y_t} = E_t\widehat{y_{t+1}} - \frac{1}{\sigma}(i_t - r - E_t\pi_{t+1}) - \frac{1}{2\sigma}E_t\pi_{t+1}^2 - \frac{1}{2}E_t\widetilde{y_{t+1}}^2. \tag{24}$$

Referring to the original graphical IS relation (in the $\widehat{y}/i$–space), the curve shifts to the right if the long-term real interest rate r, the output gap expectations $E_t\widehat{y_{t+1}}$ or the inflation expectations $E_t\pi_{t+1}$ rise. However, the slope will rise and

[19]See Appendix A.6 for the missing steps.

[20]Note that the use of the actual GDP growth rate $\widetilde{y_{t+1}}$ in Eq.(22) is merely for clarity. The relationship between $\widetilde{y_{t+1}}$ and $\widehat{y_{t+1}}$ is: $\widehat{y_{t+1}} \approx \widetilde{y_{t+1}} + \widehat{y_t}$.

[21]See Appendix A.7 for more detail.

the curve becomes flatter if the intertemporal elasticity of substitution ($1/\sigma$) rises. The second-order terms have a negative effect on $\widehat{y_t}$. However, Eq.(24) is not in reduced form since the last term still contains $\widehat{y_t}$. The formula for the conditional variance,[22] can be utilized to show the second moments' influence in detail:

$$\widehat{y_t} = E_t\widehat{y_{t+1}} - \frac{1}{\sigma}(i_t - r - E_t\pi_{t+1}) - \frac{1}{2\sigma}Var_t\pi_{t+1} - \frac{1}{2}Var_t\widetilde{y_{t+1}} \\ - \frac{1}{2\sigma}(E_t\pi_{t+1})^2 - \frac{1}{2}(E_t\widetilde{y_{t+1}})^2. \tag{25}$$

In a first step, looking only at the variances[23] and solving for the interest rate yields

$$i_t = -\sigma\widehat{y_t} + r + E_t\pi_{t+1} + \sigma E_t\widehat{y_{t+1}} - \frac{1}{2}Var_t\pi_{t+1} - \frac{\sigma}{2}Var_t\widehat{y_{t+1}} - \dots, \tag{26}$$

which states that uncertainty would shift the curve to the left compared to the original IS curve. Considering the second moment, there are two additional effects namely expected output gap growth affects the slope and a variation of the curve's shape. That is because the last term of Eq.(25) contains $\widehat{y_t}$ and $\widehat{y_t}^2$:

$$-\frac{1}{2}(E_t\widehat{y_{t+1}} - \widehat{y_t})^2 = -\frac{1}{2}(E_t\widehat{y_{t+1}})^2 + E_t\widehat{y_{t+1}} \cdot \widehat{y_t} - \frac{1}{2}\widehat{y_t}^2. \tag{27}$$

Larger values for $E_t\widehat{y_{t+1}}$ result in a (slightly) flatter IS curve and vice versa. Figure 1 illustrates the shift, the different slope, and the quadratic form.

Inserting everything in Eq.(26) gives

$$i_t = -\frac{\sigma}{2}\widehat{y_t}^2 + (\sigma E_t\widehat{y_{t+1}} - \sigma)\widehat{y_t} + r + E_t\pi_{t+1} + \sigma E_t\widehat{y_{t+1}} - \frac{1}{2}Var_t\pi_{t+1} - \frac{\sigma}{2}Var_t\widehat{y_{t+1}} \\ - \frac{1}{2}(E_t\pi_{t+1})^2 - \frac{\sigma}{2}(E_t\widehat{y_{t+1}})^2. \tag{28}$$

In the quadratic IS formula, σ is the only parameter besides r. When examining the effects of a variation in σ on the derived curve, it is useful to recapitulate the meaning of $1/\sigma$. The IES measures the strength of the relationship between i_t and $\widehat{y_{t+1}}/\widehat{y_t}$ (also y_{t+1}/y_t and C_{t+1}/C_t). A positive IES implies a positive rela-

[22]The following applies for a random variable z:

$$Var_t z_{t+1} = E_t z_{t+1}^2 - (E_t z_{t+1})^2 \Leftrightarrow E_t z_{t+1}^2 = (E_t z_{t+1})^2 + Var_t z_{t+1}.$$

[23]Note that $Var_t\widetilde{y_{t+1}} \approx Var_t(\widehat{y_{t+1}} - \widehat{y_t}) = Var_t\widehat{y_{t+1}}$ because $\widehat{y_t}$ is t-measurable and constants (in period t) do not affect Var_t.

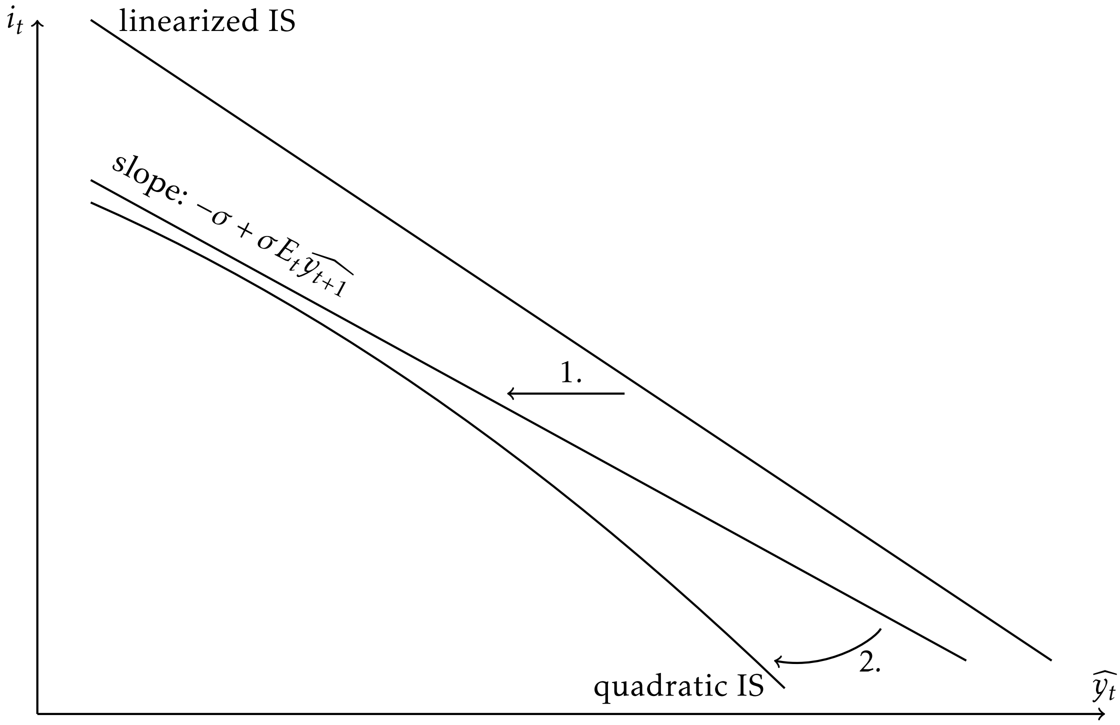

Figure 1: 1. Shift of the locus and a change in the slope (for $E_t\widehat{y_{t+1}} > 0$). 2. The quadratic form.

tionship. Also, if i_t rises, there is a negative effect on $\widehat{y_t}$ due to the substitution effect. If the IES increases (decreases) the relationship gets stronger (weaker) and the IS curve's slope should be flatter (steeper). Hence, increasing σ should lead to a steeper IS curve. The effect is indeed a more concave and steeper curve. Additionally, it shifts to the left (right) if uncertainty is relatively high (low) in comparison to the expected values.

2.3 Targeting Rule under Discretion

The central bank takes Phillips and IS curves as given and seeks to optimally set the interest rate for period t. Therefore, the central bank's targeting rule will be derived by minimizing the discounted loss function over all periods[24]

$$\min_{\pi,\, \widehat{y}} \left\{ E_t \left[\sum_{s=t}^{\infty} \beta^{s-t} \left((\pi_s - \pi^*)^2 + \delta \widehat{y_s}^2 \right) \right] \right\} \tag{29}$$

[24]The loss function can be derived by a second-order approximation of the households' welfare loss, first introduced by Rotemberg and Woodford (1999, 54–61). It can also be found in the textbooks by Galí (2015), Walsh (2010), and Woodford (2003b). In a similar vein, Kim et al. (2008, 3410) argue that utility-based welfare effects of monetary policy should include second-order or even higher-order terms.

resulting in the standard targeting rule under discretion:[25]

$$\delta\widehat{y_t} = -\kappa\pi_t \quad \Leftrightarrow \quad \widehat{y_t} = -\frac{\kappa}{\delta}\pi_t. \tag{30}$$

Every difference between the inflation rate and the central bank's target π^* results in a loss.[26] Also, every output gap leads to a loss but is reduced by a weighting factor δ, normally smaller than one. Squaring ensures that higher deviations yield disproportionately higher losses and the optimized variables will not vanish in the derivatives. Moreover, it makes the loss function symmetrical.[27]

Although the optimal interest rate is not explicitly given, all relationships between the macroeconomic variables are derived. The process is as follows: the nominal interest rate has an effect on the output gap (IS curve), which, in consequence, affects the inflation rate (NKPC). Furthermore, Eq.(30), the "leaning against the wind" condition, implies a countercyclical monetary policy intended to stabilize prices and eventually contract the economy. The degree of this contraction increases in κ and decreases in δ, the weight on output stabilization.

Finally, (16), (25), and (30) can lead to a forward-looking Taylor type rule (with uncertainty added). Plugging (30) into (16) gives

$$-\frac{\delta}{\kappa}\widehat{y_t} = \beta E_t\pi_{t+1} + \kappa\widehat{y_t} \quad \Leftrightarrow \quad \widehat{y_t} = -\frac{\kappa}{\delta+\kappa^2}\cdot\beta E_t\pi_{t+1}, \tag{31}$$

which can be utilized for (25):

$$\begin{aligned} i_t = r + \left(1 + \frac{\beta\kappa\sigma}{\delta+\kappa^2}\right)E_t\pi_{t+1} + \sigma E_t\widehat{y_{t+1}} - \frac{1}{2}Var_t\pi_{t+1} - \frac{\sigma}{2}Var_t\widehat{y_{t+1}} \\ -\frac{1}{2}(E_t\pi_{t+1})^2 - \frac{\sigma}{2}(E_t\widehat{y_{t+1}} - \widehat{y_t})^2. \end{aligned} \tag{32}$$

When examining the coefficients on first and second moments, the parameters β, δ, κ, and σ have to be taken into account. Larger values for β and κ increase the weight on expected inflation,[28] whereas larger values for σ increase not only the weight on expected inflation, but on expectation and uncertainty concerning the output gap growth, as well. Following Bauer and Neuenkirch (2015), the

[25] See Appendix A.8 for the missing steps.

[26] Note that $\pi^* = 0$ as it does not change the essential findings.

[27] See Nobay and Peel (2003, 661) for an asymmetric loss function (Linex form) that becomes quadratic in a special case.

[28] The increasing relationship holds for $\delta = 0.25$ (independent of β and σ) if $\kappa < 0.5$, which can be assumed (see Appendix A.11).

squared expected inflation rate and the squared expected output gap growth rate should not be over-interpreted here, as it takes very small values for advanced economies.

Ultimately, the difference between this approach and the conventionally derived Taylor rules lies in the negative variance term that Bauer and Neuenkirch (2015, 15–17) empirically confirmed for uncertainty in future inflation rates where central banks lower the interest rate for higher values of $Var_t \pi_{t+1}$. Branch (2014, 1042–1044) also adds variances in an empirical model for a Taylor rule. He estimates negative coefficients with a more significant (and more negative) value for the coefficient on the inflation variance.

The NKPC, the IS curve, and the targeting rule were all derived by second-order approximations. However, this implements uncertainty only in the IS curve since P_{t+1} and Y_{t+1} are non-t-measurable. Thus, besides the quadratic terms of the IS curve, all derivations follow standard approaches.

3 Persistent Shocks and Equilibrium Condition

This section adds stochastic terms to the derived curves and solves these forward to a reduced form solution for the nominal interest rate.

3.1 Adding Persistent Stochastic Shocks

Given the possibility that unforeseen events might interrupt the normal economic process (e.g., inventions, cold winters, higher oil prices, wars), stochastic shocks (in reduced-form) will be added to the existing relationships. The realistic feature of a certain duration of the event that will dwindle over time can be modeled by means of stationary AR(1) processes:[29]

$$e_t = \mu e_{t-1} + \zeta_t, \tag{33.1}$$

$$u_t = \nu u_{t-1} + \eta_t. \tag{33.2}$$

The coefficients of the shocks in period $(t-1)$, $\mu, \nu \in \,]0,1[$, declare the percentage impact of shocks that carries over to the subsequent period. Additional assumptions are normally distributed error terms with an expected value equal to zero, that is, $\zeta_t \sim \mathcal{N}(0, \sigma_e^2)$ and $\eta_t \sim \mathcal{N}(0, \sigma_u^2)$, which are also serially uncorrelated.

Adding Eq.(33.1) to the NKPC, Eq.(16), can be described as a cost shock, a cost-push shock or an inflation shock and adding Eq.(33.2) to the IS curve, Eq.(25), indicates a taste shock, a demand shock or fluctuations in the flexible-price equilibrium output level (Walsh 2010, 352):[30]

$$\pi_t = \beta E_t \pi_{t+1} + \kappa \widehat{y_t} + e_t, \tag{34.1}$$

$$\widehat{y_t} = E_t \widehat{y_{t+1}} - \frac{1}{\sigma}(i_t - r - E_t \pi_{t+1}) - \frac{1}{2\sigma} E_t \pi_{t+1}^2 - \frac{1}{2} E_t \widetilde{y_{t+1}}^2 + u_t. \tag{34.2}$$

[29] For instance, Clarida et al. (2000, 170) are also assuming a stationary AR(1) process in the context of a NKM.

[30] See Galí (2015, 128) for a further discussion of cost shocks, the type that will be most important throughout the remainder of the paper.

3.2 Equilibrium Condition

A standard approach is chosen to substitute expectations through forward solving. Inserting the targeting rule (30) into the stochastic NKPC yields

$$\pi_t = \beta E_t \pi_{t+1} - \frac{\kappa^2}{\delta}\pi_t + e_t \quad \Leftrightarrow \quad \pi_t = \frac{\beta\delta}{\delta+\kappa^2} E_t \pi_{t+1} + \frac{\delta}{\delta+\kappa^2} e_t. \tag{35}$$

Devising the same formula for $t+1$ and substituting π_{t+1} gives

$$\pi_t = \frac{\beta\delta}{\delta+\kappa^2} E_t \left[\frac{\beta\delta}{\delta+\kappa^2} E_{t+1}[\pi_{t+2}] + \frac{\delta}{\delta+\kappa^2} e_{t+1} \right] + \frac{\delta}{\delta+\kappa^2} e_t. \tag{36}$$

With $E_t[E_{t+n}[\pi]] = E_t[\pi]$ and $E_t[e_{t+n}] = \mu^n e_t$, future expectations and shocks will leave the equation:

$$\pi_t = \left(\frac{\beta\delta}{\delta+\kappa^2}\right)^2 E_t[\pi_{t+2}] + \frac{\beta\delta\mu}{\delta+\kappa^2} \cdot \frac{\delta}{\delta+\kappa^2} e_t + \frac{\delta}{\delta+\kappa^2} e_t. \tag{37}$$

After $(n-1)$ iterations, the equation converts to

$$\pi_t = \left(\frac{\beta\delta}{\delta+\kappa^2}\right)^n E_t[\pi_{t+n}] + \frac{\delta}{\delta+\kappa^2} e_t \sum_{j=0}^{n-1} \left(\frac{\beta\delta\mu}{\delta+\kappa^2}\right)^j. \tag{38}$$

Developing n towards infinity, and making use of the formula for the infinite geometric series, leaves only parameters and the cost shock:

$$\pi_t = \frac{\delta}{\delta+\kappa^2} e_t \cdot \frac{\delta+\kappa^2}{\delta+\kappa^2-\beta\delta\mu}. \tag{39}$$

Rearranging and setting $\theta = (\kappa^2 + (1-\beta\mu)\delta)^{-1}$ as an auxiliary parameter results in the equilibrium conditions[31] for π_t and $\widehat{y_t}$:

$$\pi_t = \frac{\delta}{\kappa^2 + (1-\beta\mu)\delta} \cdot e_t \; = \delta\theta e_t \tag{40.1}$$

$$\text{and} \quad \widehat{y_t} = \frac{-\kappa}{\kappa^2 + (1-\beta\mu)\delta} \cdot e_t \; = -\kappa\theta e_t. \tag{40.2}$$

[31] See also Clarida et al. (1999, 1680) for a comparison of these results to those under commitment.

Determine the expectation values[32] analogously:

$$E_t\pi_{t+1} = \delta\theta E_t e_{t+1} = \delta\mu\theta e_t \tag{41.1}$$

$$\text{and}\quad E_t\widehat{y_{t+1}} = -\kappa\theta E_t e_{t+1} = -\kappa\mu\theta e_t. \tag{41.2}$$

Solution without Uncertainty

In a first step, I solve for the target interest rate

$$i_t = r - \sigma\widehat{y_t} + \sigma E_t\widehat{y_{t+1}} + E_t\pi_{t+1} + \sigma u_t, \tag{42}$$

which can be rewritten with the equilibrium conditions (40.2), (41.1), and (41.2):

$$i_t = r + \sigma\kappa\theta e_t - \sigma\kappa\mu\theta e_t + \delta\mu\theta e_t + \sigma u_t. \tag{43}$$

Simplifying results in

$$i_t = r + ((1-\mu)\sigma\kappa + \mu\delta)\theta e_t + \sigma u_t \tag{44}$$

and finally setting $\alpha_\mu > 0$ as a summarizing parameter gives

$$i_t = r + \alpha_\mu e_t + \sigma u_t, \tag{45}$$

a reduced-form solution for the nominal interest rate that describes the static equilibrium behavior under optimal discretion. The central bank's optimized interest rate in period t can be expressed through the long-run real interest rate and both shocks, which are weighted by a composition of parameters. Since these coefficients are positive, larger shocks correspond to higher interest rates.[33] Galí (2015, 133–134) refers to this equation type as instrument rule. In contrast to targeting rules (see Eq.(30), "practical guides for monetary policy"), Eq.(45) is not easy to implement.[34] It requires real-time observation of variations in the cost-push shock and knowledge of the model's parameters, including the efficient interest rate r.

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[32] $E_t e_{t+1} = E_t[\mu e_t + \zeta_{t+1}] = \mu E_t e_t + \underbrace{E_t\zeta_{t+1}}_{=0} = \mu e_t.$

[33] See also Walsh (2010, 364) for a more detailed discussion.

[34] The paper by Svensson and Woodford (2005) discusses the "targeting" vs. "instrument" topic in more detail.

Model with Uncertainty

After including the second-order terms, however, Eq.(45) will be examined theoretically in order to understand how shocks and persistence correspond to i_t in the equilibrium.

The basic procedure is to solve the IS curve for the interest rate and replace all variables with shocks. The difference between this approach and standard approaches is the quadratic terms, thus lower interest rates should be expected. Beginning with the expected value of the squared inflation ($E_t\pi_{t+1}^2$), Eq.(40.1) in period $t+1$ gives

$$\pi_{t+1} = \delta\theta e_{t+1} = \delta\theta\left(\mu e_t + \zeta_{t+1}\right), \tag{46}$$

by using the former shock definition with persistence and a normally distributed error term. Therefore,

$$E_t\pi_{t+1}^2 = E_t\left[(\delta\theta)^2\left(\mu e_t + \zeta_{t+1}\right)^2\right] = (\delta\theta)^2 E_t\left[\mu^2 e_t^2 + 2\mu e_t\zeta_{t+1} + \zeta_{t+1}^2\right], \tag{47}$$

where the middle term equals zero, since e_t can be treated as a constant in E_t and $E_t\zeta_{t+1} = 0$. Inserting the variance, again with Eq.(22), yields

$$(\delta\theta\mu)^2 e_t^2 + (\delta\theta)^2\left(Var_t\zeta_{t+1} + (E_t\zeta_{t+1})^2\right). \tag{48}$$

The variance is defined as σ_e^2 and hence,

$$E_t\pi_{t+1}^2 = (\delta\theta)^2\left(\mu^2 e_t^2 + \sigma_e^2\right). \tag{49}$$

Doing the same for the expected value of the squared output growth rate[35] ($E_t\widehat{y_{t+1}}^2 = E_t(\widehat{y_{t+1}} - \widehat{y_t})^2$), Eq.(40.2) in period $t+1$ gives

$$\widehat{y_{t+1}} = -\kappa\theta e_{t+1} = -\kappa\theta\left(\mu e_t + \zeta_{t+1}\right) \tag{50}$$

and therefore,

$$E_t(\widehat{y_{t+1}} - \widehat{y_t})^2 = (\kappa\theta)^2\left((1-\mu)^2 e_t^2 + \sigma_e^2\right). \tag{51}$$

[35] Note that the output gap can also be replaced by the inflation rate with the standard targeting rule (30) to obtain the same results. See Appendix A.10 for the missing steps.

The equilibrium condition under uncertainty is now

$$i_t = r + \alpha_\mu e_t - \frac{1}{2}\left(\left((1-\mu)^2\sigma\kappa^2 + \mu^2\delta^2\right)\theta^2 e_t^2 + \left(\sigma\kappa^2 + \delta^2\right)\theta^2\sigma_e^2\right) + \sigma u_t \tag{52}$$

and finally setting $\alpha_e > 0$ and $\alpha_\sigma > 0$ as summarizing parameters gives

$$i_t = r + \alpha_\mu e_t - \frac{1}{2}\left(\alpha_e e_t^2 + \alpha_\sigma\sigma_e^2\right) + \sigma u_t, \tag{53}$$

a reduced-form solution for the nominal interest rate that describes the static equilibrium behavior under uncertainty.[36] Compared to the approach in Clarida et al. (1999), a negative term and an additional parameter $\left(\sigma_e^2\right)$ enters the condition. The term entails a generally lower interest rate level. Moreover, a larger cost shock variance also corresponds to lower values for i_t, an essential result.[37]

[36]Going one step further, e_t and u_t could be replaced by the error terms:

$$i_t = r + \alpha_\mu \sum_{k=0}^{\infty}\mu^k\zeta_{t-k} - \frac{1}{2}\left(\alpha_e\left(\sum_{k=0}^{\infty}\mu^k\zeta_{t-k}\right)^2 + \alpha_\sigma\sigma_e^2\right) + \sigma\sum_{k=0}^{\infty}\nu^k\eta_{t-k}.$$

This visualizes the past (known) shocks that are discounted by μ and ν.

[37]The equation in its static form does not directly contain ν and σ_u^2. This is due to the simplified targeting rule and the resulting assumption that $\widehat{y}$ and π can be represented only through cost shocks.

4 Numerical Simulation

Table 1 shows the baseline (BL) values and the overall range used when taking all simulations into account.[38] Every value is assumed to be obtained on a quarterly basis. In order to cover even extreme scenarios, e_t initially ranges from -0.5% to 2.5%.

Table 1: Overview of all Parameters

Parameter	BL Calibration	Applied Range	Description
β	0.99	0.99	Discount factor
κ	0.04	0.01 - 0.25	Slope of the NKPC
σ	1	0.5 - 5	Reciprocal value of the IES
δ	0.25	0.25	Weight on output fluc.
μ	0.6 - 0.8	0.6 - 0.85	Cost shock persistence
σ_e^2	0.0001	0.00005 - 0.0005	Cost shock variance
e_t	-0.005 - 0.025	-0.005 - 0.025	Cost shock

4.1 Equilibrium Condition

In the baseline calibration, shown in Table 1, $\beta = 0.99$, $\kappa = 0.04$, $\sigma = 1$, $\delta = 0.25$, $\sigma_e^2 = 0.0001$, μ reaches from 0.6 to 0.8 and e_t from -0.5% to 2%. Since ν and σ_u^2 play no role when the central bank acts under discretion, u_t is assumed to be zero. The optimal interest rate would react one-to-one and there would be no gain of further insights.

Figure 2 shows the results of the model with uncertainty using a variety of persistence and cost shock combinations. The interest rate takes values from -1.1% to 10.2%. It is assumed that negative interest rates are possible and that the zero lower bound does not represent an obstacle. Indeed, central banks can raise a tax on deposits made by commercial banks.[39] When the model calibrates negative values for i_t, it could also be interpreted as an unconventional policy

[38]See Appendix A.11 for parameter discussion and literature review.

[39]The concise paper by Bassetto (2004) derives a framework in which the central bank commits to negative nominal interest rates and discusses the equilibrium condition in such a situation.

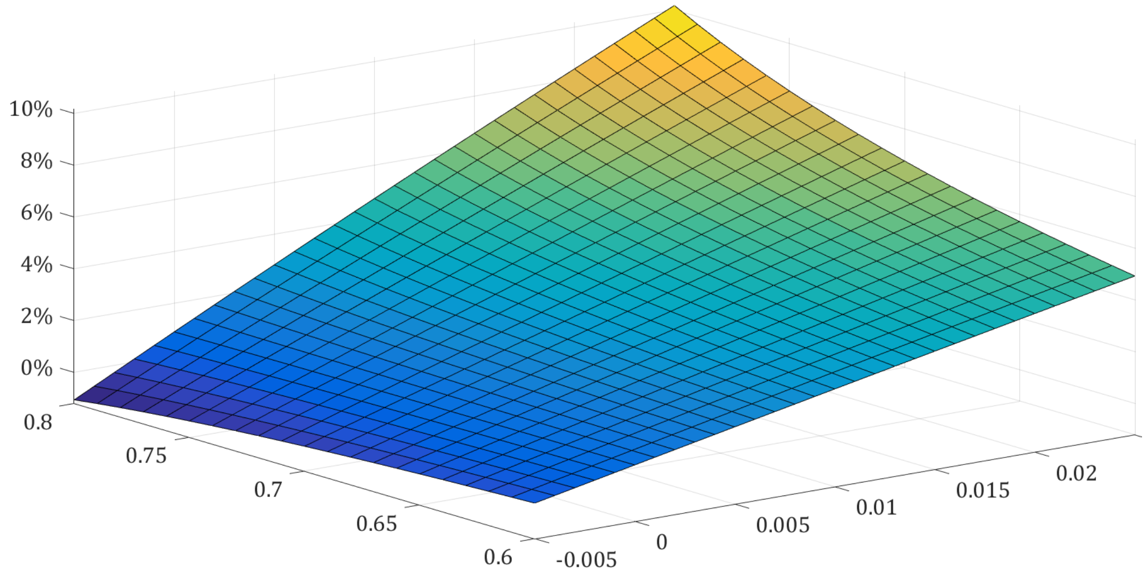

Figure 2: Corresponding interest rate in the equilibrium condition. Horizontal axes: Persistence μ and cost shock e_t. Vertical axis: Interest rate i_t.

(i.e., quantitative easing) by the monetary authorities.[40] The lowest interest rates occur hand-in-hand with highly persistent negative cost shocks, a fairly extreme scenario since the only major developed country to have faced deflationary tendencies over a prolonged period of time is Japan. But even in the latter case, the negative cost shocks were closer to zero. As expected, the highest values come with large cost shocks. For a low persistence, regardless of the shocks, the resulting interest rate varies very little.

Model Comparison

To isolate the partial effect of the parameters, the interest rate differences after subtracting the values with (see Figure 2) and without uncertainty are shown, whereas values for i_t are always higher in the latter case. Due to small interest rate differences, the vertical axis in Figures 3 to 6 is scaled in basis points (100 basis points = one percentage point).

Figure 3 gives a broad overview on the effect of uncertainty. There is a significant amount of persistence/shock combinations that support the estimations by Bauer and Neuenkirch (2015). In particular, highly persistent shocks affect the interest rate outcome in the equilibrium behavior. In this case, the interest rate difference reaches from 10 to 60 basis points. Figure 4 can be understood as a

[40]The Wu-Xia shadow rate does exactly that (see Wu and Xia (2016)) and is negative since mid-2009 for the federal funds rate.

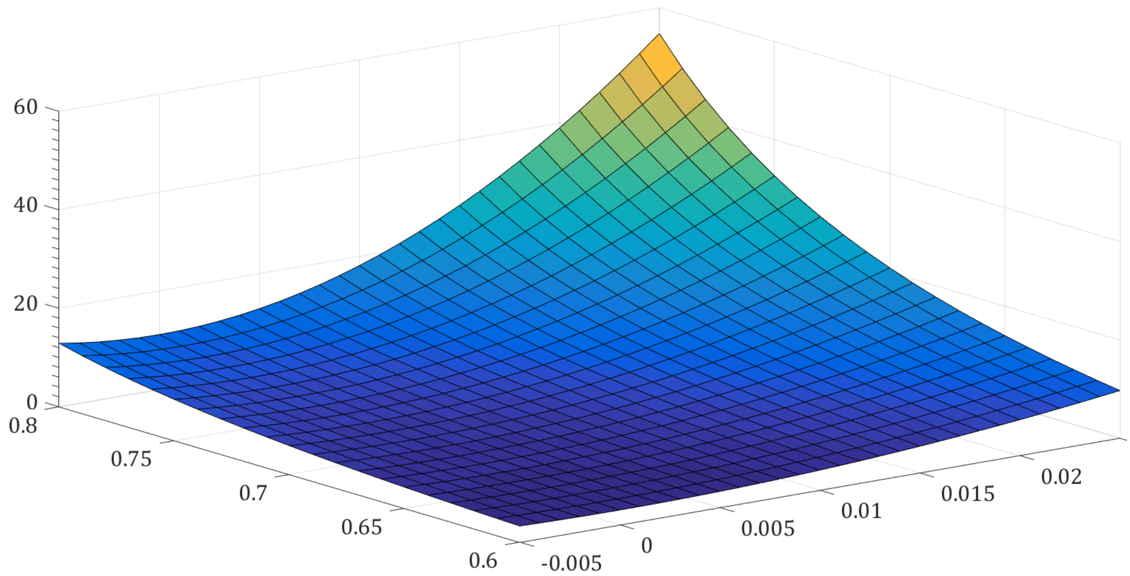

Figure 3: Differences between both cases (with and without uncertainty) in the equilibrium condition. Horizontal axes: Persistence μ and cost shock e_t. Vertical axis: Difference of interest rate i_t in basis points.

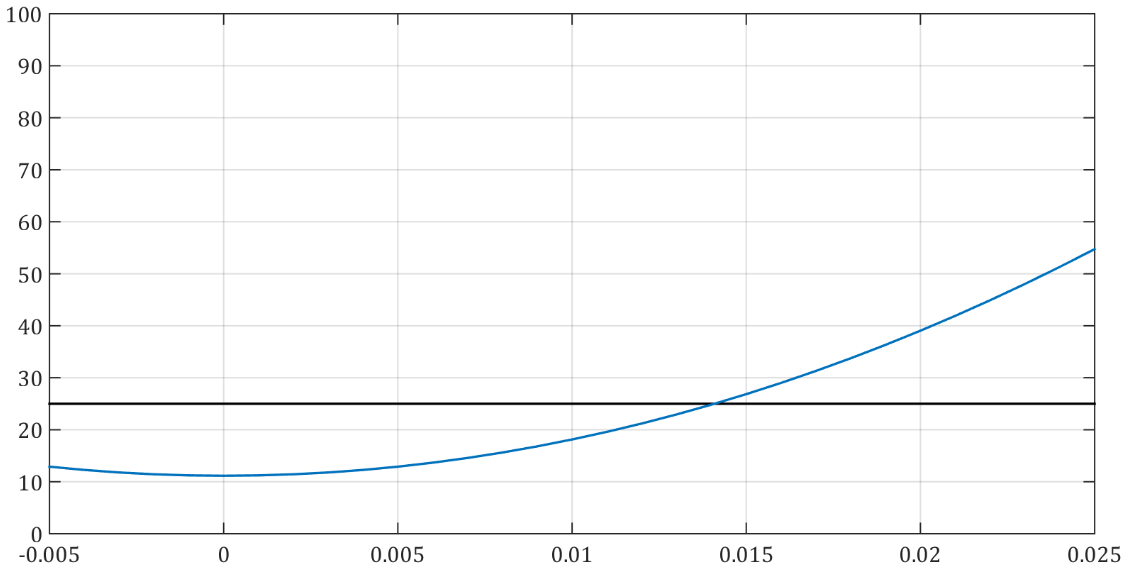

Figure 4: Differences between both cases (with and without uncertainty) in the equilibrium condition ($\mu = 0.8$). Horizontal axis: Cost shock e_t. Vertical axis: Difference of interest rate i_t in basis points.

cross section of Figure 3 with $\mu = 0.8$, a realistic assumption when reviewing the literature, such as Smets and Wouters (2003). It reveals, as one of the main findings from a theoretical point of view, that accounting for uncertainty results in lower policy rates, even during tranquil times. A black line is drawn at 25 basis points to show the empirical conclusion by Bauer and Neuenkirch (2015, 21).[41]

4.2 Impulse Response Analysis

First, we examine the macro variables' short- and medium-term adjustments in the newly derived framework. In a subsequent step, the latter will be compared to the basic NKM.

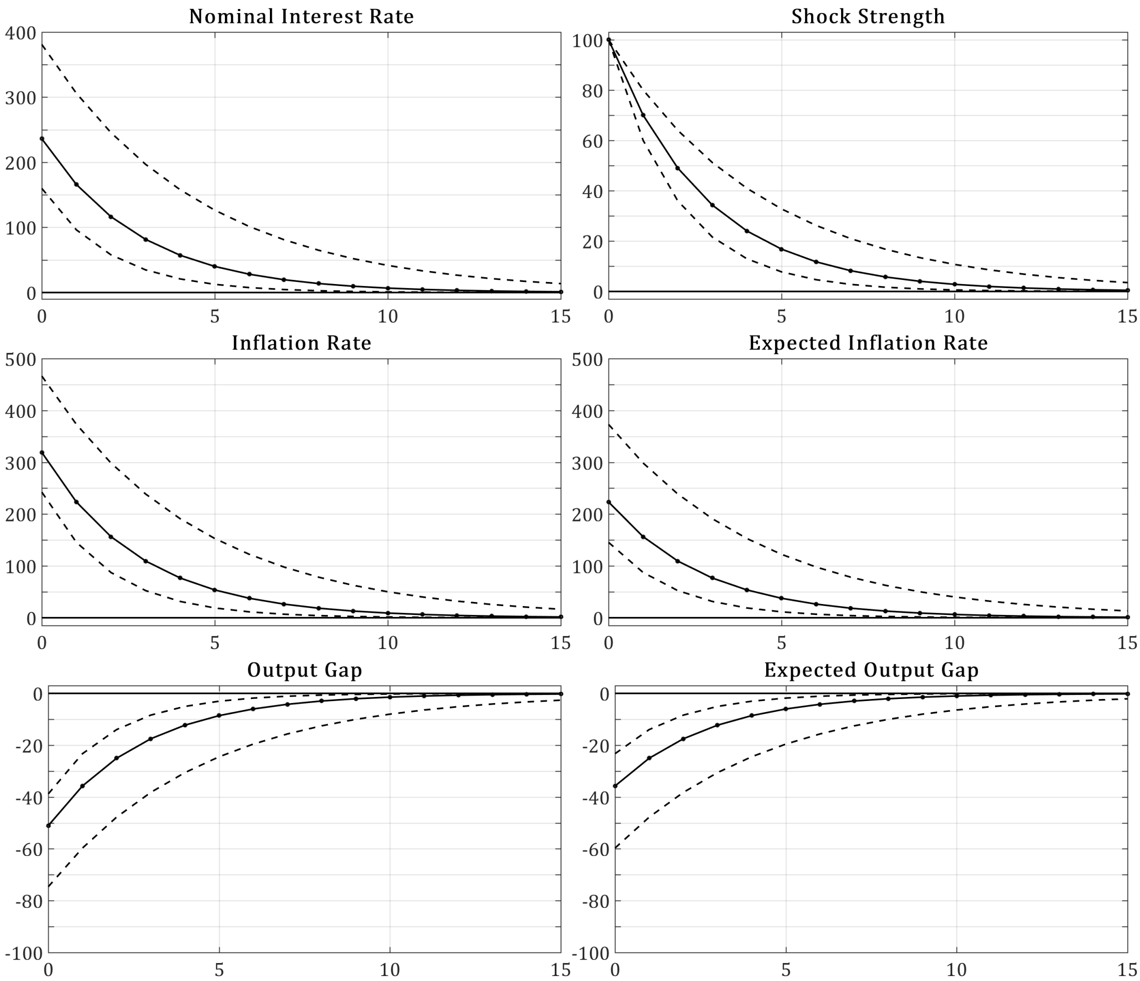

Figure 5: Dynamic responses to a cost shock by 100 basis points. Horizontal axes: Timeline in quarters. Vertical axes: Responses of i_t, e_t, π_t, $E_t\pi_{t+1}$, $\widehat{y_t}$, and $E_t\widehat{y_{t+1}}$ for $\mu \in \{0.6, 0.7, 0.8\}$ in basis points.

Figure 5 shows the adjustment over time to the steady state in the baseline case (see Table 1). The dashed lines indicate the scenarios of (relatively) high and low persistent shocks. In these scenarios, the upper (lower) course corresponds

[41] Note that Bauer and Neuenkirch (2015) have no assumption regarding the level of shock persistence.

to the high (low) persistence for the nominal interest rate, the shock strength, and the (expected) inflation rate. The opposite is the case with regard to the (expected) output gap. All values adjust normally, but with quantitative differences if the level of persistence is varied. In the median case, the nominal interest rate has to be raised by almost 2.5% and should then sluggishly adjust to the steady state (depending on the real interest rate). The inflation rate and output gap follow their respective expectation values. The initial inflation rate is ranged between 2.5% and 5%, and the output gap starts at around −0.5%.

Model Comparison

Similar to Section 4.1 the following graphics show the "gap" in i_t when accounting for uncertainty.

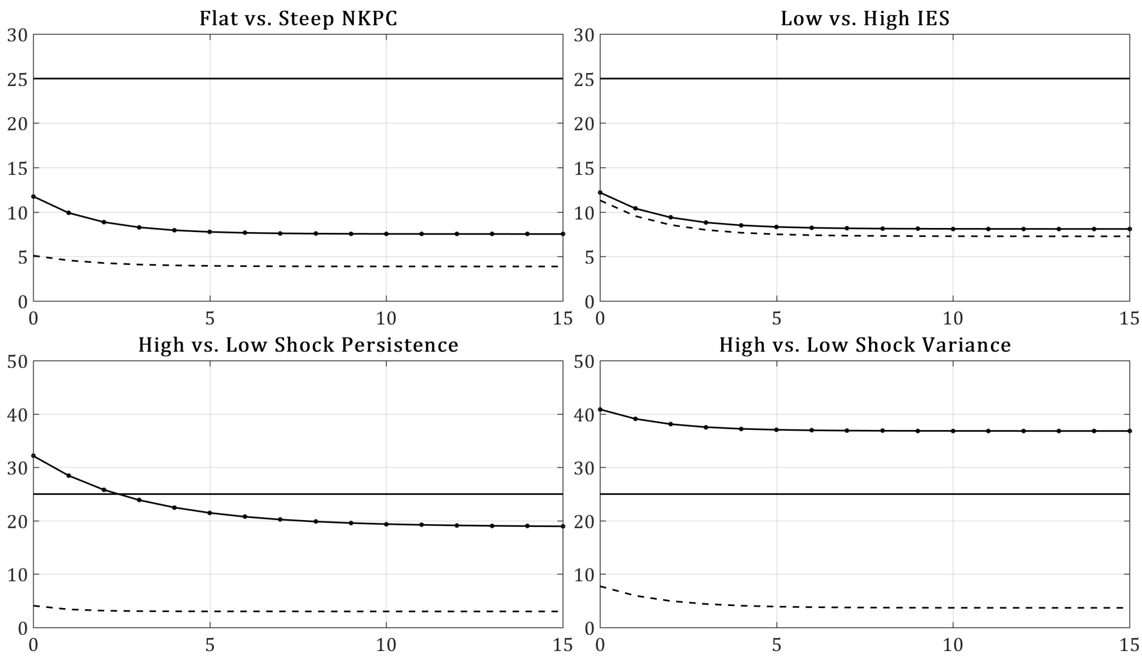

Figure 6: Comparing dynamic responses to a cost shock by 100 basis points with $\mu = 0.75$. Horizontal axes: Timeline in quarters. Vertical axes: Difference of interest rate i_t (with and without uncertainty) in basis points.

Figure 6 compares the NKM with and without uncertainty and shows the resulting differences of the nominal interest rate in each case. In addition, different scenarios are positioned opposite each other: Slope of the NKPC with 0.01 (black dotted) and 0.25 (dashed), IES with 0.5 (black dotted) and 5 (dashed), shock persistence with 0.85 (black dotted) and 0.6 (dashed), shock variance with 0.0005 (black dotted) and 0.00005 (dashed). The cases with high shock persistence and high shock variance play a very important role showing a difference of up to 30 and 40 basis points, respectively. Also, with a very flat Phillips curve (in contrast to a steep NKPC) an effect comes to light (around 10 basis points). Comparable effects can be observed in the different IES cases, but variations in elasticity play a negligible role. Although these examples indicate that there is no obligatory difference between the model with uncertainty and without uncertainty, (highly) persistent shocks and, in particular, increasing levels of uncertainty show distinctive variations.

5 Conclusion

This theoretical paper explores a variety of situations in which uncertainty is incorporated in the New Keynesian framework. The analysis focuses on how the equilibrium behaves when confronted with a wide range of parameter values. Our analysis reveals several points of interest. First, interest rates are generally lower when taking uncertainty into account. Under reasonable assumptions, accounting for uncertainty leads to lower interest rates of roughly 25 basis points. Second, when there is a higher degree of cost shocks (positive or negative) and shocks are more persistent, this difference in interest rates increases. We also show that a steeper NKPC decreases the impact of uncertainty. Third, over time, the impact of uncertainty on the nominal interest rate decreases and the adjustment critically depends on the degree of persistence. Our theoretical analysis also confirms the results found in the empirical literature.

There are some open avenues left for future research. First, a targeting rule derived under commitment could be taken into account. Second, due to the negative interest rate in the equilibrium and because of the more prominent role of unconventional monetary policy in recent years, the model could include a zero lower bound when considering this type of policy. Third, calibrating the shock variance and the underlying distribution, which is essential for the resulting uncertainty, might also be considered as an additional topic to explore.

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Appendix

A.1 Consumers – Calculation Steps

$\partial\mathcal{L}/\partial C_\tau$ can be obtained by using the chain rule:

$$P_\tau - \lambda\frac{\varepsilon}{\varepsilon-1}\left(\int_0^1 C_\xi^{\frac{\varepsilon-1}{\varepsilon}}\,d\xi\right)^{\frac{\varepsilon}{\varepsilon-1}-1}\cdot\underbrace{\frac{\varepsilon-1}{\varepsilon}C_\tau^{\frac{\varepsilon-1}{\varepsilon}-1}}_{\text{derivative of sub-function}} = 0 \tag{A1.1}$$

$$\Leftrightarrow \qquad P_\tau - \lambda\left(\int_0^1 C_\xi^{\frac{\varepsilon-1}{\varepsilon}}\,d\xi\right)^{\frac{1}{\varepsilon-1}}\cdot C_\tau^{-\frac{1}{\varepsilon}} = 0. \tag{A1.2}$$

First, exponentiate the integral with ε and $1/\varepsilon$ for rearranging the first-order condition. Then insert C from the constraint. It follows that

$$P_\tau = \lambda C_\tau^{-\frac{1}{\varepsilon}}C^{\frac{1}{\varepsilon}} \quad\Leftrightarrow\quad P_\tau = \lambda\left(\frac{C}{C_\tau}\right)^{\frac{1}{\varepsilon}} \tag{A2.1}$$

$$\Leftrightarrow\quad \frac{P_\tau}{\lambda} = \left(\frac{C_\tau}{C}\right)^{-\frac{1}{\varepsilon}} \quad\Leftrightarrow\quad \left(\frac{P_\tau}{\lambda}\right)^{-\varepsilon} = \frac{C_\tau}{C}. \tag{A2.2}$$

To obtain Eq.(5), solve Eq.(4) for C_τ and insert the result for all firms in the constraint, Eq.(1):

$$C = \left(\int_0^1\left(\left(\frac{P_\xi}{P}\right)^{-\varepsilon}C\right)^{\frac{\varepsilon-1}{\varepsilon}}d\xi\right)^{\frac{\varepsilon}{\varepsilon-1}} \Leftrightarrow\; C = \left(\frac{1}{P}\right)^{-\varepsilon}C\left(\int_0^1 P_\xi^{1-\varepsilon}\,d\xi\right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{A3.1}$$

$$\Leftrightarrow\quad P^{-\varepsilon} = \left(\int_0^1 P_\xi^{1-\varepsilon}\,d\xi\right)^{\frac{\varepsilon}{\varepsilon-1}} \quad\Leftrightarrow\; P = \left(\int_0^1 P_\xi^{1-\varepsilon}\,d\xi\right)^{\frac{1}{1-\varepsilon}}. \tag{A3.2}$$

A.2 Firms – Calculation Steps

Eq.(6) can be written in more detail. Using Eq.(4) with Y and rearranging leads to

$$\max_{P_\tau}\left\{\left(\frac{P_\tau}{P}\right)^{1-\varepsilon}Y - K\left(\left(\frac{P_\tau}{P}\right)^{-\varepsilon}Y\right)\right\}. \tag{A4}$$

The first-order condition is now straightforward, using the chain rule:

$$\frac{\partial}{\partial P_\tau} = (1-\varepsilon)\left(\frac{P_\tau}{P}\right)^{-\varepsilon}\cdot\frac{Y}{P} - K'(Y_\tau)\cdot(-\varepsilon)\left(\frac{P_\tau}{P}\right)^{-\varepsilon-1}\cdot\frac{Y}{P} = 0. \tag{A5}$$

Simplifying and denoting the optimal price with P_τ^* yields

$$(\varepsilon-1)\left(\frac{P_\tau^*}{P}\right)^{-\varepsilon} = K'(Y_\tau)\cdot\varepsilon\left(\frac{P_\tau^*}{P}\right)^{-\varepsilon-1} \tag{A6.1}$$

$$\Leftrightarrow \qquad 1 = \left(\frac{\varepsilon}{\varepsilon-1}\right)K'(Y_\tau)\left(\frac{P_\tau^*}{P}\right)^{-1} \tag{A6.2}$$

$$\Leftrightarrow \qquad P_\tau^* = \left(\frac{\varepsilon}{\varepsilon-1}\right)K'(Y_\tau)\cdot P. \tag{A6.3}$$

However, perfect substitutes let the monopolistic structure vanish and show the typical polypolistic result:

$$\lim_{\varepsilon\to\infty}\left(\frac{\varepsilon}{\varepsilon-1}\right)K'(Y_\tau)\cdot P = K'(Y_\tau)\cdot P = P_\tau^*. \tag{A7}$$

Now, with a cost function in real terms of quantities Y_τ defined as

$$K(Y_\tau) = \frac{c_{var}}{\psi+1}Y_\tau^{\psi+1} + c_{fix}, \tag{A8}$$

where c_{fix} are the fix costs, c_{var} is a measure for the variable costs and ψ represents the elasticity of marginal costs, Eq.(7) becomes a micro-funded AS curve that takes the form of a power function:

$$P_\tau^* = \left(\frac{\varepsilon}{\varepsilon-1}\right)c_{var}Y_\tau^{\psi}\cdot P. \tag{A9}$$

A.3 Log-Linearization

It is convenient to use log-linearized variables instead of level variables in order to solve the model analytically. Also, some interpretations of the results, in terms of elasticity and growth rates, become quite useful. So both Eq.(4) and Eq.(A9) can be approximated through log-linearization around the steady state. Thus, the approximation becomes more precise with small growth rates. However, some preparation is necessary. Let Z be a state variable that can change over time and Z_{ss} its long-term value. When defining

$$z \equiv \ln Z - \ln Z_{ss}, \tag{A10}$$

z becomes a good approximation of $\widehat{z}$, the growth rate around the steady state. Also, a first-order Taylor approximation "in reverse" shows the relationship between z and $\widehat{z}$:

$$\widehat{z} \approx \ln(1+\widehat{z}) = \ln\left(1+\frac{Z-Z_{ss}}{Z_{ss}}\right) = \ln Z - \ln Z_{ss}. \tag{A11}$$

Furthermore, in the steady state, long-term values for individual variables are by definition the same as for those on aggregated level, thus $Z_{\tau ss} = Z_{ss}$. The state would otherwise include endogenous forces. And finally, the long-run marginal costs equal the multiplicative inverse of the firms' mark-up:[42]

$$c_{var} Y_{ss}^{\psi} = \frac{\varepsilon - 1}{\varepsilon}. \tag{A12}$$

An explanation is the long-run version of Eq.(A9) and hence $P_{\tau ss} = P_{ss}$. Now this can be applied to the previous results. First, Eq.(4), the AD curve will be log-linearized. Taking logs, expanding with the log long-term values, and using (A10) gives

$$
\begin{aligned}
& & \ln Y_\tau = \ln Y + \varepsilon(\ln P - \ln P_\tau) & \quad \text{(A13.1)}\\
&\Leftrightarrow & \ln Y_\tau - \ln Y = -\varepsilon(\ln P_\tau - \ln P) & \quad \text{(A13.2)}\\
&\Leftrightarrow & \ln Y_\tau - \ln Y_{ss} - (\ln Y - \ln Y_{ss}) = -\varepsilon(\ln P_\tau - \ln P_{ss} - (\ln P - \ln P_{ss})) & \quad \text{(A13.3)}\\
&\Leftrightarrow & y_\tau - y = -\varepsilon(p_\tau - p) & \quad \text{(A13.4)}\\
&\Leftrightarrow & y_\tau = -\varepsilon p_\tau + \varepsilon p + y, & \quad \text{(A13.5)}
\end{aligned}
$$

a linearized AD curve in terms of growth rates with the slope of $-1/\varepsilon$. A higher elasticity of substitution would result in a flatter curve, so a change in the firm's price growth p_τ would have a stronger effect on production growth y_τ.

Next, with the use of (A12), the AS curve type Eq.(A9), can be rewritten in a similar way:

$$
\begin{aligned}
& & \ln P_\tau^* = \ln\left(\frac{\varepsilon}{\varepsilon-1}\right) + \ln c_{var} + \psi \ln Y_\tau + \ln P & \quad \text{(A14.1)}\\
&\Leftrightarrow & \ln P_\tau^* - \ln P = \ln\left(\frac{\varepsilon}{\varepsilon-1}\right) + \ln c_{var} + \psi(\ln Y_\tau - \ln Y_{ss} + \ln Y_{ss}) & \quad \text{(A14.2)}\\
&\Leftrightarrow & p_\tau^* - p = \psi y_\tau + \ln\left(\frac{\varepsilon}{\varepsilon-1}\right) + \ln c_{var} + \psi \ln Y_{ss} & \quad \text{(A14.3)}
\end{aligned}
$$

[42] Other authors simply define this property, see e.g. (Galí 2015, 57).

$$\Leftrightarrow \qquad p_\tau^* - p = \psi y_\tau + \ln\left(c_{var} Y_{ss}^{\psi}\right) - \ln\left(\frac{\varepsilon - 1}{\varepsilon}\right) \tag{A14.4}$$

$$\Leftrightarrow \qquad p_\tau^* - p = \psi y_\tau + \underbrace{\ln\left(\frac{c_{var} Y_{ss}^{\psi}}{(\varepsilon - 1)/\varepsilon}\right)}_{=0}. \tag{A14.5}$$

The latter expression shows the assumption that the log deviations of marginal costs from their long-run trend values are linear in the amount of ψ. When the firm's optimized price growth p_τ^* is equal to the aggregated price growth p, then there is no growth in the firm's production.

Having log-linearized both demand and supply side, Figure A1 sums up.

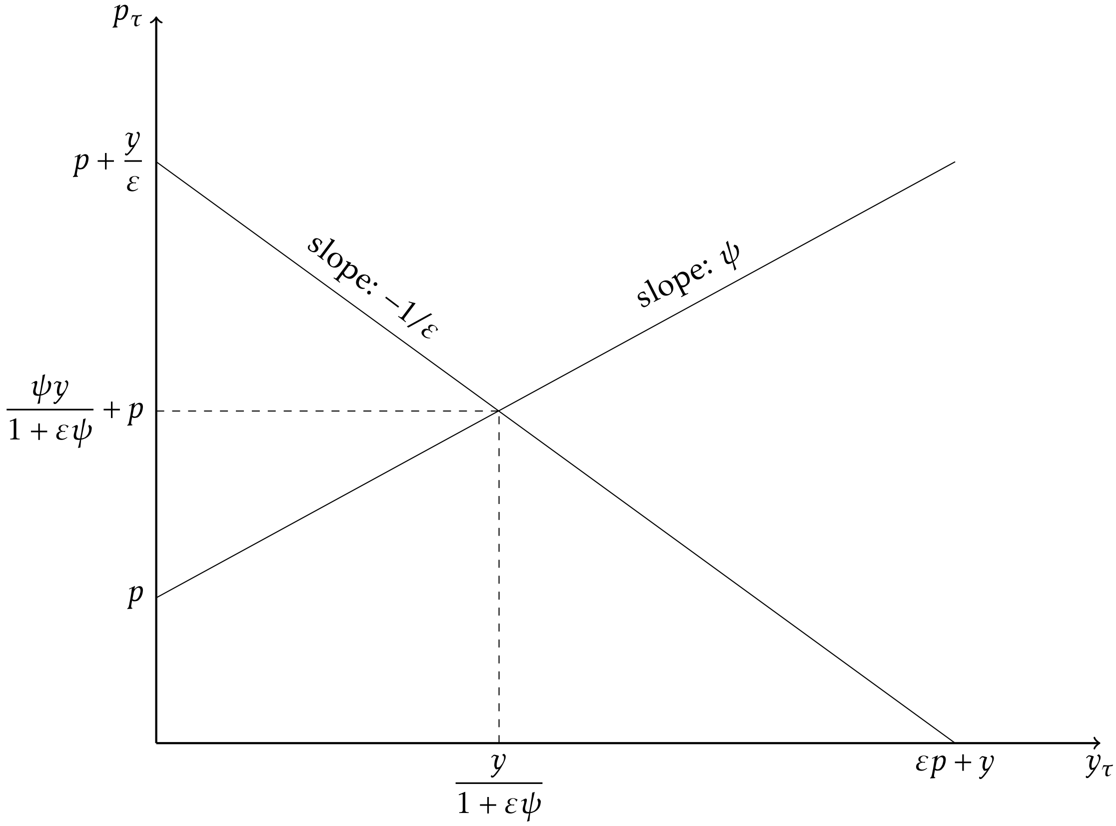

Figure A1: Graphical results of households' and firms' static optimization.

Finally, inserting (A13.5) in (A14.5) combines all the results and gives

$$p_\tau^* - p = \psi(-\varepsilon p_\tau^* + \varepsilon p + y) \tag{A15.1}$$

$$\Leftrightarrow \qquad p_\tau^* - p = -\psi\varepsilon(p_\tau^* - p) + \psi y \tag{A15.2}$$

$$\Leftrightarrow \qquad (1 + \psi\varepsilon)(p_\tau^* - p) = \psi y \tag{A15.3}$$

$$\Leftrightarrow \qquad p^*_\tau - p = \left(\frac{\psi}{1+\psi\varepsilon}\right) y. \tag{A15.4}$$

A.4 Calvo Pricing – Calculation Steps

Dividing the first-order condition by $2k$, using the fact that x_t is t-measurable, and expanding the sum gives

$$\sum_{j=0}^{\infty} (\beta\phi)^j x_t - \sum_{j=0}^{\infty} (\beta\phi)^j E_t p^*_{t+j} = 0. \tag{A16}$$

Excluding x_t from the sum, using the formula for an infinite geometric series, and multiplying by $(1-\beta\phi)$ gives

$$x_t = (1-\beta\phi) \sum_{j=0}^{\infty} (\beta\phi)^j E_t p^*_{t+j}. \tag{A17}$$

Again, using t-measurability ($E_t p^*_t = p^*_t$) and excluding the first summand provides a sum from $j=1$ to infinity that can be substituted in a subsequent step:

$$x_t = (1-\beta\phi)\left[\sum_{j=1}^{\infty} (\beta\phi)^j E_t p^*_{t+j} + p^*_t\right]. \tag{A18}$$

Furthermore, Eq.(A17) can be rewritten for $t+1$ (since firms optimize in each period),

$$E_t x_{t+1} = (1-\beta\phi) \sum_{j=1}^{\infty} (\beta\phi)^{j-1} E_t p^*_{t+j} \tag{A19.1}$$

$$\Leftrightarrow \quad \beta\phi E_t x_{t+1} = (1-\beta\phi) \sum_{j=1}^{\infty} (\beta\phi)^{j} E_t p^*_{t+j}, \tag{A19.2}$$

for eliminating the sum in (A18):

$$x_t = \beta\phi E_t x_{t+1} + (1-\beta\phi) p^*_t. \tag{A20}$$

Inserting condition (11) leads to the expression

$$\frac{p_t - \phi p_{t-1}}{1-\phi} = \beta\phi \frac{E_t p_{t+1} - \phi p_t}{1-\phi} + (1-\beta\phi) p^*_t \tag{A21.1}$$

$$\Leftrightarrow \quad p_t - \phi p_{t-1} = \beta\phi(E_t p_{t+1} - \phi p_t) + (1-\phi)(1-\beta\phi)p_t^*, \tag{A21.2}$$

that only contains parameters and variants of the variable p. Then, with the definition of (A10) and first-order Taylor expansion, the inflation rate π can be expressed through differences of p. In the same way, the conditional expectation value for period $t+1$ can be expressed with

$$E_t p_{t+1} - p_t \approx E_t \pi_{t+1}. \tag{A22}$$

Since this approximation is sufficiently exact for small values of π, an equality sign will be used for all following calculations. Now (A21.2) can be rearranged to insert approximations π and Eq.(A22):

$$\phi(p_t - p_{t-1}) = \beta\phi(E_t p_{t+1} - \phi p_t) + (1-\phi)(1-\beta\phi)p_t^* - (1-\phi)p_t \tag{A23.1}$$

$$\Leftrightarrow \quad \pi_t = \beta E_t \pi_{t+1} + \frac{(1-\phi)(1-\beta\phi)}{\phi} p_t^* - \frac{1-\phi}{\phi} p_t + \beta(1-\phi)p_t. \tag{A23.2}$$

A.5 Intertemporal Optimization – Calculation Steps

The optimization problem has the constraint

$$C_t \cdot P_t + B_{t+1} = W_t + (1 + i_{t-1}) \cdot B_t, \tag{A24}$$

where W is the nominal wage and B the nominal value of bonds. The latter provides the link between two periods. Depending on the definition of the interest rate, the period can vary. Here it has been chosen in a way so that the interest from period t enters the Euler condition. Dynamic Programming uses the additively separable utility function and the envelope theorem to set up optimality conditions for two consecutive periods. The procedure can be divided into three parts. The first part is to write a value function, the Bellman equation. Under the assumption that the second term of the expanded utility

$$U(C_t) + E_t\left[\sum_{s=t+1}^{\infty} \beta^{s-t-1} U(C_s)\right] \tag{A25}$$

is maximized in period t, the Bellman equation is

$$V(B_t) \equiv \max_{C_t} \{U(C_t) + \beta V(B_{t+1})\}. \tag{A26}$$

The expected value vanishes since B_{t+1} is determined by variables in period t in the constraint. Differentiating with respect to C_t gives the first-order condition

$$\frac{d}{dC_t}U(C_t) + \beta\frac{d}{dC_t}V(B_{t+1}) = U'(C_t) + \beta V'(B_{t+1}) \cdot \frac{dB_{t+1}}{dC_t} = 0, \tag{A27}$$

which results in

$$U'(C_t) = P_t\beta V'(B_{t+1}). \tag{A28}$$

Eq.(A28) relates the marginal utility to the marginal value in the following period, the time preference, and prices in the same period. Therefore, a higher β and P_t results in a lower C_t.

In the next part, the envelope theorem is used to differentiate the value function (by inserting the optimized C_t^*) with respect to the costate variable B_t:

$$V(B_t) = U(C_t^*) + \beta V(B_{t+1}) \tag{A29.1}$$

$$\Rightarrow \quad \frac{dV}{dB_t} = \beta V'(B_{t+1}) \cdot \frac{dB_{t+1}}{dB_t} \tag{A29.2}$$

$$\Leftrightarrow \quad V'(B_t) = \beta V'(B_{t+1}) \cdot (1 + i_{t-1}). \tag{A29.3}$$

Eq.(A29.3) reveals the relationship of the marginal value functions.

In a third and last step, the first-order condition (A28) can be used to replace the value functions in Eq.(A29.3) with the marginal utility in both periods t and $t-1$:

$$\frac{U'(C_{t-1})}{P_{t-1}\beta} = \beta \cdot \frac{U'(C_t)}{P_t\beta} \cdot (1 + i_{t-1}) \tag{A30.1}$$

$$\Rightarrow \quad \frac{U'(C_t)}{P_t} = \beta(1 + i_t)E_t\left[\frac{U'(C_{t+1})}{P_{t+1}}\right]. \tag{A30.2}$$

The time shift yields the Euler condition.

A.6 Jensen's Inequality – Calculation Steps

$f(EX) \geq E[f(X)]$ holds for concave functions, i.e. the logarithm and Jensen's inequality still holds for the conditional expected value. Since the function's curvature is sufficiently small, the accuracy is comparable to log-linearization for small growth rates. Moreover, the exactness increases for larger values because of $(\ln(x))'' \to 0$ for increasing x. However, resulting values will always be underestimated.

$$\ln E_t\left[\frac{Z_{t+1}}{Z_t}\right] = \ln E_t\left[\exp\left(\ln\left(\frac{Z_{t+1}}{Z_t}\right)\right)\right] \approx \ln E_t\left[1 + \ln\left(\frac{Z_{t+1}}{Z_t}\right)\right] \tag{A31.1}$$

$$= \ln\left(1 + E_t\left[\ln\left(\frac{Z_{t+1}}{Z_t}\right)\right]\right) \approx E_t\left[\ln\left(\frac{Z_{t+1}}{Z_t}\right)\right]. \tag{A31.2}$$

A.7 Second-Order Taylor Approximation

The Taylor series (in $\mathbb{R}$) helps in finding a polynomial to substitute a certain function $f(x)$ (i.e. exponential, logarithm, etc.) around a point x_0. The generalized formula of the degree n in the compact sigma notation is

$$Taylor(n) = \sum_{j=0}^{n} \frac{f^{(j)}(x_0)}{j!}(x - x_0)^j, \tag{A32}$$

where $f^{(j)}$ denotes the jth derivative with $f^{(0)} = f$ as a special case. Thereby, larger values for n give better approximations of the original function $f(x)$. In (23.1), $f(x) = \ln(1 + x)$ and $n = 2$. Formula (A32) simplifies to

$$Taylor(2) = \ln(1 + x_0) + \frac{1}{1 + x_0}(x - x_0) - \frac{1}{2(1 + x_0)^2}(x - x_0)^2. \tag{A33}$$

The result in (23.1) appears with $x_0 = 0$ and $\widetilde{y_{t+1}}$ (π_{t+1} respectively) as the argument of the function:

$$\ln(1 + \widetilde{y_{t+1}}) \approx \widetilde{y_{t+1}} - \frac{1}{2}\widetilde{y_{t+1}}^2. \tag{A34}$$

A.8 Standard Targeting Rule – Calculation Steps

The Lagrangian has to be differentiated with respect to $\widehat{y_t}$, π_t, and i_t, since the central bank sets the nominal interest rate:

$$\mathscr{L}(\pi_t, \widehat{y_t}, i_t) = E_t\Bigg[\sum_{s=t}^{\infty} \beta^{s-t}\left(\pi_s^2 + \delta\widehat{y_s}^2\right) - \chi_s(\pi_s - \beta\pi_{s+1} - \kappa\widehat{y_s})$$
$$-\varphi_s\left(\widehat{y_s} - \widehat{y_{s+1}} + \frac{1}{\sigma}(i_s - r - \pi_{s+1}) + \frac{1}{2\sigma}\pi_{s+1}^2 + \frac{1}{2}\widetilde{y_{s+1}}^2\right)\Bigg]. \tag{A35}$$

First-order conditions:

$$\frac{\partial \mathscr{L}}{\partial \pi_t} = 2\pi_t - \chi_t \qquad = 0 \tag{A36.1}$$

$$\frac{\partial \mathscr{L}}{\partial \widehat{y_t}} = 2\delta\widehat{y_t} + \chi_t\kappa - \varphi_t(1 + \widehat{y_t} - E_t\widehat{y_{t+1}}) = 0 \tag{A36.2}$$

$$\frac{\partial \mathscr{L}}{\partial i_t} = -\frac{\varphi_t}{\sigma} \qquad = 0. \tag{A36.3}$$

Condition (A36.2) follows with Eq.(27). From condition (A36.3) follows that $\varphi_t = 0$, hence the minimized loss will not change if the IS curve shifts, as the central bank can counteract it one by one through resetting the nominal interest rate. Combining (A36.1) and (A36.2), the standard targeting rule under discretion arises.

A.9 Optimal Interest Rate for Positive Inflation Targets

When the Lagrangian attains the "leaning against the wind" condition, it is extended with π^* (as in (29), the loss function). Therefore, the standard targeting rule changes to

$$\pi_t - \pi^* = -\frac{\delta}{\kappa}\widehat{y_t}, \tag{A37}$$

whereby the optimal output gap,

$$\widehat{y_t} = -\frac{\beta\kappa}{\delta + \kappa^2}E_t\pi_{t+1} + \frac{\pi^*\kappa}{\delta + \kappa^2}, \tag{A38}$$

comprises an additional term. After inserting (A38) in the IS curve, the interest rule also has an additional (negative) term. This would lead to a generally lower interest level.

A.10 Equilibrium Condition – Calculation Steps

Eq.(51) and Eq.(52) in more detail:

$$E_t(\widehat{y_{t+1}} - \widehat{y_t})^2 = E_t\left[((-\kappa\theta)(\mu e_t + \zeta_{t+1}) - (-\kappa\theta)e_t)^2\right] \tag{A39.1}$$

$$= E_t\left[(-\kappa\theta)^2(\mu e_t + \zeta_{t+1} - e_t)^2\right] \tag{A39.2}$$

$$= (-\kappa\theta)^2 E_t\left[((\mu - 1)e_t + \zeta_{t+1})^2\right] \tag{A39.3}$$

$$= ((-\kappa\theta)(\mu - 1))^2 e_t^2 + (-\kappa\theta)^2\left(Var_t\zeta_{t+1} + (E_t\zeta_{t+1})^2\right) \tag{A39.4}$$

$$= \kappa^2\theta^2(\mu-1)^2 e_t^2 + \kappa^2\theta^2\sigma_e^2 \tag{A39.5}$$
$$= (\kappa\theta)^2\left((1-\mu)^2 e_t^2 + \sigma_e^2\right) \tag{A39.6}$$

and

$$i_t = r + \alpha_\mu e_t - \frac{1}{2}(\delta\theta)^2\left(\mu^2 e_t^2 + \sigma_e^2\right) - \frac{\sigma}{2}(\kappa\theta)^2\left((1-\mu)^2 e_t^2 + \sigma_e^2\right) + \sigma u_t \tag{A40.1}$$
$$= r + \alpha_\mu e_t - \frac{1}{2}\left((\delta\theta)^2\mu^2 e_t^2 + (\delta\theta)^2\sigma_e^2 + \sigma(\kappa\theta)^2(1-\mu)^2 e_t^2 + \sigma(\kappa\theta)^2\sigma_e^2\right)$$
$$+ \sigma u_t \tag{A40.2}$$
$$= r + \alpha_\mu e_t - \frac{1}{2}\left(\left((1-\mu)^2\sigma\kappa^2 + \mu^2\delta^2\right)\theta^2 e_t^2 + \left(\sigma\kappa^2 + \delta^2\right)\theta^2\sigma_e^2\right) + \sigma u_t. \tag{A40.3}$$

A.11 Parameter Discussion

Eq.(45) includes all parameters of the model.[43] This subsection gives a brief overview over possible values, which are used to graphically depict the equilibrium conditions.

The discount parameter β is typically close to 1. Galí (2015, 67) and Rotemberg and Woodford (1997, 321) set β equal to 0.99 (quarterly), whereas Jensen (2002, 939) uses this under an annual interpretation. Walsh (2010, 362) also sets it to 0.99. Galí and Gertler (1999, 207) estimate a value of 0.988. To keep the framework close to the actual interest setting of the central bank, all calculations are carried out quarterly and β will be set to 0.99.

The slope of the NKPC κ takes values close to zero and usually lower than 1. Roberts (1995, 982) estimates in his original NKPC article $\kappa \approx 0.3$. On a quarterly basis, Walsh (2010, 362) sets 0.05, Galí and Gertler (1999, 13) estimate 0.02, and McCallum and Nelson (2004, 47) suggest 0.01–0.05. Jensen (2002, 939) calibrates an annual value of 0.142, whereas Clarida et al. (2000, 170) set 0.3 (yearly) and give a range of 0.05 to 1.22 in the literature. In the baseline simulation, κ is set to 0.04.[44]

Woodford (2003a, 165) states that a value of 1 is customary in the RBC literature for σ, the multiplicative inverse of the IES (see, e.g. Clarida et al. (2000, 170), Galí (2015, 67), Yun (1996, 359)). A slightly larger value (1.5) is set by Jensen (2002, 939) and Smets and Wouters (2003, 1143) estimate 1.4. An insightful metadata study by Havranek et al. (2015) estimates a mean IES of 0.5 ($\sigma = 2$)

[43] Note that variances σ_e^2 and σ_u^2 are only indirectly included.
[44] Note that this implies $\kappa = 0.16$ on a yearly basis.

across all countries. However, they report that more developed countries have a higher IES (lower σ). Therefore, σ will be set to 1.

The weight on output fluctuations δ is set to 0.25 in almost all the literature (see, e.g. Walsh (2010, 362), 939), McCallum and Nelson (2004, 47), Jensen (2002, 939)). The latter reports values from 0.05 to 0.33 in other papers. Thus, $\delta = 0.25$ will also be assumed for the simulation.

Walsh (2003, 275) allows values up to 0.7 for μ, the cost shock persistence. Clarida et al. (2000, 170) set 0.27 (yearly) and Galí and Rabanal (2004, 48) estimate 0.95. Generally, Smets and Wouters (2003, 1142–1143) estimate persistencies of 0.8 and higher, which is confirmed by Smets and Wouters (2007). Thus, μ will be treated as a variable in the range of $0.6 - 0.85$. The smallest value 0.6 implies 0.1296 on an annual basis.

For the standard deviation of a cost shock, Sims (2011, 17) sets 0.01 ($\sigma_e^2 = 0.0001$), Jensen (2002, 939) sets 0.015 ($\sigma_e^2 = 0.000225$), and Galí and Rabanal (2004, 48) estimate 0.011 ($\sigma_e^2 = 0.000121$). McCallum and Nelson (2004, 47) set an annualized standard deviation of 0.02 ($\sigma_e^2 = 0.0004$). The conservative value of 0.0001 will be taken for the simulation.

Table A1: Symbols

Letter	Description
α	Summarizing parameters ($\alpha_\psi, \alpha_y, \alpha_\pi, \alpha_\mu, \alpha_e, \alpha_\sigma$)
β	Discount factor (time preference)
δ	Weighting on output gap in loss function
ε	Elasticity of substitution
ζ	Error term of cost shock
η	Error term of demand shock
θ	Auxiliary parameter
κ	Slope of NKPC
μ	Cost shock persistence
ν	Demand shock persistence
π	Inflation
σ	Reciprocal value of the IES
τ	Firm index
ϕ	Price stickiness
ψ	Parameter in cost function
c	Constants in cost function (c_{fix}, c_{var})
e	Cost shock
i	Nominal interest rate
k	Cost parameter in Calvo pricing
p	Log-linearized price around the steady state
r	Long-run real interest rate
u	Demand shock
x	Calvo price
y	Log-linearized output growth rate around the steady state
$\widehat{y}$	Growth rate of output gap around the steady state
$\widetilde{y}$	Output growth rate
B	Bonds
C	Consumption
$K(.)$	Cost function
P	Price; Price level
$U(.)$	Utility function
W	Wage
Y	Output