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Sensitivity Analysis for Dynamic Microsimulation Models

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Abstract

Microsimulations usually contain various transition processes such as births and deaths, relocations, and change in household characteristics. The estimation, organisation, and implementation of these processes can have a substantial impact on the simulation outcomes. We propose to evaluate a dynamic discrete-time stochastic microsimulation using the concept of variance based sensitivity analysis which is well established for indicator assessment in survey statistics. This method is suitable for measuring and comparing the impact of different input factors on the output, even when analytical methods – given the complexity in a dynamic framework – cannot be applied.

1   Introduction

In many areas of decision making, such as transport, infrastructure and political consulting, microsimulation modelling is a powerful tool for conducting scenario-based analysis. Usually, microsimulation models use agents or representatives to consider differing individual behaviour. With today's computing power, complex dynamic microsimulations become more feasible. Instead of using a subset of the population or a few agents, many dynamic microsimulation studies simulate the development of each individual unit within a total population.

One of the major tasks in assessing microsimulation models is to address the different sources of uncertainty. Goedemé, Van den Bosch, Salanauskaite, and Verbist (2013) stress the necessity to estimate variances and confidence intervals in microsimulation models based on the sampling variance of the input data. Lappo (2015) uses established methods of uncertainty measurement in static microsimulation models to consider sampling variability in inequality and poverty indicators. An application to dynamic microsimulations has not been adopted due to the increasing complexity caused by *the added uncertainty relating to the estimation of temporal and behavioural effects* (Lappo, 2015). The enormous amount of potential influence factors render analytical approaches unsuitable for most microsimulation models (Sharif et al., 2012). For this reason, Sharif et al. (2012) suggest a Monte-Carlo based method to estimate confidence intervals regarding previously selected factors.

In order to measure and compare the impact of various particular input factors, sensitivity analysis is a commonly applied method in composite indicator research (see, e.g. Münnich & Seger, 2014). In the context of microsimulation, this was suggested by various researchers with recommendations for its application. Dowling, Holland, and Huang (2002) suggest sensitivity analysis for the targeted assessment of the reliability of microsimulations by varying uncertain inputs or assumptions. Brown et al. (2011) mention sensitivity analysis as a validation tool that is used to evaluate the simulation results regarding extreme assumptions and parameters of the underlying models. Rutter, Zaslavsky, and Feuer (2011) propose the use of sensitivity analysis in case of not directly observable parameters to explore their plausibility. So far, however, there is no explicit documentation of an implementation strategy. Furthermore, there is neither a general description nor a visualisation of the outcome of a sensitivity analysis.

If there is an indication for a poorly performing microsimulation model, sensitivity analysis could help to identify the components that need to be improved. Thus, sensitivity analysis are an useful supplement to formal variance estimation techniques (National Research Council, 1991). In the 12 lessons for microsimulation modelling of the SAGE research group, sensitivity analysis are named to measure the impact of different parameters on the results and to analyse the influence of macroeconomic indicators (Zaidi & Rake, 2001). Harding, Keegan, and Kelly (2010) refer to the use of sensitivity analysis in combination with alignment methods where the individual transition probabilities are calibrated to different target values. The aim is to determine the impact of policies assuming different developments on birth rate, wage growth or labour force. Lastly, Marois, Sabourin, and Bélanger (2017) mention that sensitivity analysis can be used to illustrate simulation results under different scenarios and uncertainties.

Our contribution is the application and usability of sensitivity analysis to assess uncertainty in input factors as a tool for interpretation purposes. The sensitivity analysis is applied to a dynamic discrete-time microsimulation model to investigate the change in

nursing needs and the care level over time.

First, we briefly introduce the concept of sensitivity analysis and how it can be applied in microsimulations. In Chapter 3 we describe the microsimulation model used for assessing the development of nursing needs and the care level. Then we analyse the outcomes of our microsimulation using the proposed sensitivity analysis approach in Chapter 4. Last, a conclusion is drawn.

2   Sensitivity Analysis

Sensitivity analysis describes techniques for providing reliable information about *which of the uncertain input factors is more important in determining the uncertainty in the output of interest* (Saltelli, Tarantola, Campolongo, & Ratto, 2004). The aim is to measure the influence of changing model parameters and other factors on the results (Loucks, Van Beek, Stedinger, Dijkman, & Villars, 2005). This is done by decomposing the output variance into parts due to the uncertain input factors.

Within the framework of microsimulation modelling, sensitivity analysis is applied to uncertainty measurement as well as interpretation and visualisation of the results (Harding et al., 2010; Rutter et al., 2011). The decomposition of the output variance contributes to the transparency of the analysis, since the most influential uncertain input factors can be easily identified. In case of unclear or competing model structures or scenarios, the choice of the model itself can also be implemented as a source of uncertainty (Saltelli, Tarantola, & Campolongo, 2000). For these purposes, we propose to use variance-based methods of sensitivity analysis as follows.

In line with Saltelli et al. (2008) let $f(\cdot)$ be a microsimulation model with a K-dimensional pairwise independent input factor $X = \{X_1, ..., X_k, ..., X_K\}$. Those factors can include different scenarios, types of models, parameters and data sources. Furthermore there is a univariate outcome Y which can be written as a function of X:

$$Y = f(X) = f(X_1, X_2, ..., X_K).$$

The pairwise independence of input factors does not necessarily indicate the absence of mutual influence on the results. The total or unconditional variance of Y can be decomposed into the sum of the conditional variances V_i (first-order effect of X_i on Y) and the sum of the joint effects V_{ij} (second-order or two-way effects). Effects of higher orders, like the third-order effect V_{ijl}, can be derived analogous (Saltelli et al., 2008).

$$\text{Var}(Y) = \sum_{i \in \{1,...,K\}} V_i \quad + \sum_{\substack{j \in \{1,...,K\} \\ i<j}} V_{ij} \quad + \ldots + V_{12...K}$$

$$V_i = \text{Var}_{X_i}\left(E_{\mathbf{X}_{\{1,...,K\}\setminus\{i\}}}(Y \mid X_i)\right)$$

$$V_{ij} = \text{Var}_{X_{ij}}\left(E_{\mathbf{X}_{\{1,...,K\}\setminus\{i,j\}}}(Y \mid X_i, X_j)\right) - \text{V}_i - \text{V}_j$$

From this first and second-order effects, the so-called main effects, interaction effects, and total effects are calculated as the ratio of the first/second-order effect to the total variance.

Main Effects:

Main effects (or first-order sensitivity index) measure the effect of the variation of the k-th input factor onto the outcome variable excluding the interaction effects. The main sensitivity index S_k^M for the factor k is obtained by dividing the conditional variance V_k by the unconditional variance $\text{Var}(Y)$.

$$S_k^M = \frac{V_k}{\text{Var}(Y)} \tag{1}$$

The main effects are always in the in the unit interval $[0,1]$ with higher values indicating more relevant variables (Saltelli et al., 2008). If the number of input factors K is large, usually only the main and total effects are computed due to the exponential growth of the dimensions. With a K-dimensional input factor, the number of indices is $2^K - 1$ (Iooss & Lemaître, 2015).

Interaction Effects:

Simplifying, factors interact when their effect on the outcome is unequal to the sum of their single effects. The interaction effect for the factors k and l can be written as follows:

$$S_{k,l}^M = \frac{V_{k,l}}{\text{Var}(Y)}. \tag{2}$$

The sum of all main and interaction effects is always equal to 1:

$$\sum_i S_i \quad + \quad \sum_i \sum_{j>i} S_{i,j} \quad + \quad \sum_i \sum_{j>i} \sum_{l>j} S_{i,j,l} S_{i,j,l} + ... + S_{1,2,...,k} = 1. \tag{3}$$

Total Effects:

The total effects correspond to the sum of main effects with all interaction effects including:

$$S_k^T = \frac{E_{\mathbf{X}_{\{1,...,K\}\setminus\{k\}}} \left(\text{Var}_{X_k}(Y \mid \mathbf{X}_{\{1,...,K\}\setminus\{k\}}) \right)}{\text{Var}(Y)}. \tag{4}$$

For the implementation of the sensitivity analysis, we use the R-package `multisensi` (Lamboni, Makowski, & Monod, 2010). More detailed information about the derivation of the variance components for variance based sensitivity analysis can be found in Iooss and Lemaître (2015) and Saltelli et al. (2008).

3 Description of the Microsimulation

3.1 Structure

For the base dataset, we use a synthetic population of Germany, which was created using anonymised population register data from the census project 2011 (Münnich, Gabler, Ganninger, Burgard, & Kolb, 2012). This dataset contains socio-demographic and socio-economic variables at micro level derived from several surveys, notably the German Microcensus, with a hierarchical breakdown to persons within households.

We focus on the city of Trier with approx. 105,000 inhabitants in 55,00 households in 2011. For the simulation carried out here, the base dataset is considered as a closed

population (Zagheni, 2015). Individuals can only enter the population through birth and leave by death. For reasons of computing time reduction, we omit a migration module. Ignoring migration flows leads to unrealistic developments regarding the population structure, which is particularly apparent by the lack of emigrating and immigrating students. This limitation should be unproblematic as the focus is on the application of sensitivity analysis.

All persons in the population get an individual probability $p_j \in [0, 1]$ for an event j to occur. The probabilities are based on statistical models, mainly binary and multinomial logit regression models, and administrative data tables. To estimate p_j, we use the public-use-file of the German Microcensus panel and the scientific-use-file of the German Microcensus (Bihler, Herter-Eschweiler, & Wirth, 1996).

3.2   Modules

All events of state changes within the microsimulation progress are organised in specific modules which have to be executed in a previously determined order for each simulation run. The modules used in this simulation include the processes of fertility, mortality, movings/separations, new partnerships, relationship status, care/level of care, education, vocational training, and employment. The order is arranged as shown in Figure 1.

Fertility
   The fertility module is based on a two-step modelling approach. First of all, the events of giving birth based on the familiar and personal situation of women aged 15-49 are simulated. Afterwards twin births – conditional on a birth considering the age of the bearing women – are determined.

Mortality
   To simulate deaths, we estimate model-based probabilities using socioeconomic factors as well as the level of care. The probabilities are subsequently calibrated to the age- and gender-specific values from mortality tables using logit-scaling (Stephensen, 2016).

Leaving households
   We differenciate between two possibilities for individuals to leave remaining households: separations and movings out of parental houses/flat sharing communities. To simulate separations of cohabiting partners, model-based probabilities are applied to execute the event of a separation, which only is possible for individuals living in partnerships. In case of associated children, a household-specific allocation progress decides about the residence. With different probabilities, the children remain with the mother, the father or are allocated amongst both parents. All persons over the age of 18 who are living together with their parents or other adults in a flat-sharing community are capable of leaving and establishing a new household.

Partnerships
   Probabilities for new partnerships are estimated for all single males aged 18 or older not living in their parental household. After the identification of all single males that get into a relationship, suitable women are selected using a two-step matching approach. In the first step, we match all selected males to non-single males in the population. The non-single males and their spouses act as proxies. In the second

5

step, single woman are matched to the spouses of the previously selected non-single males. The women selected in the second step are then partnered with the single males. To take account of different scalings, we use Gowers distance for calculating a distance matrix (Handl, 2002). Furthermore, to avoid a decreasing bias caused by repeatedly matched individuals, we execute the matching without replacement.

Relationship status

All marriages are unambiguous allocated in the base dataset using personal-unique marriage-IDs. Thus, marriages can be matched even after spatial separations. Probabilities of new marriages are estimated for unmarried couples living in the same household. Divorces are only possible for spouses living separated. The death of a person determines the widowhood of her or his spouse.

Care/level of care

First, we estimate the probability of becoming in need of care or leaving the state of care dependency and afterwards the level of care. So we enable both, changes in the care status and the level of care during the simulation process.

Education and Vocational training

Transition probabilities of the highest degree of education and vocational education are results of ordered logit models accounting for the ordinal scale of the variables. All models are split up according to the initial state to prevent implausible transitions. First, the school leaving certificate is simulated, then the training certificate, considering the school leaving certificate. This makes it possible to take into account different combinations of school-leaving qualifications and vocational qualifications already acquired.

The order of the modules is also considered in the modelling process due to the choice of exogenous variables. Hence, the employment status of the previous period is used as an explanatory variable for the modelling of vocational training qualification. For the modelling of the employment status, in turn, the vocational training status of the same period is used.

3.3 Variable of Outcome

Since this paper aims to show the application of sensitivity analysis in a dynamic microsimulation context, we have to define the univariate outcome variable Y. We have chosen a more complex target value because the influence of individual factors is more difficult to quantify. The focused variable is the ratio of living alone in need for care to the number of economically active people in Trier. It can be assumed that the regional necessity of formal care highly depends on the household structures (Sole-Auro & Crimmins, 2014). The presence of further family members in a household facilitates nursing by unpaid family carers. However, the rise of living alone care-dependent population leads to a decreasing potential for providing informal care at home and to an increasing need for public services (Wetzstein, Rommel, & Lange, 2015). Hence, the trends in living conditions and household compositions are an indicator for the future need for care institutions as well as for the evolution of public care expenditure and the private healthcare sector.
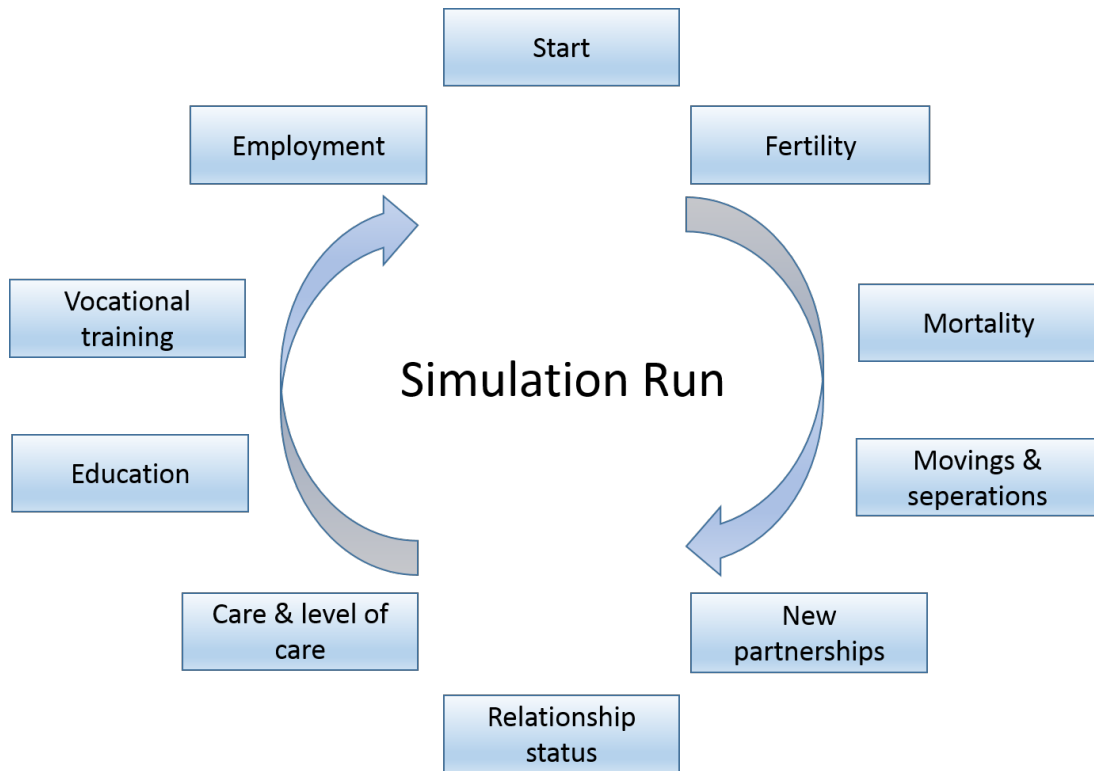
Figure 1: Sequence of Modules of one Simulation run.

3.4 Input Factors and Simulation Setup

To examine the influence of specific input factors on the univariate outcome Y, we set up various scenarios by varying the ratios of state transitions in the modules for births, partnerships, and separations:

Births:

- constant ratio of births to fertile women
- increasing ratio of births to fertile women
- decreasing ratio of births to fertile women

Partnerships:

- constant ratio of new partnerships to singles
- increasing ratio of new partnerships to singles
- decreasing ratio of new partnerships to singles

Separations:

- constant ratio of sparations to couples
- increasing ratio of separations to couples
- decreasing ratio of separations to couples

For the baseline scenarios, the transition probabilities were estimated using the original models without any modification. Deviating from the baseline scenario, for the increasing (decreasing) share scenarios, the aligned ratios were monotonically increased (decreased) up to 150% (50%) of the baseline value. This dynamic could come from external experts with contrary opinions about the future development. Alternatively, this could also be derived from a confidence interval from an external forecast or the uncertainty of the modelling process.

Additionally, the baseline models were estimated twice: once under consideration of survey weights and once without. In the weighted case, both the survey- and longitudinal-weights were included in the modelling process. This is especially important as most models used in microsimulation stem from survey data, and unbiased or at least consistent estimates may be of great importance.

For all combinations of input factors and scenarios, the microsimulation was computed 20 times. Overall, this means a total of $3 \cdot 3 \cdot 3 \cdot 2 \cdot 20 = 1080$ simulation runs for the components of births, separations, and partnerships with three scenarios each, using both weighted and unweighted models and 20 repetitions. The simulation periods are set to 60 years as we want to emphasize the use of the sensitivity analysis as a tool which is also capable of considering the temporal component in microsimulation models.

4   Sensitivity Analysis on the Simulation Outcome

In a microsimulation context it is necessary to consider the Monte-Carlo error which is defined as the error produced within the stochastic transition processes (Bélanger & Sabourin, 2017). When repeating a simulation runs, the results of certain input combinations are not deterministic and differ in each run, since the individual transitions are results of stochastic processes.

Dowling et al. (2002) compare a single microsimulation run with rolling dice only once. In order to avoid misinterpretations, microsimulations are usually performed several times with different random numbers. For all generated outcomes, the mean value of the results can be derived from different runs and then interpreted.
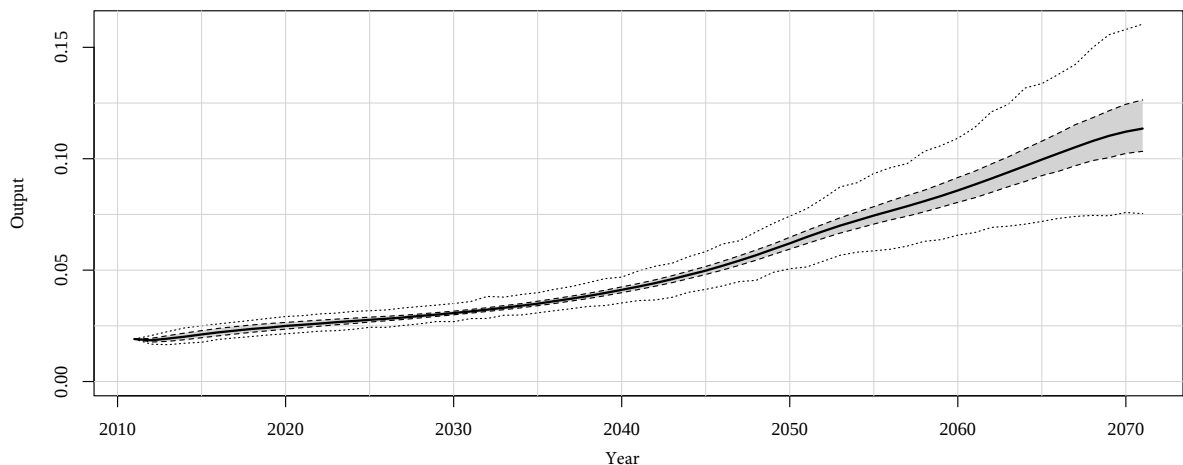

Figure 2: Visualisation of the output without differentiation

Figure 2 shows the median of all generated outputs over all combinations of input fac-

tors and random runs as a bold line, the extreme values as outer ties and the interquartile range as the grey area in between. Furthermore, the outer line shows the minimal overall outcome value within all combinations of input factors and simulation runs. Even in this extreme output, a sharp increase in the share of living alone care-depend to economically active people can be observed. This kind of presentation is useful to obtain a broad overview of all generated simulation results and thus to detect developments under the consideration of uncertainty. However, it is not possible to identify the influence of individual input factors on the results. For this purpose, the outcome can be visualised by differentiating between the input scenarios. In the upper part of Figure 3, the simulation outputs are illustrated for the birth-, partnership- and separation-scenarios. The increasing share scenarios are blue, the decreasing share ratios green and the baseline scenarios red coloured. Additionally, the results are illustrated by boxplots for the years 2040 to 2070 in the figure below.

The graphs show the strong impact of the scenarios on the outcome, with the factor births having the greatest impact in later periods. A more detailed comparison of the input factors and in particular of their interactions – especially for earlier periods – is almost impossible.
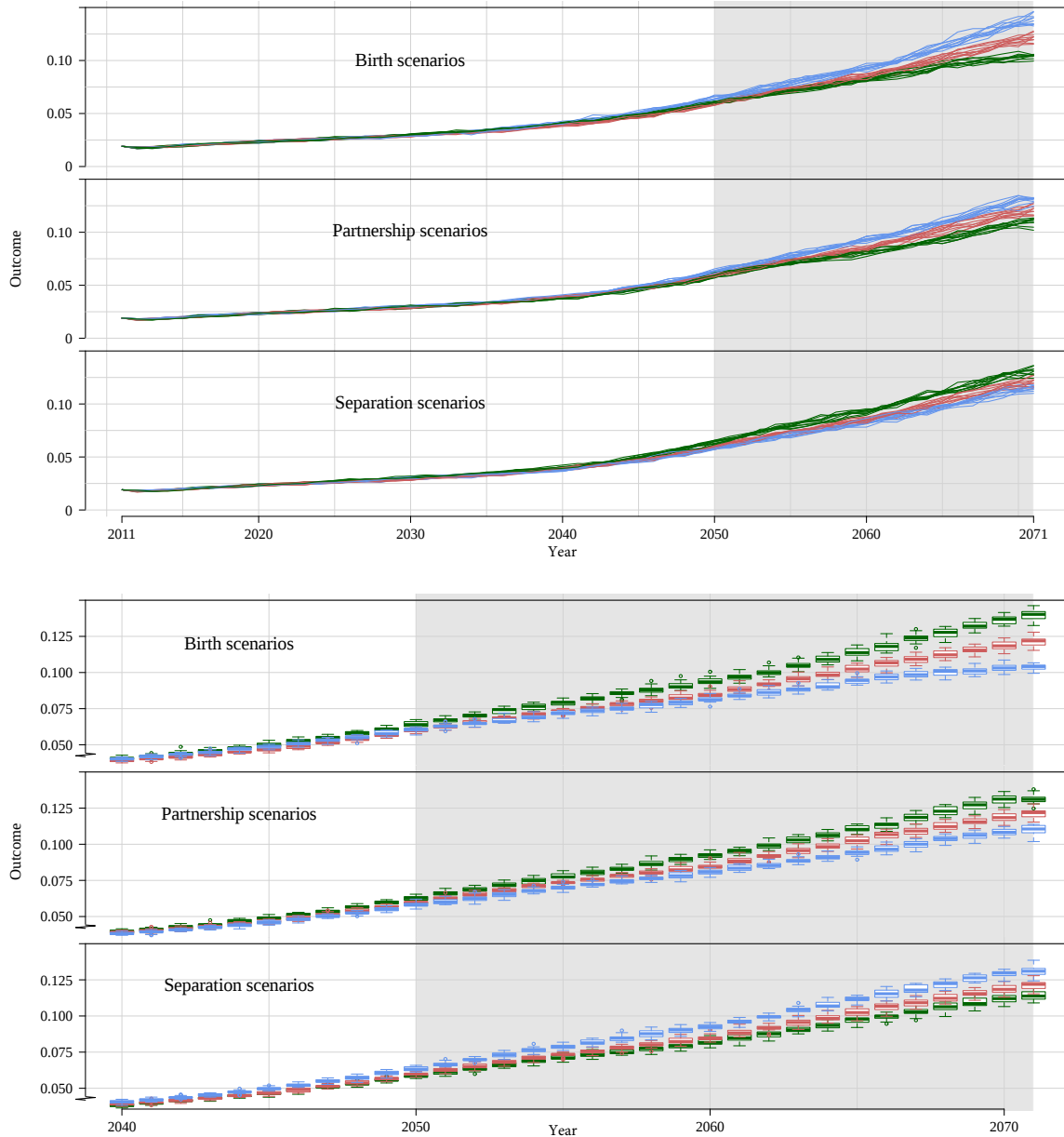
Figure 3: Scenario-specific visualisation of the outputs

4.1 Implementation

In order to compare the influence caused by the various factors on the simulation results, the main and total effects can be calculated independently for each discrete simulation period. For this purpose, the sensitivity measurements are determined after the execution of all modules for the years 2012 to 2071 with the R-function `multisensi()` from the eponymous R-package.

As shown above, variance-based sensitivity analysis distributes the total variation among the implemented factors. If, for example, only one simulation run is used for the sensitivity analysis, the sum of all sensitivity indices for the scenarios of birth (S_b), partnerships (S_s), separation (S_s) and weights (S_w) together with the interaction effects

is equal 1:

$$S_b + S_p + S_s + S_w + S_{b,p} + S_{b,s} + S_{b,w} + S_{p,s} + S_{p,w} + \ldots + S_{b,p,s,w} = 1.$$

Since the simulation includes stochastic processes, Monte-Carlo uncertainty has to be considered in the analysis. Taking into account possible impacts from the stochastic simulation component, the random seed is used as an additional influence factor S_r. This increases the number of main and interaction effects from 15 to 31. Due to the normalisation of the effects, the sum is still 1:

$$S_b + S_p + S_s + S_w + S_r + S_{b,p} + S_{b,s} + S_{b,w} + S_{p,s} + \ldots + S_{b,p,s,w,r} = 1.$$

The sum of all effects influenced by the stochastic component thus corresponds to the unexplained part of the variance:

$$S_{unexp} = S_r + S_{b,r} + S_{p,r} + S_{s,r} + S_{w,r} + S_{b,p,r} + S_{b,s,r} + \ldots + S_{b,p,s,w,r}.$$

Table 2 shows the main and total sensitivity indices in five-year steps. The cells highlighted in light grey emphasise the highest influencing factor in the respective period.

Alternatively, the main and interaction effects can be rescaled to the conditional variance components to enable a more detailed inter-temporal comparison. To calculate the variance components, the main effects $S_{i,t}$ for period t are multiplied by the variance of the results $\text{Var}(Y_t)$:

$$V_{i,t} = S_{i,t} \times \text{Var}(Y_t). \tag{5}$$

In contrast to $S_{i,t}$, $V_{i,t}$ measures the absolute impact. Thus, the value range of $V_{i,t}$ is between 0 and $\text{Var}(Y_t)$.

Table 1: Main Sensitivity Indices

	Main Sensivity Indices					
Year	2016	2021	2026	2031	2036	2041
Births	0.0009	0.0001	0.0006	0.0115	0.0879	0.1758
Separations	0.0001	0.0051	0.0295	0.0863	0.1475	0.2004
Partnerships	0.0012	0.0117	0.0402	0.1117	0.1807	0.2238
Weights	0.8718	0.7927	0.5683	0.2964	0.1341	0.0349
Random Run	0.0069	0.0112	0.0149	0.0158	0.0152	0.0103
Year	2046	2051	2056	2061	2066	2071
Births	0.2041	0.2206	0.2447	0.3057	0.3955	0.5142
Separations	0.2488	0.2561	0.2486	0.2172	0.1806	0.1231
Partnerships	0.2786	0.3081	0.3197	0.3000	0.2486	0.1938
Weights	0.0008	0.0149	0.0392	0.0610	0.0876	0.1014
Random Run	0.0059	0.0035	0.0031	0.0017	0.0008	0.0009

Table 2: Total Sensitivity Indices

Year	Total Sensivity Indices					
	2016	2021	2026	2031	2036	2041
Births	0.0882	0.1291	0.2447	0.3321	0.3704	0.3904
Separations	0.0826	0.1351	0.2779	0.4256	0.4439	0.4302
Partnerships	0.0858	0.1392	0.2908	0.4552	0.4820	0.4567
Weights	0.9336	0.8883	0.7580	0.5680	0.3741	0.2472
Random Run	0.1205	0.1756	0.3192	0.4149	0.3578	0.2752
Year	2046	2051	2056	2061	2066	2071
Births	0.2041	0.2206	0.2447	0.3057	0.3955	0.5142
Separations	0.2488	0.2561	0.2486	0.2172	0.1806	0.1231
Partnerships	0.2786	0.3081	0.3197	0.3000	0.2486	0.1938
Weights	0.0008	0.0149	0.0392	0.0610	0.0876	0.1014
Random Run	0.0059	0.0035	0.0031	0.0017	0.0008	0.0009

4.2 Visualisation and Interpretation

As already described, the objective of sensitivity analysis is to quantify the individual influence of previously selected impact factors on the outcome. In order to achieve a more precise interpretation, different ways of visualisation can be used. Figure 4 shows a comparison of the main effects for the years 2041 and 2071 using radio-polygons. In the horizon 2041, the modelling options for partnerships, births, and separations do have more or less the same impact on the variability of the dependent variable. In contrast, for the horizon 2071, the factor of births dominates the variability. For both horizons, the inclusion/exclusion of weights does not change the output as much as any other factor.

In this simulation the simulation runs do not induce much of variability, indicating a quite stable simulation path given the input factors. This kind of visualisation can be helpful, especially for the direct comparisons of sub-populations or, as in this case, different years. However, this type of visualisation can quickly become confusing when the number of factors increases.

To illustrate both, the influence caused by the factors and the development of the temporal impact, stacked surfaces of the main effects are a suitable option (see Lamboni et al., 2010). Such a illustration can be seen in Figure 5, which shows the main effects at each point in time of the simulation. Furthermore, the sum of second-order interaction effects corresponds to the grey and the set of all effects influenced by the Monte-Carlo variation S_{unexp} to the remaining transparent area.

The dependence of sensitivity indices on the number of simulated periods is clearly visible. In comparison to all factors considered, the largest relative part of the variance is caused by the factor weights until period 2021. From the years 2021 to 2061, the largest impact on the outcome can be observed from the factor separations and subsequently from the factor births. Small values for the sensitivity indices of the factor births in a short simulation period appear realistic assuming that newborns affect neither the working nor the care-dependent population. In contrast, the impact of the factor births predominates

Figure 4: Radiopolygon: Main Sensitivity Indices
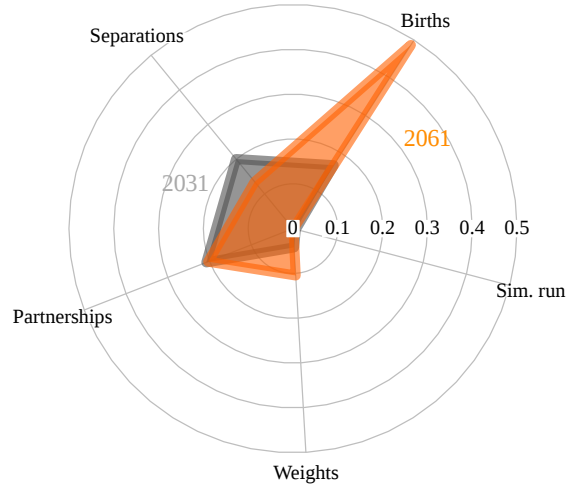
in the further process which could be reasoned by the double effect of births on a long-term run. Both the labour force and household constellations get influenced by the development of the birth rate. Thus, a decreasing birth rate leads to a reduction of the economically active population and simultaneously to a reduction in the informal care potential of adult children. Initially, the share of the interaction effects on the output variance is low with a rising trend and fairly constant from the 20th period onwards.

In general, a higher value of the sensitivity index indicates a strong impact of the corresponding factor, but there are reasons for cautious interpretation, especially for comparisons over time. High values of sensitivity indices cannot be equated with large influence on the output Y. This becomes particularly clear with the example of weights, whose effect after 60 periods is much higher than at the beginning, even the main effect in period 5 is almost eight times as large as in period 60. If the total impact on variation is of interest, the sensitivity indices have to be rescaled to the variance components, as shown in Figure 6. This shows that the total impact of the weights in the first 20 years is lower than the total impact of partnerships after 50 years, although the sensitivity index is much higher. The correct interpretation is that a majority of variance in 2015 can be explained by the use of weighted and unweighted regression models, but the whole variation induced by this input factor is quite small in sum of the total variability of the output. Therefore, when interpreting sensitivity indices, it is essential to account for the total variance of the respective simulation period.

A disadvantage of stacked surfaces plots is that if one factor overly dominates sensitivity, the others are almost invisible. In addition, microsimulation models with many input factors may overfill the plot because the relative influence depends on the number of factors considered in the analysis. In the case of conditional surfaces plots, the other factors can be analysed again by conditioning or fixing dominant factors.

Figure 7 shows sensitivity indices conditional surfaces for a selection of different scenario combinations. In the first column, weights are included in the microsimulation as
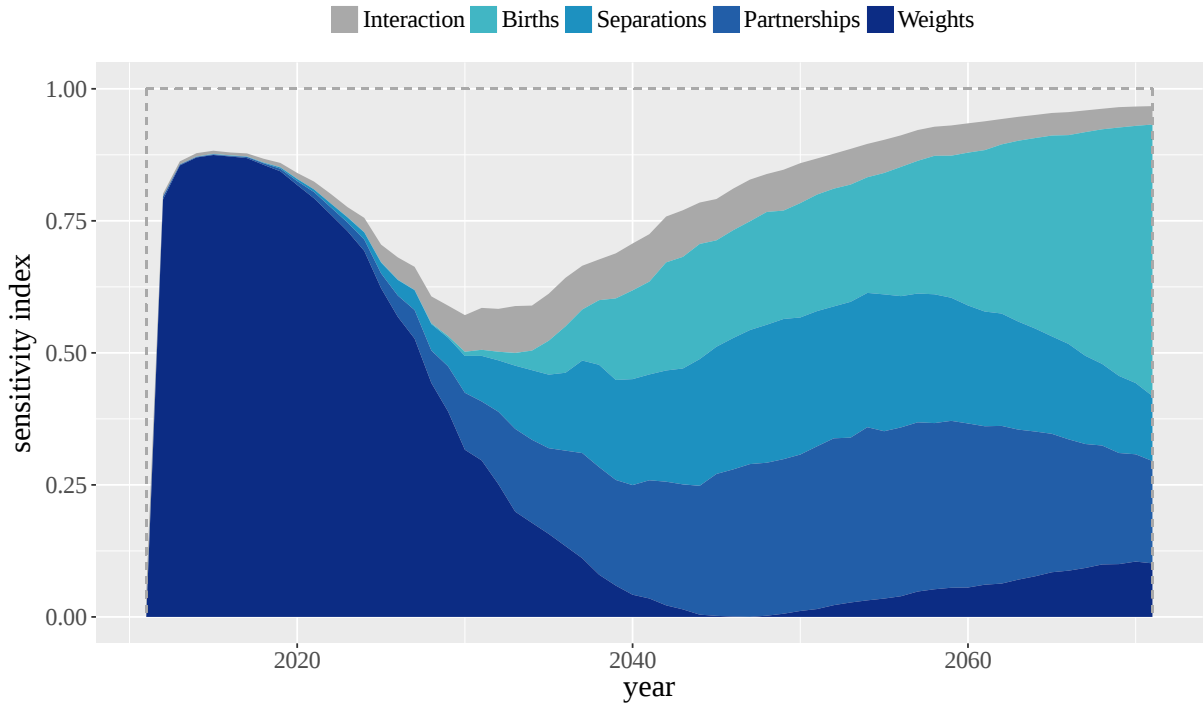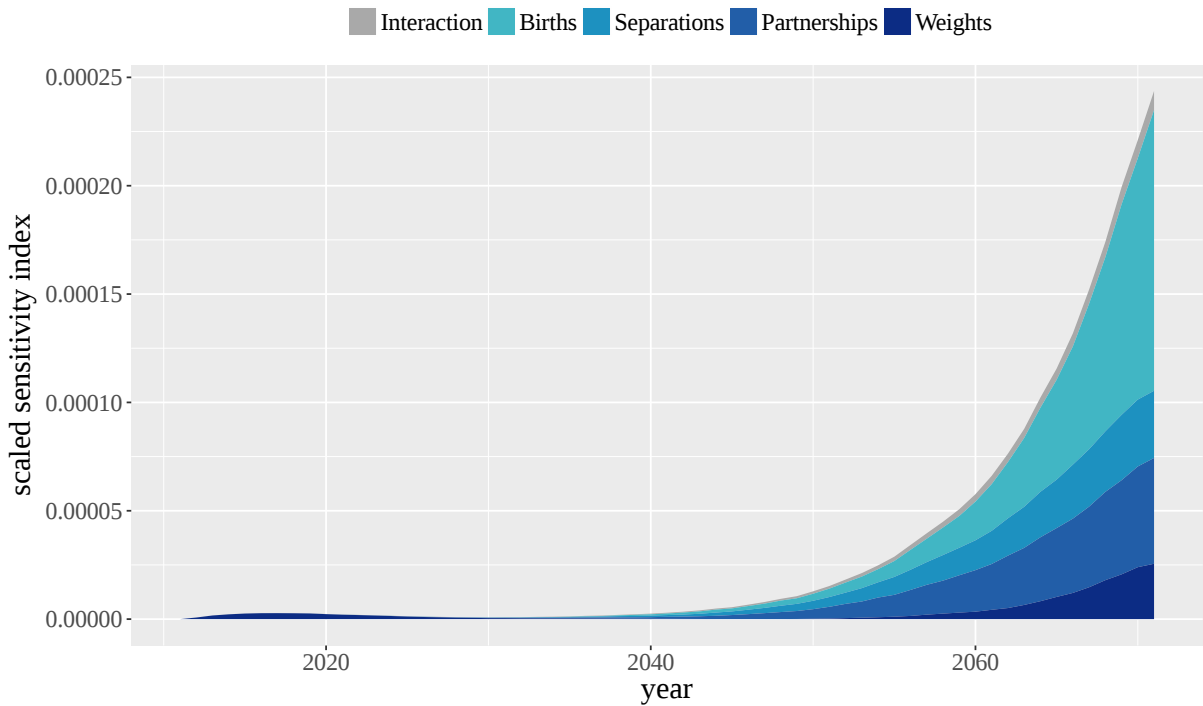
Figure 5: Sesitivity Indices

Figure 6: Sensitivity Indices normalised to variance

a factor, while in the second and third column respectively the unweighted and weighted models were used. In the first row, all other factors are considered as input factors, while in the second to fourth rows, the diagrams are conditioned on the constant models of birth, partnership, and separation. Therefore, the figure in the upper left field contains
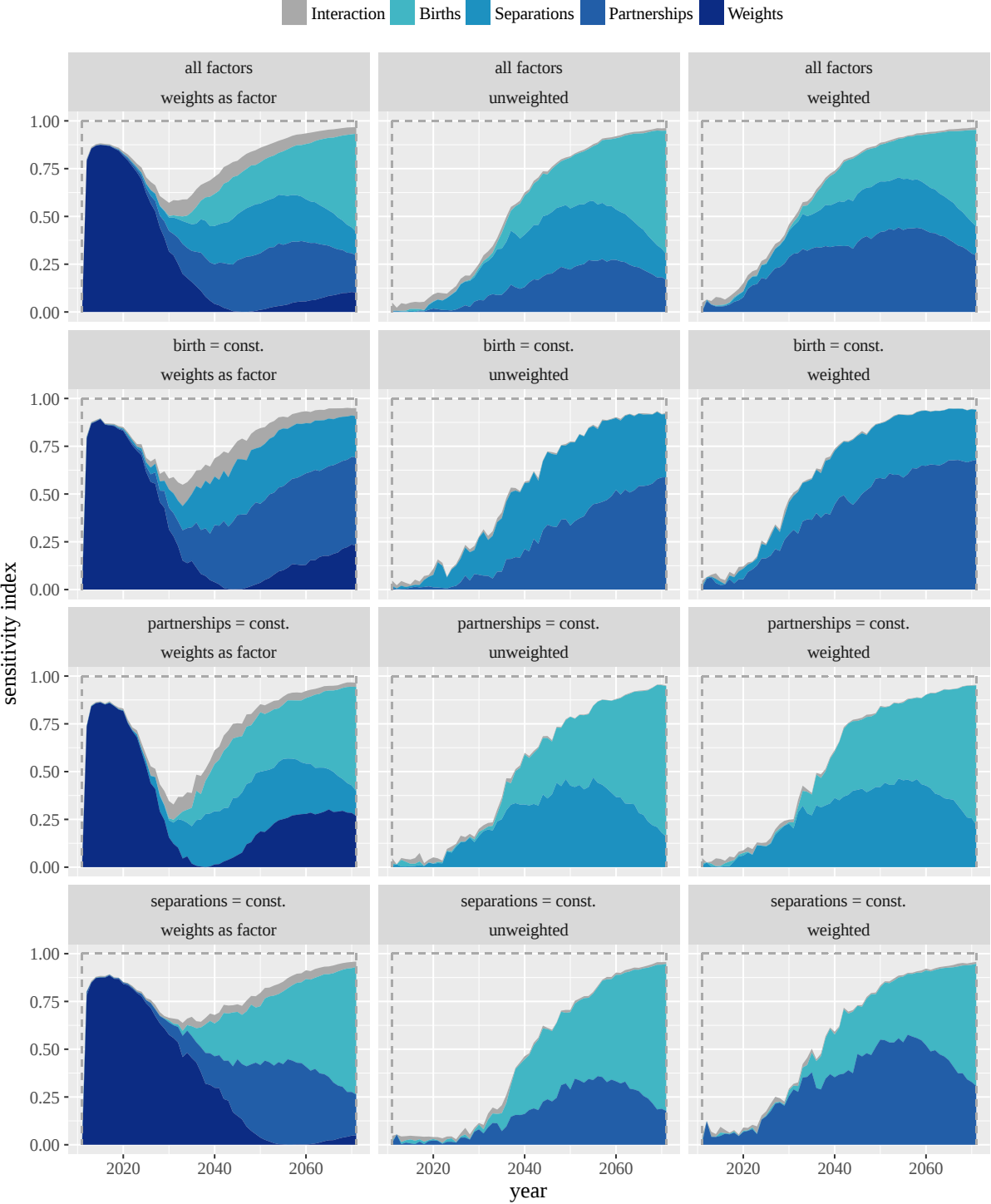
all factors, i.e. it is the same as in Figure 5.

Figure 7: Sesitivity Indices

Looking at the sensitivity regarding the input factors partnership and separations and holding the birth input factor constant, it may seem that partnerships are simply more relevant than separations if they are not dependent on the inclusion of survey weights. However looking at the second and third column, a more differentiated analysis is possible. It can be seen that using the weighted model leads to a much higher impact of partnerships

in the medium run than of separations. An observation that does not hold in the same strength in the case of unweighted models. The same applies if the input factors births and partnerships are compared with a simultaneous fixation of the factor separations in the last row of Figure 7. Again the relative impact of partnerships in weighted models is much higher.
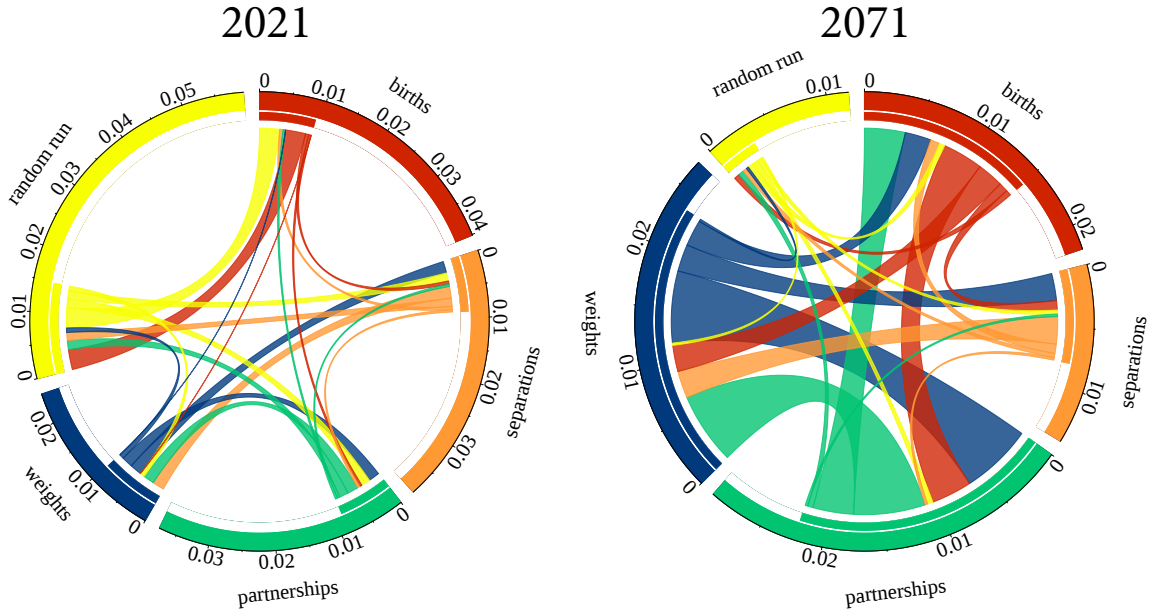

Figure 8: First- and second-order interaction effects

So far, the second-order interaction effects have only been summarised without specifying the relationship between the individual factors. Therefore, no differentiated statements can be made about the interaction of two factors. Circular plots, which are mainly used in the field of biostatistics (Zhang, Meltzer, & Davis, 2013), are suitable possibilities for visualisation and comprehensive examination of dependencies of different factors. The length of the outer area of the circular plots shown in Figure 8 corresponds to the sum of all second- and third-order effects containing the respective factor. Additionally, the sum of second-order effects is presented exclusively in the shorter inner bar of the circle. To emphasise the composition of the second-order interaction effects, connecting lines are drawn between the factors, whereby the thickness is proportional to the effect size. To indicate the share of interaction effects between birth and partnership, for example, the uniformly thick red and green lines between both factors have to be added.

Comparing the interactions after 10 and 60 simulated years, a sharp reduction of the effects including the factor of the random run is observable. The interactions of this factor are almost equivalent, so that no systematic effect can be derived. This is a desirable result, as the random run should provide independent realisations of the possible simulation paths. Similar to the main effects, statements about the absolute influence cannot be made. Therefore, a factor related and relative interpretation of the effect strengths is indispensable.

5 Conclusion

Due to the high degree of complexity and the associated uncertainty, the measurement and comparison of the impact of model components on the simulation outcomes is a major challenge in the field of microsimulation modelling, especially when applying a dynamic ageing process. There are already methods to estimate Monte-Carlo based confidence intervals (Sharif et al., 2012). However, these methods have their limitations, as usually only selected influence factors and not all sources of uncertainty can be taken into account. The selection of relevant uncertainty factors itself requires further knowledge about the impact and strength of individual factors which in turn must be examined in advance. Furthermore, no comparative statements can be made about the influence of individual factors.

We propose the use of sensitivity analysis in the form of main-, total- and interaction-effects which can be determined by decomposing the total variation of an outcome variable into various conditional variance components regarding the model inputs. With the visualisation methods outlined here, the effects of different input factors can be compared simultaneously for all simulated periods. Taking into account the time-dependent variation, a more comprehensive interpretation of the results can be achieved by rescaling the effects to variance components or conditioning the analysis to specific factors. Additionally, the impact based on the interaction of two input factors can be accurately visualised using circular plots.

Data availability

The final dataset used as base population for the simulation is onsite available on request at the research institute for official and survey statistics (RIFOSS) at Trier University.

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