Substitution Bias in the Measurement of Import and Export Price Indices: Causes and Correction

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Abstract: The import and export price indices of an economy are usually compiled by some Laspeyres type index. It is well known that such an index formula is prone to substitution bias. Therefore, also the terms of trade (ratio of export and import price index) are likely to be distorted. The underlying substitution bias accumulates over time. The present paper introduces a simple and transparent retrospective correction approach that removes the substitution bias and produces meaningful long-run time series of import and export price levels and, therefore, of the terms of trade. Furthermore, an empirical case study is conducted that demonstrates the efficacy and versatility of the correction approach.

Keywords: distortion, official statistics, terms of trade, time series.

1 Introduction

Besides the consumer price index and the producer price index, the national statistical offices (NSOs) usually publish a monthly or quarterly export price index and import price index. The latter two indices are used in the indexation of various types of international contracts and they are also required in the national accounts as deflators of nominal values of exports and imports. These are necessary to derive volume estimates of GDP by the expenditure approach. In the assessment of an economy's inflationary trends, special attention is paid to the import price index, because it is considered as an early indicator of increasing or weakening inflationary pressure. Correspondingly, the export price index is an early indicator of the inflationary pressure in the destination countries of the exports.

The terms of trade index of an economy is usually defined as the ratio of the economy's export price index and import price index. Changes in the terms of trade translate into changes of the real income of the economy's population.

Bias in the measurement of the import and export indices leads to flawed economic statistics and to costly errors in public and private economic decision making. Therefore, the "Export and Import Price Index Manual" published by the IMF (2009) provides guidelines for an unbiased measurement of the export and import price indices. In practice, however, the NSOs must make compromises to ensure a cost efficient and timely publication of the newest index numbers. Therefore, most NSOs rely on some type of Laspeyres index, even though such indices are known to generate (upper-level) substitution bias (e.g., Dridi and Zieschang., 2004, p. 169; IMF, 2009, pp. 413-439). In chained Laspeyres type indices the substitution bias accumulates over time.

Therefore, the central contribution of the present paper is a fully worked out retrospective correction approach that, with some delay, provides more reliable index numbers than those produced by a chained Laspeyres type index. Our correction approach produces meaningful long-run time series of import and export prices and, therefore, of the terms of trade. The approach is simple and transparent. It can be applied not only to an import or export price index, but also to a consumer or producer price index.

The first pillar of our correction approach is a retrospectively computed price index that compares the prices of the two latest weight reference periods, that is, the two latest periods for which detailed information about the relative importance of the various products is available. Because of its symmetric treatment of the two weight reference periods, we use the Törnqvist formula, though other index formulas that treat the two periods in a symmetric fashion would be equally appropriate (e.g., Walsh, Marshall-Edgeworth, Fisher index). Chaining consecutive Törnqvist indices instead of Laspeyres indices removes the long-run substitution bias. However, it does not correct the index numbers of the periods between the weight reference periods. To revise also these index numbers, we construct from the ratio of the Laspeyres and Törnqvist index a correction factor the impact of which gradually increases between the two weight reference periods. This correction factor is the second pillar of our approach.

The second contribution of the present paper is an empirical case study that not only estimates a lower bound of the long-run substitution bias in officially published import and export price indices, but also demonstrates how the correction approach can be implemented to avoid the long-run bias as well as the short-run bias arising between the weight reference periods. We have opted for the trade data of the Federal Statistical Office of Germany (Destatis). These data are publicly available, they comprise price data collected from producers and wholesalers (instead of the less reliable unit values compiled from customs sources), and the official compilation procedure of Destatis is documented in accessible publications (Statistisches Bundesamt, 2019, pp. 6-7).

Theoretically, if the substitution bias in the export and import price indices was of equal magnitude, the terms of trade index would remain unbiased. However, we show that there is hardly any substitution bias in the German export price index, while the upward substitution bias in the German import price index is substantial.

The explanation of this finding is the third contribution of this paper. Our analysis reveals that the difference between the bias in the export and import price index is mostly driven by the volatility of the prices of oil and gas and by the fluctuations in the euro's exchange rate against the dollar.

Our findings are relevant not only for Destatis, but for all NSOs that use infrequently chained Laspeyres type indices for their measurement of import and export prices. Without an appropriate retrospective correction of these indices, the substitution bias in the officially published index numbers accumulates over time.

The paper proceeds as follows. Section 2 presents in stylized form the interpretation and official computation of the import and export price indices and explains why they are likely to be biased. The retrospective correction approach is introduced in Section 3. Its application to the German foreign trade data is presented in Section 4. Section 5 concludes.

2 Calculation of the Terms of Trade

Suppose that during a reference period $t = 0$ a country imports only one single good i and exports only one other good j. Let p_i^0 be the euro price of the imported good and z_j^0 the euro price of the exported good. Then, the terms of trade of the reference period (ToT^0) are defined by the ratio of these two prices: $\mathrm{ToT}^0 = z_j^0/p_i^0$. Correspondingly, the terms of trade of some comparison period $t = 1$ are $\mathrm{ToT}^1 = z_j^1/p_i^1$. The change in the terms of trade between the reference and the comparison period can be expressed by the following ratio:

$$\frac{\mathrm{ToT}^1}{\mathrm{ToT}^0} = \frac{z_j^1/p_i^1}{z_j^0/p_i^0} = \frac{z_j^1/z_j^0}{p_i^1/p_i^0} \ . \tag{1}$$

Real world economies export and import millions of different goods. Hence, formula (1) is too simple. However, the basic idea of this formula, that is, to set the change in export prices in relation to the change in import prices, can be transferred to the case of many goods. In such a context, the intertemporal change of the export and import prices can be measured by price index formulas. Applying the Laspeyres formula, the change in the export prices is given by

$$E_{\text{La}}^{0 \to 1} = \frac{\sum_{j=1}^{N} z_j^1 x_j^0}{\sum_{j=1}^{N} z_j^0 x_j^0} \,, \tag{2}$$

where x_j^0 is the quantity of good j exported during the reference period and N is the number of exported goods. The subscript “La” stands for “Laspeyres” and the superscript “$0 \to 1$” indicates that the index measures the average price change between the reference period 0 and the comparison period 1. Analogously, the price index of the import prices is

$$I_{\text{La}}^{0 \to 1} = \frac{\sum_{i=1}^{M} p_i^1 m_i^0}{\sum_{i=1}^{M} p_i^0 m_i^0} \,, \tag{3}$$

where m_i^0 is the quantity of good i imported during the reference period and M is the number of imported goods.

The change in the terms of trade between the reference and the comparison period is derived from the quotient of the price indices (2) and (3):

$$\text{ToT}^{0 \to 1} = \frac{\text{ToT}^1}{\text{ToT}^0} = \frac{E_{\text{La}}^{0 \to 1}}{I_{\text{La}}^{0 \to 1}} \,. \tag{4}$$

This ratio is the terms of trade index. It can be interpreted as a generalisation of the right-hand side equality of Equation (1) to the case of many goods. Even though $\text{ToT}^{0 \to 1}$ defined by (4) measures “the *change* in the terms of trade between the reference and the comparison period”, it is conventionally denoted as “*the* terms of trade of the comparison period”. We follow this convention.

The terms of trade index (4) can also be written in the form that corresponds to the left-hand side equality in Equation (1):

$$\text{ToT}^{0 \to 1} = \frac{\left(\sum_{j=1}^{N} z_j^1 x_j^0\right) \Big/ \left(\sum_{i=1}^{M} p_i^1 m_i^0\right)}{\left(\sum_{j=1}^{N} z_j^0 x_j^0\right) \Big/ \left(\sum_{i=1}^{M} p_i^0 m_i^0\right)} \,. \tag{5}$$

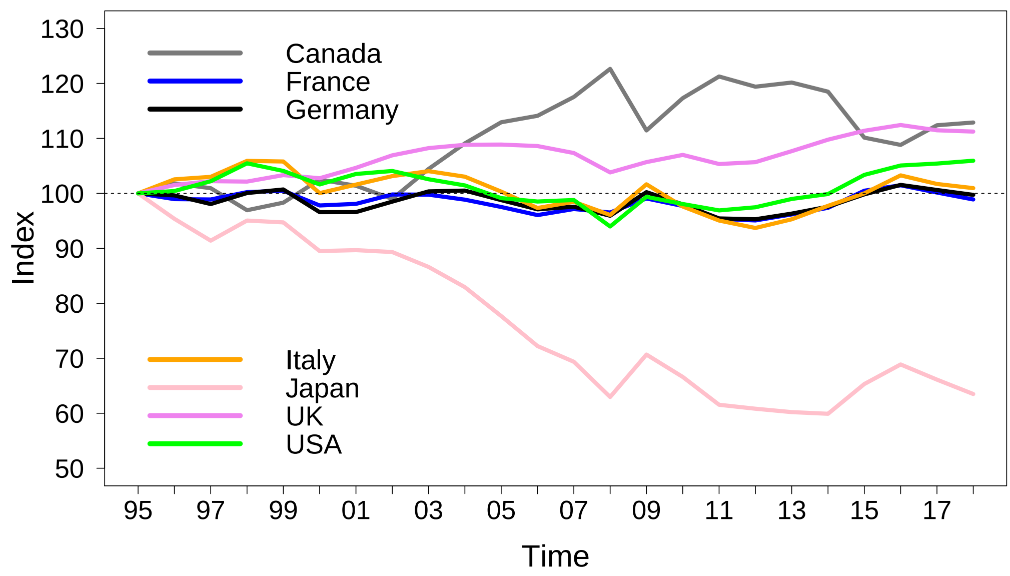
Interpreting the exported quantities of the reference period, $(x_1^0, \ldots, x_N^0)$, as the “reference period export basket” and the imported quantities of the reference period, $(m_i^0, \ldots, m_M^0)$, as the “reference period import basket”, the denominator of Equation (5) measures the purchasing power of a reference period export basket measured in units of reference period import baskets. The numerator indicates the purchasing power of the reference period export basket during the comparison period. Again, this purchasing power is measured in units of reference period import baskets. Therefore, the terms of trade index (5) indicates the change in the purchasing power of the reference period export basket, measured in units of reference period import baskets.

Figure 1 shows the officially published terms of trade of the G7-countries from 1995 to 2018.[1] The Canadian terms of trade are largely driven by the price of oil and gas. The

[1] The statistical methodologies of the NSOs of the G7-countries (Canada, France, Germany, Italy,

terms of trade of all other G7-countries are negatively correlated with the Canadian ones. Between 2000 and 2007, there was a strong devaluation of the Japanese yen against the euro, though not against the dollar. Despite the yen's subsequent appreciation until 2012, the Japanese terms of trade remained at their lower level.

Figure 1: Terms of Trade of the G7 countries (1995 = 100) from 1995 to 2018. Source: Own calculations based on data of OECD.



3 Retrospective Correction of Substitution Bias

Bias in the terms of trade arises, when the export price index, E_{La}, or the import price index, I_{La}, or both price indices are distorted – provided that the two distortions do not cancel. Of course, such distortions can occur already during data collection and processing. These problems are well known and extensively discussed in IMF (2009, pp. 287-297). Therefore, it is assumed here that the available price and quantity data are accurate and that the only remaining source of bias is the choice of the index formula.

In IMF (2009, pp. 413-439) it is argued that imports and exports can be viewed from a resident's perspective or a non-resident's perspective. For each perspective, economic theory makes predictions about the direction of the measurement bias arising from the Laspeyres index formula.

Japan, UK and US) are sketched out in Statistics Canada (2019), INSEE (2019), Peter (2009, 2014, 2019), Statistisches Bundesamt (2004), Istat (2019), Bank of Japan (2019), ONS (2017), and Bureau of Labor Statistics (2018a,b, 2020).

When the resident's perspective is applied, the import quantities and prices collected by NSOs reflect the residents' cost minimizing consumer behavior (including firms purchasing their inputs). This side increases the purchases of products that become relatively less expensive and they reduce the purchases of products that become relatively more expensive. This consumer behavior would result in a negative correlation between intertemporal price and quantity changes and, therefore, in upward substitution bias of the Laspeyres index.

Furthermore, the observed export quantities and prices reflect the revenue maximization of the residents' producer side. This side increases the output of products that become relatively more expensive and they reduce the output of products that become relatively less expensive. Therefore, the intertemporal price and quantity changes are positively correlated. In a Laspeyres index, this would lead to downward substitution bias.

In sum, in the resident's perspective the numerator in the terms of trade index (4) would understate the average change in export prices, while the denominator would overstate the average change in import prices. Therefore, the measured terms of trade would exhibit downward bias.

When a non-resident's perspective were applied, the direction of the bias would be reversed. The price change in export prices would be overrated and the price change in import prices would be underrated, leading to an upward bias in the terms of trade.

These considerations are merely predictions that are based on economic theory. An empirical examination of these predictions requires a measurement approach that can be expected to produce unbiased index numbers. A deviation between these index numbers and the Laspeyres numbers is an indication of the direction and extent of the actual substitution bias of the Laspeyres index.

The Laspeyres index (3) is often expressed in the following equivalent form:

$$I_{\mathrm{La}}^{0\to 1} = \sum_{i=1}^{M} s_i^0 \, \frac{p_i^1}{p_i^0} \, , \tag{6}$$

with

$$s_i^0 = \frac{p_i^0 m_i^0}{\sum_{j=1}^{M} p_j^0 m_j^0} \, .$$

Therefore, the Laspeyres index can be interpreted as a weighted arithmetic mean of price ratios where the weights are the expenditure shares of the reference period.

The Laspeyres index is not the only index formula that is prone to substitution bias. The Paasche index,

$$I_{\mathrm{Pa}}^{0\to 1} = \frac{\sum_{i=1}^{M} p_i^1 m_i^1}{\sum_{i=1}^{M} p_i^0 m_i^1} = \left(\sum_{i=1}^{M} s_i^1 \left(\frac{p_i^1}{p_i^0} \right)^{-1} \right)^{-1} \, , \tag{7}$$

has the same design flaw, though its substitution bias points in the opposite direction. The right-hand side of (7) expresses the Paasche index as the weighted harmonic mean of the price ratios where the weights are the expenditure shares of the comparison period.

A large number of price indices avoid the substitution bias of the Laspeyres and Paasche index. Examples are the Walsh, Marshall-Edgeworth, Fisher and Törnqvist index. These index formulas utilize not only the set of reference period quantities or the set of comparison period quantities, but both sets of quantities. Usually, these four formulas generate very similar index numbers. Therefore, we confine our analysis to the Törnqvist index. It is defined by

$$I_{\text{Tö}}^{0\to1} = \exp\left(\sum_{i=1}^{M} \frac{1}{2}\left(s_i^0 + s_i^1\right) \ln\left(\frac{p_i^1}{p_i^0}\right)\right) . \tag{8}$$

The compilation of this price index requires not only the expenditures $p_i^0 m_i^0$ and the price ratios p_i^1/p_i^0, but also the expenditures $p_i^1 m_i^1$. However, the collection and compilation of the latter expenditures is a labor intensive process that would significantly delay the publication of the indices (IMF, 2009, p. 58). Therefore, most NSOs are reluctant to implement such an index formula.

Nevertheless, the IMF (2009, p. 56) points out that a retrospective revision of the index numbers would be feasible when the requisite data on updated weights become available. The present paper argues that such a revision is indispensable for unbiased long-run indices. Therefore, it develops a suitable method for this task.

For example, suppose that in January 2010 (shorthand notation 1/10) a survey was conducted providing us with the import expenditure weights for that month, $s_i^{1/10}$. Therefore, this month is our first *weight reference* period. For February 2010 (2/10) and all subsequent months we calculate a monthly Laspeyres import price index that measures the average price change between the *price reference* (and first weight reference) period January 2010 and some comparison period t:

$$I_{\text{La}}^{1/10\to t} = \sum_{i=1}^{M} s_i^{1/10} \frac{p_i^t}{p_i^{1/10}} , \qquad t = 1/10\,,\, 2/10\,,\ldots\ . \tag{9}$$

For the comparison period $t = 1/10$, this index yields $I_{\text{La}}^{1/10\to1/10} = 1$, that is, the reference price level of the index series is Jan. 2010 = 1. Therefore, January 2010 is not only the price and weight reference period, but also the *index reference* period.[2]

Suppose that January 2015 is the next weight reference period, but that the expenditure weights relating to that month become available only in July 2018. Therefore, January 2010 remains the price (and index) reference period also for the Laspeyres indices compiled between January 2015 and July 2018. In July 2018, when the expenditure weights of January 2015 become available, we can conduct a retrospective revision of past index numbers. We propose to conduct this revision in three stages. The first and second stage revise the index numbers of January 2015 to July 2018, while the third stage revises the index numbers of January 2010 to December 2014. The three stages yield a consistent

[2] Our distinction between a price, weight and index reference period follows the terminology advocated in ILO et al. (2004, p. 165).

time series of price levels that stretches from January 2010 to July 2018. It can be easily continued without harming its consistency.

Stage 1: We begin the revision by computing a new series of Laspeyres index numbers for January 2015 to July 2018. This new series uses as price, weight and index reference period January 2015 instead of January 2010:

$$I_{\mathrm{La}}^{1/15 \rightarrow t} = \sum_{i=1}^{M} s_i^{1/15} \, \frac{p_i^t}{p_i^{1/15}} \,, \qquad t = 1/15\,,\, 2/15\,, \ldots\,,\, 7/18\,. \tag{10}$$

This new series of Laspeyres index numbers can be expected to exhibit considerably less substitution bias than the original series, because the quantity information is more up to date (January 2015 instead of January 2010). However, some substitution bias remains, because we still apply the Laspeyres index formula. Only when the results of the next weight reference period will become available, this bias can be addressed.

Stage 2: To rebase the new series to the index reference period January 2010, advocates of the Laspeyres index would multiply the index numbers compiled by (10) by the Laspeyres index $I_{\mathrm{La}}^{1/10 \rightarrow 1/15}$. However, we know that this Laspeyres index exhibits substantial substitution bias. Therefore, we recommend to apply the Törnqvist index (8) instead:

$$I_{\mathrm{Tö}}^{1/10 \rightarrow 1/15} = \exp\left(\sum_{i=1}^{M} \frac{1}{2} \left(s_i^{1/10} + s_i^{1/15} \right) \ln \left(\frac{p_i^{1/15}}{p_i^{1/10}} \right) \right) \,. \tag{11}$$

In view of the upward substitution bias of the original Laspeyres index, we expect that $I_{\mathrm{Tö}}^{1/10 \rightarrow 1/15} < I_{\mathrm{La}}^{1/10 \rightarrow 1/15}$. The rebased series is obtained from

$$I^{1/10 \rightarrow t} = I_{\mathrm{Tö}}^{1/10 \rightarrow 1/15} \cdot I_{\mathrm{La}}^{1/15 \rightarrow t} \,, \qquad t = 1/15\,,\, 2/15\,, \ldots\,,\, 7/18\,. \tag{12}$$

Therefore, $I^{1/10 \rightarrow 1/15} = I_{\mathrm{Tö}}^{1/10 \rightarrow 1/15}$. The rebased series of price index numbers relates each of the price levels between January 2015 and July 2018 to Jan. 2010 = 1. Overall, we expect a significant downward revision of the original price levels compiled by (9).

The first two stages of revision affect only the time series of import price levels from January 2015 to July 2018. The price levels of the previous months have not yet been revised. Quite likely, the original price level of December 2014, $I_{\mathrm{La}}^{1/10 \rightarrow 12/14}$, is larger than the downwards revised price level of January 2015, $I_{\mathrm{Tö}}^{1/10 \rightarrow 1/15}$. To obtain a consistent time series stretching from January 2010 to July 2018, the price level of the index reference period January 2010 should remain at 1, but the price levels of February 2010 to December 2014 must be revised. This is the third and most challenging stage of the retrospective revision.

Stage 3: In the course of Stage 2, we replaced the Laspeyres index $I_{\mathrm{La}}^{1/10 \rightarrow 1/15}$ by the Törnqvist index $I_{\mathrm{Tö}}^{1/10 \rightarrow 1/15}$. Obviously, we cannot make the same replacement for the previous months ($t = 2/10$ to $t = 12/14$), because the quantities of these months are unknown. Quantity information is available only for the two weight reference periods January 2010 and January 2015. However, we can utilize in our retrospective revision of these earlier index numbers the ratio $\left(I_{\text{Tö}}^{1/10\rightarrow 1/15}\big/ I_{\text{La}}^{1/10\rightarrow 1/15}\right)$ as a correction factor:

$$I^{1/10\rightarrow t} = I_{\text{La}}^{1/10\rightarrow t} \cdot \left(\frac{I_{\text{Tö}}^{1/10\rightarrow 1/15}}{I_{\text{La}}^{1/10\rightarrow 1/15}}\right)^{\lambda_t} , \qquad t = 1/10\,,\, 2/10\,, \ldots\,,\, 1/15\,. \tag{13}$$

The parameter λ_t represents the impact that we concede to the correction factor. For the comparison period $t = 1/15$, the correction factor should exert its full impact $\left(\lambda_{1/15} = 1\right)$ such that formula (13) yields $I^{1/10\rightarrow 1/15} = I_{\text{Tö}}^{1/10\rightarrow 1/15}$. However, for the initial months $(t = 2/10,\ 3/10, \ldots)$ the situation is different. During these comparison periods, the import basket of month $t = 1/10$ is far less outdated than in month $t = 1/15$ and, therefore, the substitution bias of $I_{\text{La}}^{1/10\rightarrow 2/10}$, say, tends to be much smaller than that of $I_{\text{La}}^{1/10\rightarrow 1/15}$. Accordingly, during the initial months, the impact λ_t should be smaller than 1. In fact, for the comparison period $t = 1/10$, the correction factor should have no impact $\left(\lambda_{1/10} = 0\right)$. Otherwise, we would get $I^{1/10\rightarrow 1/10} \neq 1$. As the comparison period t moves away from the price reference period 1/10, the impact λ_t should gradually increase above 0 until, in $t = 1/15$, it finally reaches its maximum value 1.

For a formal definition of the impact λ_t we introduce the counter variable s. In the first month, $t = 1/10$, this integer has the value $s = 1$, in month $t = 2/10$ the value $s = 2$, and so on. In month $t = 1/15$ the counter reaches its maximum value $s = 61$. Denoting this maximum value by S, we can define the impact λ_t by

$$\lambda_t = \frac{s-1}{S-1}\,. \tag{14}$$

Formulas (13) and (14) yield the desired series of revised price index numbers for the months January 2010 to January 2015. Combining this series with the revised index numbers that were compiled during Stages 1 and 2 of the revision process, yields a consistent series of revised index numbers stretching from January 2010 to July 2018. It is consistent in the sense, that, for month $t = 1/15$, formulas (12) and (13) yield the same index number, namely $I_{\text{Tö}}^{1/10\rightarrow 1/15}$.

Formulas (13) and (14) are not the only conceivable method for Stage 3 of the revision process. Some alternative options are explored in Auer and Shumskikh (2020).[3]

4 Application to German Foreign Trade Data

Different NSOs apply different compilation methods for their export and import price indices. Our correction approach is adaptable to a wide range of such methods. To

[3] Instead of a correction factor, these alternative options would replace the Laspeyres index $I_{\text{La}}^{1/10\rightarrow 1/15}$ by a modified version of the Fisher, Marshall-Edgeworth, Törnqvist, or Walsh index, or some other index formula that incorporates the quantities of both, the price reference and the comparison period (e.g., Theil index). The modified Fisher index represents a refined version of a proposal by Diewert et al. (2009, pp. 128-139).

verify this claim and to get an impression of the magnitude of the bias inherent in official compilation procedures, we adapt our approach to the method and the trade data of Destatis, the Federal Statistical Office of Germany.

An important difference between the calculation outlined in Section 2 and the method of Destatis is the choice of the period lengths. In Section 2, all periods had a uniform length, namely one month. By contrast, the price, weight and index reference periods of Destatis are years, while the comparison periods are months. The resulting complications arising in the Destatis method are described in the Appendix. There we also demonstrate how our correction approach can be adapted to these complications.

For January 1995 to May 2019, we have monthly price levels of 30 categories of German imports and 28 categories of German exports. In addition, we know the categories' expenditure weights for the years 1995, 2000, 2005, 2010 and 2015.

As documented in Pötzsch (2004) and Peter (2009, 2014, 2019), the officially published long-run import price index of Destatis (Statistisches Bundesamt, 2019, pp. 8-9) incorporates Stage 1 of our revision process but not Stages 2 and 3. For example, in September 2018 Destatis published the index numbers for the comparison months $t = 1/10$ to $t = 7/18$. They were compiled by the Laspeyres index with weight and index reference year 2010, $I_{\mathrm{La}}^{2010 \to t}$. However, the index numbers for the comparison months $t = 1/15$ to $t = 7/18$ were only preliminary. As soon as the survey results of the year 2015 became available, Destatis replaced these index numbers by revised index numbers that were compiled by the Laspeyres index with price, weight and index reference year 2015, $I_{\mathrm{La}}^{2015 \to t}$ (Peter, 2019, p. 37). This revision is equivalent to Stage 1 of our three-stage revision process outlined in Section 3.

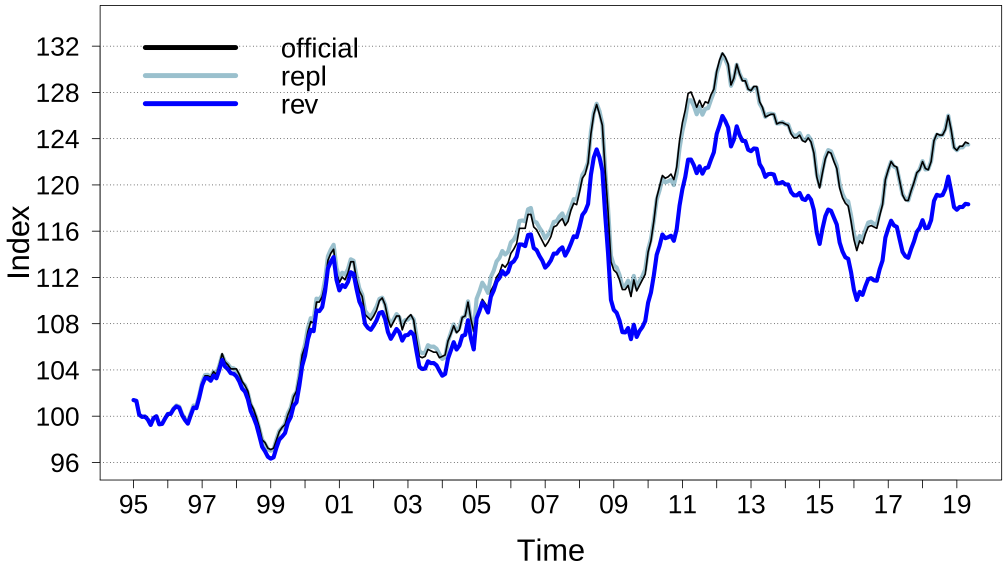
Our first task is to replicate the compilation process of Destatis and to compile a consistent time series of price levels relating to the index reference year 1995. The replicated index is denoted by $I_{\mathrm{repl}}^{95 \to t}$. Following the exposition in the Appendix, we use the following formulas:

$$
\begin{aligned}
t &= 1/95, \ldots, 1/00: & I_{\mathrm{repl}}^{95 \to \mathrm{t}} &= I_{\mathrm{La}}^{95 \to t} \\
t &= 1/00, \ldots, 1/05: & I_{\mathrm{repl}}^{95 \to \mathrm{t}} &= I_{\mathrm{repl}}^{95 \to 1/00} \cdot I_{\mathrm{Pa}}^{1/00 \to 00} \cdot I_{\mathrm{La}}^{00 \to t} \\
t &= 1/05, \ldots, 1/10: & I_{\mathrm{repl}}^{95 \to \mathrm{t}} &= I_{\mathrm{repl}}^{95 \to 1/05} \cdot I_{\mathrm{Pa}}^{1/05 \to 05} \cdot I_{\mathrm{La}}^{05 \to t} \\
t &= 1/10, \ldots, 1/15: & I_{\mathrm{repl}}^{95 \to \mathrm{t}} &= I_{\mathrm{repl}}^{95 \to 1/10} \cdot I_{\mathrm{Pa}}^{1/10 \to 10} \cdot I_{\mathrm{La}}^{10 \to t} \\
t &= 1/15, \ldots, 5/19: & I_{\mathrm{repl}}^{95 \to \mathrm{t}} &= I_{\mathrm{repl}}^{95 \to 1/15} \cdot I_{\mathrm{Pa}}^{1/15 \to 15} \cdot I_{\mathrm{La}}^{15 \to t} .
\end{aligned}
$$

Our results are depicted in Figure 2 and compared to the (rebased) official import price index published by Destatis. Even though our data only relate to rather broad categories and do not include all subcategories of the official import price index of Destatis, our replicated import price index (labelled as "repl"), is very close to the official one (labelled as "official").

Our second task is to apply our correction approach and to compute the retrospectively revised import price index, $I_{\mathrm{rev}}^{95 \to t}$. Since the replicated index includes Stage 1 of the correction approach, any deviations between the revised index $I_{\mathrm{rev}}^{95 \to t}$ and the replicated

Figure 2: German import price index (1995 = 100) for January 1995 to May 2019. Source: Own calculations based on data of Destatis.



index $I_{\text{repl}}^{95 \to t}$ can be attributed to Stages 2 and 3.

Our compilations follow the process outlined in the Appendix:

$$t = 1/95, \ldots, 1/00: \qquad I_{\text{rev}}^{95 \to t} = I_{\text{La}}^{95 \to t} \cdot \left(\frac{I_{\text{Tö}}^{95 \to 00}}{I_{\text{La}}^{95 \to 1/00} \cdot I_{\text{Pa}}^{1/00 \to 00}} \right)^{\lambda_t}$$

$$t = 1/00, \ldots, 1/05: \qquad I_{\text{rev}}^{95 \to t} = I_{\text{Tö}}^{95 \to 00} \cdot I_{\text{La}}^{00 \to t} \cdot \left(\frac{I_{\text{Tö}}^{00 \to 05}}{I_{\text{La}}^{00 \to 1/05} \cdot I_{\text{Pa}}^{1/05 \to 05}} \right)^{\lambda_t}$$

$$t = 1/05, \ldots, 1/10: \qquad I_{\text{rev}}^{95 \to t} = I_{\text{Tö}}^{95 \to 00} \cdot I_{\text{Tö}}^{00 \to 05} \cdot I_{\text{La}}^{05 \to t} \cdot \left(\frac{I_{\text{Tö}}^{05 \to 10}}{I_{\text{La}}^{05 \to 1/10} \cdot I_{\text{Pa}}^{1/10 \to 10}} \right)^{\lambda_t}$$

$$t = 1/10, \ldots, 1/15: \qquad I_{\text{rev}}^{95 \to t} = I_{\text{Tö}}^{95 \to 00} \cdot I_{\text{Tö}}^{00 \to 05} \cdot I_{\text{Tö}}^{05 \to 10} \cdot I_{\text{La}}^{10 \to t} \cdot \left(\frac{I_{\text{Tö}}^{10 \to 15}}{I_{\text{La}}^{10 \to 1/15} \cdot I_{\text{Pa}}^{1/15 \to 15}} \right)^{\lambda_t}$$

$$t = 1/15, \ldots, 5/19: \qquad I_{\text{rev}}^{95 \to t} = I_{\text{Tö}}^{95 \to 00} \cdot I_{\text{Tö}}^{00 \to 05} \cdot I_{\text{Tö}}^{05 \to 10} \cdot I_{\text{Tö}}^{10 \to 15} \cdot I_{\text{La}}^{15 \to t},$$

with

$$\lambda_t = \begin{cases} 0 & \text{for} \quad s = 1, 2, \ldots, 12 \\ \dfrac{s - 12}{61 - 12} & \text{for} \quad s = 13, 14, \ldots, 60\,. \end{cases}$$

In month $t = 1/95$, the value of the counter s is equal to 1. In the subsequent months it increases until in month $t = 12/99$ it reaches the value 60. In the following month,

$t = 1/00$, the value of s is reset to 1. The same reset happens in months 1/05 and 1/10. Price changes within a weight reference year ($s = 1, 2, \ldots, 12$) are not revised ($\lambda_t = 0$).

The formulas generate the retrospectively revised import price index $I_{\text{rev}}^{95\to t}$ depicted in Figure 2 (labelled as "rev"). The index deviates from the replicated index $I_{\text{repl}}^{95\to t}$ ("repl") and, therefore, from the official index of Destatis ("official"). The deviation increases over time and, in 2019, reaches more than five percentage points. This value can be considered as a lower bound of the accumulated (upper-level) long-run substitution bias in the official price index.

This reinforces the case for a revision that does not stop at Stage 1, but includes also Stages 2 and 3. Only this comprehensive revision can avoid the accumulating long-run substitution bias inherent in chained Laspeyres indices. The revision requires no additional data. It exclusively draws on information that is used in the original price index compilation.

To this point, we were exclusively concerned with the import price index. If the export price index exhibited the same bias, the terms of trade index (ratio of the export price index and import price index) would remain unbiased. Unfortunately, such a compensatory effect is unlikely. The reason is illustrated in the upper part of Figure 3. It shows the export and import price indices of Germany as compiled by Destatis. Both indices are normalized to Jan. 1980 = 100.

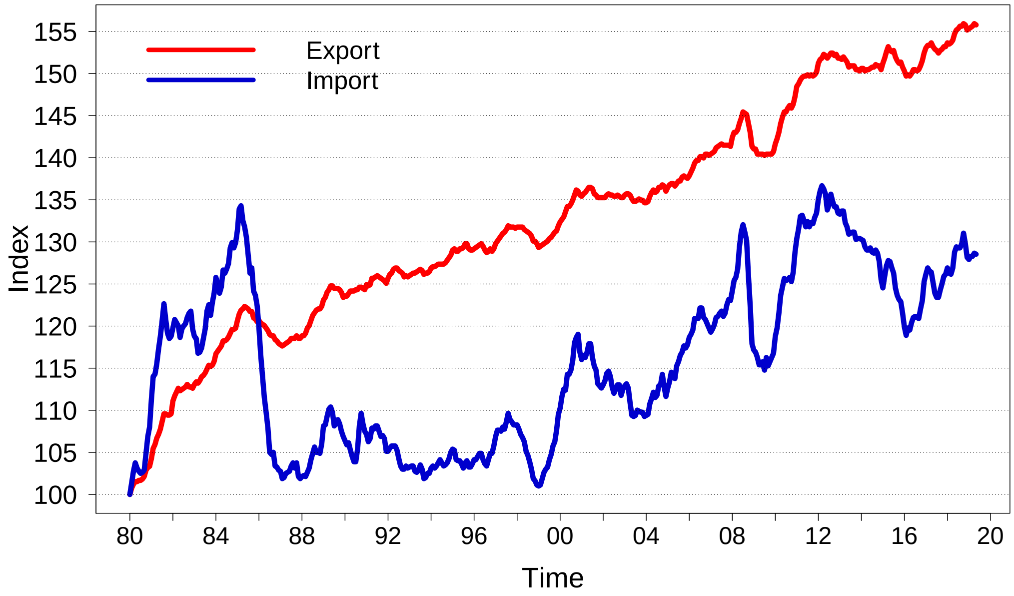
The export price index rises very evenly, while the import price index rises much more erratically.[4] In the following, we explain why a more erratic index is more vulnerable to substitution bias. The line of reasoning starts with an empirical observation. Our data reveal that for a given pair of consecutive months the change in the import index (measured in percent where the sign is eliminated) is positively correlated with the coefficient of variation of the intertemporal price relatives of the 30 categories of German imports. For the time span for which we have price data on the 30 import categories (January 1995 to May 2019), the (Pearson) correlation coefficient is almost 0.54.

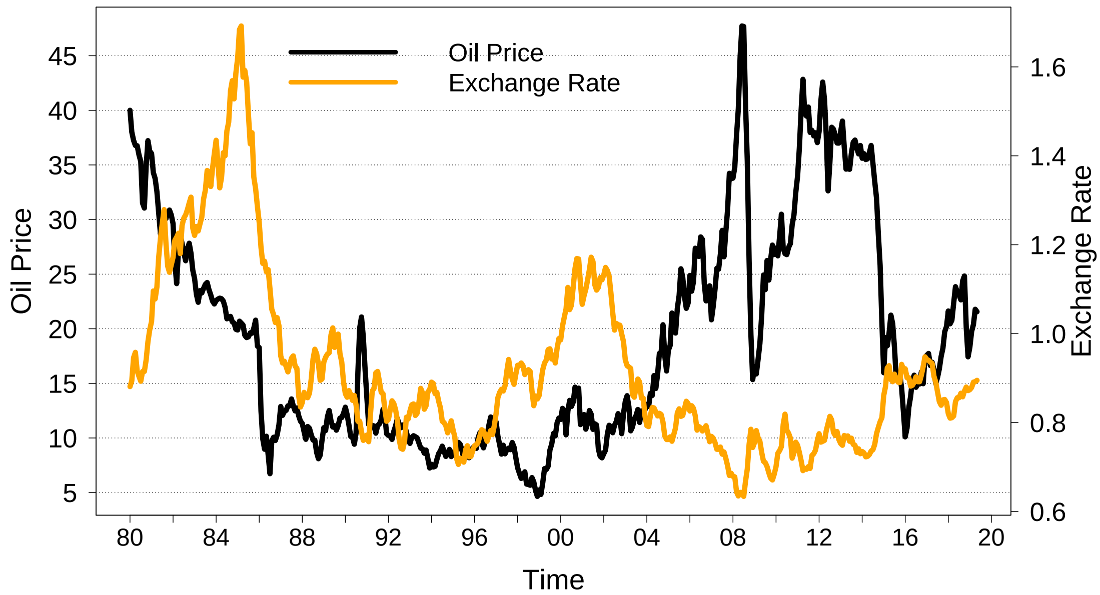
From the work of Bortkiewicz (1923, pp. 374-376) we know that the relative divergence between the Laspeyres and Paasche index depends on three factors: the coefficient of variation of the intertemporal quantity relatives, the coefficient of variation of the intertemporal price relatives, and the coefficient of linear correlation between the price and quantity relatives. Thus, for a given correlation, the divergence between the Laspeyres and Paasche index tends to increase with the volatility of the prices and quantities. An increasing Paasche-Laspeyres spread translates into an increasing Törnqvist-Laspeyres spread, because the Törnqvist index closely approximates the geometric average of the Laspeyres and Paasche index (known as the Fisher index).

Since the Törnqvist-Laspeyres spread is interpreted as an indication of substitution bias, the previous considerations can be condensed to a simple conjecture: The larger the volatility of an index, the larger the substitution bias. Therefore, the rather steadily evolving export price index is less likely to suffer from substitution bias than the much more volatile import price index.

[4] Also in France, Italy and Japan the import price index is much more volatile than the export price index.

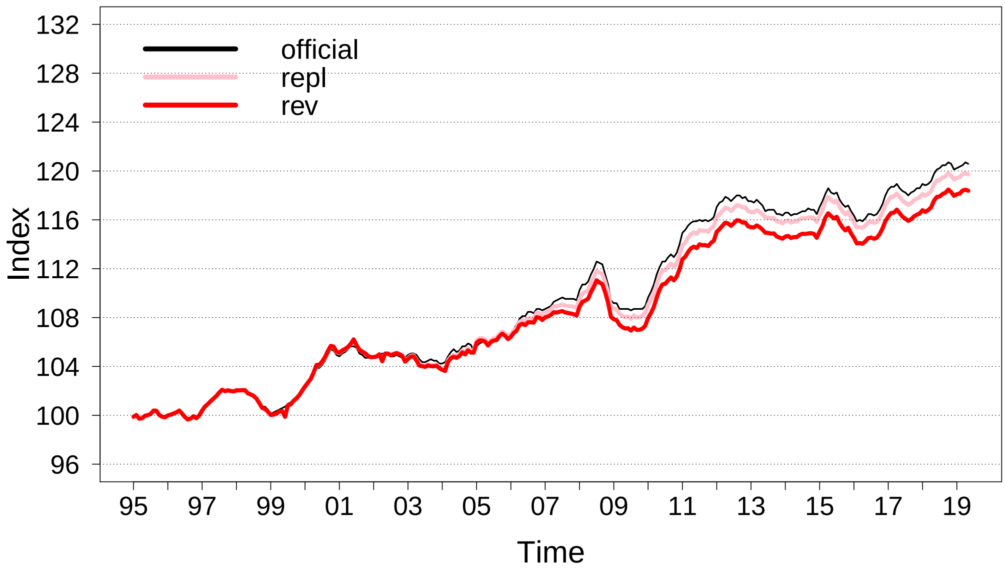
Figure 3: German export and import price indices (Jan. 1980 = 100), real oil prices (in dollars of 1980) and exchange rate (€/$) for January 1980 to May 2019. Source: Destatis, Deutsche Bundesbank, World Bank and Bureau of Labor Statistics.





To verify this conjecture, we first replicate the official export price index of Destatis, $E^{95 \to t}_{\text{repl}}$, then compile the retrospectively revised export price index, $E^{95 \to t}_{\text{rev}}$, and finally compare the difference of the two indices to the difference that we computed in the context of the import price index. We follow the same procedures that we used for the import price index. The results are depicted in Figure 4.

Figure 4: German export price index (1995 = 100) from January 1995 to May 2019. Source: Own calculations based on data of Destatis.

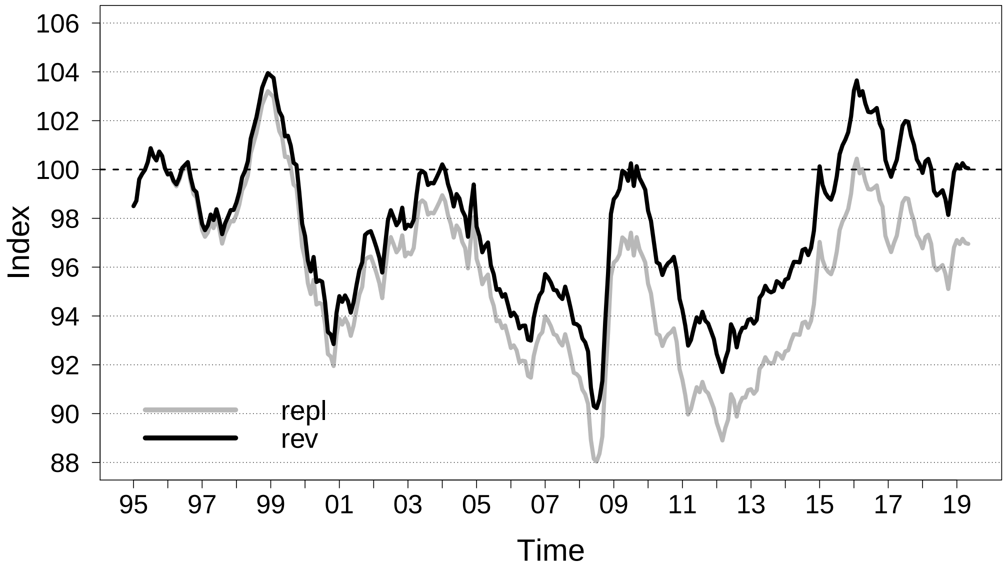


We find our claim confirmed. The revision is smaller than in the import price index, indicating that also the accumulated long-run substitution bias in the official export price index is smaller. The direction of the bias is upwards, hinting at a negative correlation between price and quantity changes. This negative correlation is more in line with the behavior of purchasers than with the behavior of producers or sellers. In other words, our empirical findings related to the German export prices corresponds to the non-resident's perspective of economic theory (IMF, 2009, pp. 414).

The preceding results have important implications for the German terms of trade index, that is, for the ratio of the export and import price index. Since the export price index exhibits only a minor upward bias, the substantial upward bias in the import price index translates into a substantial downward bias in the German terms of trade index. This is depicted in Figure 5. The terms of trade index compiled from the revised indices, $E_{\text{rev}}^{95 \to t}$ and $I_{\text{rev}}^{95 \to t}$, deviates from the terms of trade index compiled from the replicated indices, $E_{\text{repl}}^{95 \to t}$ and $I_{\text{repl}}^{95 \to t}$. The deviation suggests that in 2019 the accumulated bias in the official terms of trade reaches roughly four percentage points.

The bias in the terms of trade can be attributed to the difference in the volatility of the import and export price index. What causes the difference in volatility? Until the introduction of the euro in January 1999, German imports had to be converted into Deutsche Mark before they entered the import price index. As a consequence, the German import

Figure 5: German terms of trade (1995 = 100) from January 1995 to May 2019. Source: Own calculations based on data of Destatis.



price index depended not only on the price changes in the countries of origin, but also on the exchange rate of the Deutsche Mark against the foreign currencies, most importantly against the dollar. Therefore, until December 1998 we expect a strong positive correlation between the import price index and the nominal exchange rate in price notation, that is the price of one dollar.

Figure 3 confirms this expectation. The orange line in the lower part of the graph (reference is the right axis) shows the euro-dollar exchange rate in price notation, that is, the price of one dollar expressed in euros (or units of 1.95583 Deutsche Mark before 1999). Between January 1980 and December 1998, the correlation coefficient between the import price index and the exchange rate is above 0.94.

Since January 1999 parts of the imports and exports are invoiced in euro.[5] This is likely to dampen the impact of the foreign exchange rate on the import price index. Figure 3 also confirms this second conjecture. For the time interval January 1999 to May 2019, the correlation between the import price index and the exchange rate is negative. This indicates that also other factors must be responsible for the larger volatility of the import prices as compared to the export prices.

An obvious suspect are the prices of oil and gas. These two products represent almost ten percent of the German imports as compared to an export share of less than one percent (Peter, 2019, pp. 39-40). The black line in the lower part of Figure 3 depicts the evolution

[5] Eurostat (2017) shows that in 2016 almost 50 percent of German imports from non-EU countries are invoiced in euro, while almost 60 percent of the exports in non-EU countries are invoiced in euro.

of the real oil prices (Brent oil, in dollars representing their purchasing power in 1980; reference is the left axis). The strong volatility of the real oil prices and their positive correlation with the import price index are clearly visible. The correlation coefficient for the time interval January 1999 to May 2019 is around 0.81 (before 1999 it is around 0.52).

5 Concluding Remarks

For a given pair of months, the extent of the substitution bias of Laspeyres type indices is positively correlated with the variation of the intertemporal price relatives, the variation of the intertemporal quantity relatives, and the linear correlation between the two. Chaining of such distorted indices leads to accumulated bias. In most countries, the official import and export price indices are compiled as chained Laspeyres type indices. Therefore, these indices are likely to suffer from substitution bias.

In the present study, we examined this conjecture. In a case study of the German trade statistics we compiled a lower bound for (upper-level) substitution bias in the German import price index and export price index. For the time interval January 1995 to May 2019 the accumulated upward substitution bias of the import price index is more than five percent, while the upward bias of the export price index is slightly above one percent.

Furthermore, we demonstrated that the accumulating substitution bias can be easily avoided by a three-stage retrospective revision process. This process requires no additional data and is adaptable to the specific index compilation procedures of the various national statistical offices.

The longer the intervals between the surveys providing the quantity data of the import and export price indices, the larger the expected substitution bias. Several countries rely on five-year-intervals (e.g., Canada, Germany, Italy, Japan, UK). Such countries are most likely to benefit from the implementation of a retrospective revision process. However, also in countries with shorter intervals (e.g., France, US) the proposed revision process may help to improve the reliability of the published long-run indices.

In countries like France, Germany, Italy and Japan the import price index is considerably more volatile than the export price index. For Germany, we showed that a larger volatility of the index translates into a larger variation of the price and quantity relatives and this results in a larger substitution bias. In other words, the substitution bias of the import price index is likely to exceed the bias of the export price index. As a consequence, also the terms of trade index should be biased. Our empirical analysis confirmed this conjecture for the German case.

What causes the difference in the volatility of the import and export prices? One likely reason are exchange rate fluctuations. They tend to affect the import price index more than the export price index. A second reason are the strong fluctuations in the prices of oil and gas. These two products are all but absent from the exports of France, Germany, Italy and Japan, but they represent a substantial share of these countries' imports.

Our retrospective correction approach can also be applied to other important areas of price measurement such as the consumer price index or the producer price index.

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Appendix: Retrospective Correction in Practice

In this appendix we describe how our three-stage revision process can be applied to the trade statistics of Destatis. The monthly import and export price indices of Destatis are compiled in two variants. One is based on price data collected from exporting producers and wholesale traders, while the other is compiled from customs sources. Gehle (2013, p. 932) and Lippe and Mehrhoff (2010) show that the two variants generate different results. Following the general recommendation of the IMF (2009, p. xiv), we study the variant based on price data. Until December 2004 we had to excerpt the price levels from printed publications of Destatis (Statistisches Bundesamt, 2004). Later price levels we could retrieve from the online data base “Genesis” provided by Destatis. In addition, we could compute from the online data base the categories’ expenditure weights for the years 1995, 2000, 2005, 2010 and 2015.

We begin with the monthly German import price index from January 2010 until July 2018. The official compilation method of Destatis (a monthly Laspeyres index) is sketched out in Statistisches Bundesamt (2019, pp. 6-7). Destatis knows the yearly expenditure weights of the year 2010 (shorthand notation: 10). Therefore, the price, weight and index reference period is not a month, but the year 2010, that is, 2010 = 1. The corresponding Laspeyres index is

$$I_{\mathrm{La}}^{10 \to t} = \sum_{i=1}^{M} \bar{s}_i^{10} \frac{p_i^t}{\bar{p}_i^{10}} , \qquad t = 1/10\,,\, 2/10\,, \ldots,\, 7/18\,, \tag{15}$$

with

$$\bar{s}_i^{10} = \frac{\bar{p}_i^{10} \bar{m}_i^{10}}{\sum_{j=1}^{M} \bar{p}_j^{10} \bar{m}_j^{10}} ,$$

where $\bar{p}_i^{10}$ denotes the average euro price of good i in 2010 and $\bar{m}_i^{10}$ denotes the total imported quantity of good i in 2010. Therefore, $\bar{s}_i^{10}$ is the import share of good i in 2010.

In July 2018, the expenditure weights of the year 2015 become available. Therefore, Destatis revises the Laspeyres indices calculated for January 2015 to July 2018. The revised series uses the year 2015 as price, weight and index reference period:

$$I_{\mathrm{La}}^{15 \to t} = \sum_{i=1}^{M} \bar{s}_i^{15} \frac{p_i^t}{\bar{p}_i^{15}} , \qquad t = 1/15\,,\, 2/15\,, \ldots,\, 7/18\,. \tag{16}$$

This new series must be connected to the index numbers computed by the Laspeyres index (15). Direct chaining does not work here, because the comparison period of the Laspeyres index (15) is a month (for chaining it would be January 2015, that is, $t = 1/15$), while the index reference period of the Laspeyres index (16) is the year 2015. Therefore, Destatis introduces the following Paasche index that binds the price level of January 2015 to the price level of the year 2015:

$$I_{\mathrm{Pa}}^{1/15 \to 15} = \frac{\sum_{i=1}^{M} \bar{p}_i^{15} \bar{m}_i^{15}}{\sum_{j=1}^{M} p_j^{1/15} \bar{m}_j^{15}} = \left(\sum_{i=1}^{M} \bar{s}_i^{15} \left(\frac{\bar{p}_i^{15}}{p_i^{1/15}} \right)^{-1} \right)^{-1} = \frac{1}{I_{\mathrm{La}}^{15 \to 1/15}} . \tag{17}$$

This Paasche index is more convenient than the Laspeyres index $I_{\mathrm{La}}^{1/15 \to 15}$, because the Paasche index requires the readily available expenditure weights of the year 2015, while the Laspeyres index would require the unknown expenditure weights of January 2015.

Based on the three formulas (15), (16) and (17), Destatis calculates a consistent series of monthly index numbers covering the time span January 2010 to July 2018. To this end, Destatis multiplies the price index numbers of January 2010 to December 2014 compiled by formula (15) by the so-called "Verkettungsfaktor" $[I_{\mathrm{La}}^{10 \to 1/15} \cdot I_{\mathrm{Pa}}^{1/15 \to 15}]^{-1}$ (e.g., Statistisches Bundesamt, 2017, p. 6; Statistisches Bundesamt, 2019, p. 6). Note that the first factor of the "Verkettungsfaktor" is index (15) with comparison period $t = 1/15$ and the second factor is index (17). The Verkettungsfaktor is the reciprocal of the price change between the years 2010 and 2015. The index numbers of January 2015 to July 2018 are directly computed by formula (16). The index reference period of the resulting series is the year 2015, that is, 2015 = 1.

To rebase this series to the index reference year 2010, we multiply the whole series by $[I_{\text{La}}^{10 \to 1/15} \cdot I_{\text{Pa}}^{1/15 \to 15}]$, that is, by the inverse of the “Verkettungsfaktor”. As a result, the index numbers of January 2010 to December 2014 are compiled by formula (15), while the index numbers of January 2015 to July 2018 are compiled by:

$$I^{10 \to t} = I_{\text{La}}^{10 \to 1/15} \cdot I_{\text{Pa}}^{1/15 \to 15} \cdot I_{\text{La}}^{15 \to t} , \qquad t = 1/15\,,\, 2/15\,, \ldots\,, 7/18\;. \tag{18}$$

For the comparison month $t = 1/15$, the Laspeyres index (15) and formula (18) give $I_{\text{La}}^{10 \to 1/15}$ – see the last equality in (17). Therefore, formulas (15) and (18) generate a consistent time series with the reference price level 2010 = 1. This series differs from the official time series only by a constant factor, namely the “Verkettungsfaktor”.

Formula (15) is a Laspeyres index and, therefore, prone to upward substitution bias. Formula (18) is a chain index comprising two upwardly biased Laspeyres indices and a Paasche index. The Paasche index is prone to downward bias. However, the bias is probably much smaller than the combined upward bias of the two Laspeyres indices, because the time distance between January 2015 and the full year 2015 (Paasche index) is much smaller than that between the year 2010 and January 2015 or later months (Laspeyres indices). Therefore, not only formula (15), but also formula (18) is likely to exhibit severe upward bias (this is empirically confirmed in Section 4).

When the expenditure weights of the year 2015 have become available to Destatis, it is possible to reduce the substitution bias. To this end, we conduct the retrospective three-stage revision process outlined in Section 3. To adapt this process to the methodology of Destatis, only one modification is necessary. It relates to the denominator of the correction factor in Stage 3.

Stage 1: Using formula (16), a new series of Laspeyres indices for January 2015 to July 2018 can be compiled. This part of the revision process is already implemented in the official index compilations of Destatis.

Stage 2: To rebase the new series of Laspeyres index numbers compiled in Stage 1 to the price level 2010 = 1, we compute the following Törnqvist index:

$$I_{\text{Tö}}^{10 \to 15} = \exp\left(\sum_{i=1}^{M} \frac{1}{2} \left(\bar{s}_i^{10} + \bar{s}_i^{15} \right) \ln\left(\frac{\bar{p}_i^{15}}{\bar{p}_i^{10}} \right) \right) \;. \tag{19}$$

Next, we replace in the chain index (18) the first two links by the Törnqvist index (19) and obtain the following chain index:

$$I^{10 \to t} = I_{\text{Tö}}^{10 \to 15} \cdot I_{\text{La}}^{15 \to t} , \qquad t = 1/15\,,\, 2/15\,, \ldots,\, 7/18\;. \tag{20}$$

For January 2015 to July 2018, this chain index yields more reliable results than the chain index (18).

Stage 3: We want to revise the index numbers of January 2010 to December 2014 compiled by the Laspeyres index (15). The new series of index numbers must be consistent with the revised index number of January 2015 compiled by the Törnqvist index (19). Therefore, we multiply the Laspeyres index (15) by a correction factor that is constructed in analogy to the correction factor in formula (13). The result is the following index formula:

$$I^{10\to t} = I_{\mathrm{La}}^{10\to t} \cdot \left(\frac{I_{\mathrm{Tö}}^{10\to 15}}{I_{\mathrm{La}}^{10\to 1/15} \cdot I_{\mathrm{Pa}}^{1/15\to 15}} \right)^{\lambda_t} , \qquad t = 1/10,\ 2/10, ..., 12/14 , \tag{21}$$

with

$$\lambda_t = \begin{cases} 0 & \text{for} \quad s = 1, 2, \ldots, 12 \\ \dfrac{s-12}{61-12} & \text{for} \quad s = 13, 14, \ldots, S\ . \end{cases} \tag{22}$$

For the twelve months of the year 2010 ($s = 1, 2, \ldots, 12$) the impact λ_t is 0. Therefore, the correction factor has the value 1, that is, no correction of the Laspeyres index $I_{\mathrm{La}}^{10\to 1/11}$ occurs until December 2010. This is intended, because only the expenditure weights of the year 2010 and not the expenditure weights of the year 2015 should be included in the calculation of price changes *within* the year 2010. In January 2011 ($s = 13$) the impact λ_t is equal to 1/49. The correction factor then deviates minimally from 1, leading to a small correction of the Laspeyres index $I_{\mathrm{La}}^{10\to 1/11}$. As t increases, the counter s and the impact λ_t also gradually increase. Only in January 2015, the counter s would reach its maximum value 61 and, therefore, the impact λ_t its maximum value 1. In this last month, formula (21) would simplify to

$$I^{10\to 1/15} = \frac{I_{\mathrm{Tö}}^{10\to 15}}{I_{\mathrm{Pa}}^{1/15\to 15}}$$

which, in view of the last equality in (17), is identical to formula (20). Therefore, using formula (21) for the comparison months January 2010 to December 2014, and using formula (20) for all subsequent months, generates a consistent time series of price levels.

Quite likely, in 2023 the expenditure weights for the year 2020 will become available. Since the expenditure weights of the year 2020 add no relevant information for measuring the price changes between 2010 and 2015, there is no need to revise the index numbers of January 2010 to January 2015. However, for the index numbers of the subsequent months, the expenditure weights of the year 2020 contain valuable new information. The new import price index numbers of January 2020 and all subsequent months (Stages 1 and 2) are compiled by the chain index

$$I^{10\to t} = I_{\mathrm{Tö}}^{10\to 15} \cdot I_{\mathrm{Tö}}^{15\to 20} \cdot I_{\mathrm{La}}^{20\to t} , \qquad t = 1/20\,,\ 2/20\,, \ldots\ . \tag{23}$$

The revised import price index numbers of February 2015 to December 2019 (Stage 3) are calculated by the chain index

$$I^{10\to t} = I_{\mathrm{Tö}}^{10\to 15} \cdot I_{\mathrm{La}}^{15\to t} \cdot \left(\frac{I_{\mathrm{Tö}}^{15\to 20}}{I_{\mathrm{La}}^{15\to 1/20} \cdot I_{\mathrm{Pa}}^{1/20\to 20}} \right)^{\lambda_t} , \qquad t = 2/15\,, 3/15\,, \ldots,\ 12/19\ ,$$

where the impact λ_t is defined as in (22) with $s = 1$ in January 2015. These computations generate a consistent time series of index numbers with the index reference period 2010. The series covers the time interval from January 2010 to the months of 2023 and beyond.

The compilation and revision of the export price index can be conducted in a perfectly analogous manner. The ratio of the revised export price index and import price index of some month t yields the revised terms of trade index of that month.