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Retrospective Computations of Price Index Numbers: Theory and Application

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Retrospective Computations of Price Index Numbers: Theory and Application

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Abstract: Due to outdated weighting information, a Laspeyres-based Consumer Price Index (CPI) is prone to accumulating upward bias. Therefore, the present study introduces and examines simple and transparent revision approaches that retrospectively address the source of the bias. They provide a consistent long-run time series of the CPI and they require no additional information. Furthermore, a coherent decomposition of the bias into the contributions of individual product groups is developed. In a case study, the approaches are applied to a Laspeyres-based CPI. The empirical results confirm the theoretical predictions. The proposed revision approaches are not only adoptable to most national CPIs, but also to other price level measures such as the producer price index or the import and export price indices.

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1 Introduction

Various important fields of economic analysis rely on indicators such as real gross domestic product, real wages, real interest rates, and real public debt. To obtain a long-run time series of such an indicator, the time series of the corresponding nominal indicator is deflated by some appropriate price level measure. National statistical offices (NSOs) are entrusted to provide these price level measures in an accurate and timely manner.

The most important national price level measure is the Consumer Price Index (CPI). In their CPI compilations, NSOs relate the *comparison period*'s (or current period's) price level of final consumption to the price level of some former *price reference period* (or base period). In the applied price index formulas, the weighting of the various consumption items reflects the consumer expenditures on these items. Most NSOs apply an index derived from the Laspeyres index; in other words, a "Laspeyres type index". Usually this is a Lowe index (e.g., Australia, Canada, Switzerland, U.S.) or a Young index (e.g., Denmark, Georgia, South Africa). Only few NSOs use a proper Laspeyres index (e.g., Germany, Japan). While the weighting of a *proper* Laspeyres index exclusively depends on expenditure information of the price reference period, the weighting of a Laspeyres *type* index may draw on expenditure information from periods other than (and usually preceding) the price reference period.

Unfortunately, the Laspeyres index as well as Laspeyres type indices are known to suffer from upward substitution bias because in their item weights they fail to incorporate the substitution behavior reflected in the consumed quantities of the comparison period (e.g., ILO et al., 2004, p. 4). Among the more recent studies that provide empirical evidence of this bias are Hansen (2007), de Haan et al. (2010), Greenlees and Williams (2010), Huang et al. (2017), Klick (2018), and Herzberg et al. (2021). The more outdated the expenditure information – and, consequently, the applied item weights – the larger the risk of substitution bias and the associated accumulating upward bias in the long-run time series of monthly price levels. Such findings are worrying for institutions and analysts that rely on an unbiased long-run time series of the CPI because upward biased

CPI numbers would, for example, result in downward biased real growth rates.

To reduce the risk of biased CPI numbers, many NSOs attempt to shorten the time interval between the comparison period and the period to which the item weights relate. Another strategy is to wait until sufficiently up-to-date information on expenditures and, thus, on weighting is available and then to publish revised index numbers. However, in many countries such revisions raise legal issues. Furthermore, NSOs may worry that revisions undermine the credibility of their published numbers. Therefore, many NSOs are reluctant to retrospectively revise their initial results.

Generally, it is acknowledged that retrospectively revised numbers tend to be more accurate than original ones. Therefore, revisions are extremely valuable for scientific purposes and, thus, for designing sound economic and social policies. More NSOs might be willing to consider the provision of such revised numbers if the extra work load appears manageable. Therefore, the compilation of the revised numbers should require only information that the NSOs collect and process anyway.

The first contribution of the present paper is to present and elaborate two general approaches of such work-saving retrospective revision methods. The two approaches are denoted as the "correction approach" and the "imputation approach". For the implementation of both approaches a wide range of options is available. Therefore, they are general enough to be applied by most NSOs. The approaches effectively address the source of the accumulating substitution bias and they generate a consistent long-run time series of monthly price levels. The revised series also indicates the upward substitution bias inherent in the non-revised series.

In addition, it is shown that the imputation approach can decompose the upward substitution bias into the contributions of individual groups of items. This decomposition method is the paper's second contribution. In the future, this method may help to mitigate the bias already from the outset.

The paper's third contribution is empirical. To illustrate the relevance of the problem and the efficacy of the proposed revision approach, it is applied to the upper level aggregation of the German CPI. Even though the study relates to the CPI, its insights carry over to other price level measures such as the producer price index or the import and export price indices.

The rest of the paper is organized as follows. Section 2 recaptures the relevant concepts from index theory. The correction approach and the imputation approach are introduced in Section 3. In Section 4, an empirical application of the new revision concept is presented. The decomposition of the upward bias into the contributions of individual groups of items is developed in Section 5 and applied in Section 6. Section 7 concludes.

2 Four Types of Symmetric Price Indices

Let p_i^t and x_i^t denote the price and the quantity of item $i \in A = [1, \ldots, N]$ at time period $t \in [0, \ldots, T]$. The expenditures on item i are denoted by $v_i^t = p_i^t x_i^t$. The symbols $\sum$ and $\prod$ are the shorthand notation for the sum and the product over all N items in set A. Only for periods 0 and T the expenditure shares of the N items are known:

$$s_i^0 = \frac{v_i^0}{\sum v_j^0} \qquad \text{and} \qquad s_i^T = \frac{v_i^T}{\sum v_j^T} \ .$$

For each comparison period $t \in [0, \ldots, T]$, a Laspeyres index can be compiled that compares the average price level of the comparison period t to the average price level of the price reference period 0. The resulting sequence of Laspeyres indices, $P_{\mathrm{L}}^{0 \to t}$, is

$$P_{\mathrm{L}}^{0 \to 0} = \sum s_i^0 \frac{p_i^0}{p_i^0} = 1, \quad P_{\mathrm{L}}^{0 \to 1} = \sum s_i^0 \frac{p_i^1}{p_i^0}, \quad \ldots \quad , \quad P_{\mathrm{L}}^{0 \to T} = \sum s_i^0 \frac{p_i^T}{p_i^0} \ . \tag{1}$$

In this study, the period that provides the information for the calculation of the expenditure weights is denoted as the *expenditure reference period.* In the sequence of Laspeyres indices (1), period 0 is both, the expenditure reference period and the price reference period.[2]

[2]The expenditure reference period should not be confused with the *weight reference period* as defined in ILO et al. (2004, p. 3). The latter is the period whose *quantities* are actually used in the index. In a Laspeyres index of the form (1) the weight reference period and the expenditure reference period coincide. Another type of reference period is the *index reference period.* This is the period for which the index is set equal to 100.

Once the expenditure shares of the new expenditure reference period T become available, a retrospective price index can be compiled for each comparison period $t \in [0, \ldots, T]$. Possible candidates for such retrospective price indices are appropriately modified versions of so-called *symmetric* standard price index formulas. ILO et al. (2004, pp. 5-6) attach the label "symmetric" to price index formulas that give equal importance to the information of the first expenditure reference period 0 and the second expenditure reference period T. However, the phrase "equal importance" turns out to be rather ambiguous because it can come in different forms. Therefore, we propose to distinguish between four types of symmetry:

A: Symmetric Treatment of Quantities

The quantities, x_i, represent some combination of $x_i^0 = v_i^0/p_i^0$ and $x_i^T = v_i^T/p_i^T$. Examples are

$$\begin{aligned} \text{Marshall-Edgeworth}: \; P_{\text{ME}}^{0 \to T} &= \frac{\sum p_i^T (x_i^0 + x_i^T)}{\sum p_i^0 (x_i^0 + x_i^T)} \\ \text{Walsh}: \; P_{\text{W}}^{0 \to T} &= \frac{\sum p_i^T \sqrt{x_i^0 x_i^T}}{\sum p_i^0 \sqrt{x_i^0 x_i^T}} \; . \end{aligned} \tag{2}$$

B: Symmetric Treatment of Expenditures

The expenditures, v_i, represent some combination of v_i^0 and v_i^T. Well known examples are

$$\begin{aligned} \text{Walsh-2}: \; \ln P_{\text{W2}}^{0 \to T} &= \sum \frac{\sqrt{v_i^0 v_i^T}}{\sum \sqrt{v_j^0 v_j^T}} \ln \frac{p_i^T}{p_i^0} \\ \text{Theil}: \; \ln P_{\text{Th}}^{0 \to T} &= \sum \left[\frac{\sqrt[3]{\frac{1}{2}\left(v_i^0 + v_i^T\right) v_i^0 v_i^T}}{\sum \sqrt[3]{\frac{1}{2}\left(v_j^0 + v_j^T\right) v_j^0 v_j^T}} \right] \ln \frac{p_i^T}{p_i^0} \\ \text{Vartia}: \; \ln P_{\text{Va}}^{0 \to T} &= \sum \frac{L\left(v_i^0, v_i^T\right)}{L\left(\sum v_j^0, \sum v_j^T\right)} \ln \frac{p_i^T}{p_i^0} \; , \\ &\text{where } L\,(a, b) = \frac{a - b}{\ln a - \ln b} \text{ for } a \neq b \text{ and } L\,(a, b) = a \text{ for } a = b. \end{aligned}$$

C: Symmetric Treatment of Expenditure Shares

The expenditure shares, s_i, represent some combination of s_i^0 and s_i^T. This type of symmetry is represented by

$$
\begin{aligned}
\text{Törnqvist}: \quad \ln P_{\text{Tö}}^{0\to T} &= \sum \frac{1}{2}\left(s_i^0 + s_i^T\right) \ln \frac{p_i^T}{p_i^0} \\
\text{Walsh-Vartia}: \quad \ln P_{\text{WV}}^{0\to T} &= \sum \sqrt{s_i^0 s_i^T} \ln \frac{p_i^T}{p_i^0} \\
\text{Sato-Vartia}: \quad \ln P_{\text{SV}}^{0\to T} &= \sum \frac{L\left(s_i^0, s_i^T\right)}{\sum L\left(s_j^0, s_j^T\right)} \ln \frac{p_i^T}{p_i^0} \ .
\end{aligned} \tag{3}
$$

D: Symmetric Treatment of Indices

The indices, $P^{0\to T}$, represent some combination of an index with expenditure reference period 0 and some index with expenditure reference period T. For this purpose, the Laspeyres index, $P_L^{0\to T}$, and the Paasche index,

$$
P_{\text{P}}^{0\to T} = \frac{\sum p_i^T x_i^T}{\sum p_i^0 x_i^T} = \left[\sum s_i^T \left(\frac{p_i^T}{p_i^0}\right)^{-1}\right]^{-1} \ ,
$$

are particularly popular. They are used in price index formulas such as

$$
\begin{aligned}
\text{Fisher}: \quad P_{\text{F}}^{0\to T} &= \sqrt{P_{\text{L}}^{0\to T} P_{\text{P}}^{0\to T}} \\
\text{Drobisch}: \quad P_{\text{D}}^{0\to T} &= \frac{1}{2}\left(P_{\text{L}}^{0\to T} + P_{\text{P}}^{0\to T}\right) \ .
\end{aligned} \tag{4}
$$

Some price index formulas can be assigned to more than one type of symmetry. For example, the Walsh index can also be written in the forms

$$
P_{\text{W}}^{0\to T} = \sum \frac{\sqrt{v_i^0 v_i^T \left(p_i^T/p_i^0\right)^{-1}}}{\sum \sqrt{v_j^0 v_j^T \left(p_j^T/p_j^0\right)^{-1}}} \frac{p_i^T}{p_i^0} \tag{5}
$$

$$
= \sum \frac{\sqrt{s_i^0 s_i^T \left(p_i^T/p_i^0\right)^{-1}}}{\sum \sqrt{s_j^0 s_j^T \left(p_j^T/p_j^0\right)^{-1}}} \frac{p_i^T}{p_i^0} \ . \tag{6}
$$

This is a weighted arithmetic mean of the price ratios (p_i^T/p_i^0) where the weights depend not only on the expenditures (or expenditure shares) of the price reference period, but also on the expenditures (or expenditure shares) of the comparison period T deflated to the price reference period. Therefore, the Walsh index can be interpreted as a representative of symmetry types A, B, and C.

The listed symmetric price indices compare the prices of the comparison period T to those of the price reference period 0. In ILO et al. (2004, p. 173), the retrospective compilation of a price index such as (2), (3), or (4) is advocated. However, we need a price index formula that can relate not only period T but also all intermediate comparison periods $t \in [1, \ldots, T-1]$ to the price reference period 0. What can be achieved if neither the quantities, nor the expenditures, nor the expenditure shares of these intermediate comparison periods are known? The objective is a price index formula that provides a more reliable series of retrospective index numbers than the Laspeyres-based series (1).

3 Retrospectively Computed Price Indices

To construct a reliable series of retrospective price index numbers we recommend to use some symmetric price index formula. Unfortunately, the quantity and expenditure information of the intermediate periods ($t = 1, \ldots, T-1$) is missing. Therefore, we must make use of the available quantity or expenditure information of the two expenditure reference periods, $t = 0$ and $t = T$, and we must invoke some form of interpolation for the intermediate periods.

The construction of a retrospective price index formula can follow at least two alternative principles. The index can be defined as

- the product of the original Laspeyres index (or Laspeyres type index) and some correction factor that accounts for the gradually increasing deviation between the Laspeyres index and a symmetric price index as t progresses from 0 to T, or as

- a symmetric price index formula that imputes the missing information on x_i^t, v_i^t, or s_i^t ($t = 1, \ldots, T-1$) where the imputeted values are weighted averages of the values of periods 0 and T, and where the weight of the value of period 0 gradually decreases from 1 to 0 as t progresses from 0 to T.

We denote the two principles as *correction approach* and *imputation approach.* In both approaches, the variable $\lambda_t = t/T$ plays a crucial role. As time progresses, λ_t increases from $\lambda_0 = 0$ to $\lambda_T = 1$ and reflects in the retrospective price index formula the gradually diminishing relevance of the first expenditure reference period ($t = 0$) and the increasing relevance of the second expenditure reference period ($t = T$). The two approaches can be implemented in various ways.

Correction Approach: Using the Walsh index (2), the series of Laspeyres indices (1) can be revised by the following retrospective price index formula:

$$P^{0\to t} = \left(P_{\mathrm{L}}^{0\to t}\right) \cdot \left(\frac{P_{\mathrm{W}}^{0\to \mathrm{T}}}{P_{\mathrm{L}}^{0\to \mathrm{T}}}\right)^{\lambda_t} , \qquad t = 0 \ldots, T \, . \tag{7}$$

For $t = 0$, we have $\lambda_0 = 0$. Therefore, the correction factor $(P_{\mathrm{W}}^{0\to \mathrm{T}}/P_{\mathrm{L}}^{0\to \mathrm{T}})^{\lambda_t}$ is equal to 1 (that is, no correction), and the retrospective price index formula gives $P^{0\to 0} = P_{\mathrm{L}}^{0\to 0} = 1$. As time progresses towards period $t = T$, λ_t increases from 0 to 1, the correction factor reaches its maximum deviation from 1, and the retrospective price index gradually turns into the Walsh index: $P^{0\to T} = P_{\mathrm{W}}^{0\to \mathrm{T}}$.

Instead of the Walsh index, other symmetric price index formulas could be used to define the correction factor. For example, von Auer and Shumskikh (2020) apply the Törnqvist index (3). Using the Fisher index (4), gives

$$P^{0\to t} = \left(P_{\mathrm{L}}^{0\to t}\right) \left(\frac{P_{\mathrm{P}}^{0\to \mathrm{T}}}{P_{\mathrm{L}}^{0\to \mathrm{T}}}\right)^{\lambda_t/2} , \qquad t = 0 \ldots, T \, , \tag{8}$$

and, therefore, $P^{0\to 0} = 1$ and $P^{0\to T} = P_{\mathrm{F}}^{0\to \mathrm{T}}$. The same two values are generated by a retrospective price index formula that is proposed in de Haan et al. (2010). They replace in the correction factor of formula (8) the Laspeyres index, $P_{\mathrm{L}}^{0\to \mathrm{T}}$, by the chained index $P_{\mathrm{L}}^{0\to \mathrm{t}} P_{\mathrm{P}}^{\mathrm{t}\to \mathrm{T}}$.

Instead of the multiplicative correction of the Laspeyres index, $P_{\mathrm{L}}^{0\to t}$, applied in (7), one could use an additive one:

$$P^{0\to t} = \left(P_{\mathrm{L}}^{0\to t}\right) + \lambda_t \left(P_{\mathrm{W}}^{0\to \mathrm{T}} - P_{\mathrm{L}}^{0\to \mathrm{T}}\right) , \qquad t = 0 \ldots, T \, . \tag{9}$$

Again, one obtains $P^{0 \rightarrow 0} = 1$ and $P^{0 \rightarrow T} = P_{\text{W}}^{0 \rightarrow \text{T}}$.

Imputation Approach: When the Walsh index (2) is applied, the unknown quantities, x_i^t $(t = 1, \ldots, T-1)$, must be substituted by imputed values, $\hat{x}_i^t$. If we assume that the rate of change of x_i^t is constant between periods 0 and T, the imputed values are given by the weighted geometric average

$$\hat{x}_i^t = \left(x_i^0\right)^{1-\lambda_t} \left(x_i^T\right)^{\lambda_t} \ , \tag{10}$$

and we obtain $P^{0 \rightarrow 0} = 1$ and $P^{0 \rightarrow T} = P_{\text{W}}^{0 \rightarrow \text{T}}$. Alternatively, other symmetric price indices can be used. Furthermore, the imputed values, $\hat{x}_i^t$, can be obtained from a weighted arithmetic average:

$$\hat{x}_i^t = (1 - \lambda_t) x_i^0 + \lambda_t x_i^T \ . \tag{11}$$

The same options are available, if the missing values of the expenditures, v_i^t, or the expenditure shares, s_i^t, have to be imputed.

An early application of the imputation approach can be found in de Haan et al. (2010). They propose to use the Törnqvist index (3) or the Fisher index (4) and to impute the unknown expenditure shares, s_i^t $(t = 1, \ldots, T-1)$, by $\hat{s}_i^t = (1 - \lambda_t) s_i^0 + \lambda_t s_i^T$.

In this section we have presented two different construction principles for a retrospective price index, the correction approach and the imputation approach. For both approaches, a large set of symmetric price indices is available. In the correction approach, a multiplicative or an additive form of correction can be applied. In the imputation approach, the imputed values can be obtained from weighted arithmetic or geometric averages. Since $\lambda_T = 1$, the index number relating to the comparison period $t = T$ (the second expenditure reference period) is independent from the choice between the correction and the imputation approach.

Both, the correction approach and the imputation approach have intuitive appeal. The correction approach appears somewhat less intrusive than the imputation approach. Therefore, in the following section, we demonstrate how the correction approach can be implemented in the German CPI. On the other hand, the imputation approach allows for an additive decomposition of the overall price change. This property is useful in various ways. For example, it helps to identify those groups of items that cause the upward substitution bias of the Laspeyres index. This identification is a prerequisite for reducing the bias already from the outset. Therefore, in Section 6 the imputation approach is applied to identify the critical items in the German CPI.

4 Retrospective Correction of the German CPI

Destatis (shortened form of "Statistisches Bundesamt") compiles proper Laspeyres indices. The most recent price and expenditure reference period of Destatis is the year 2015. The previous ones were the years 2010, 2005, and so on. For example, the Laspeyres index of the comparison period January 2000 was published in February 2000 and used the year 1995 as price and expenditure reference period. The same price and expenditure reference period was used for the subsequent months.

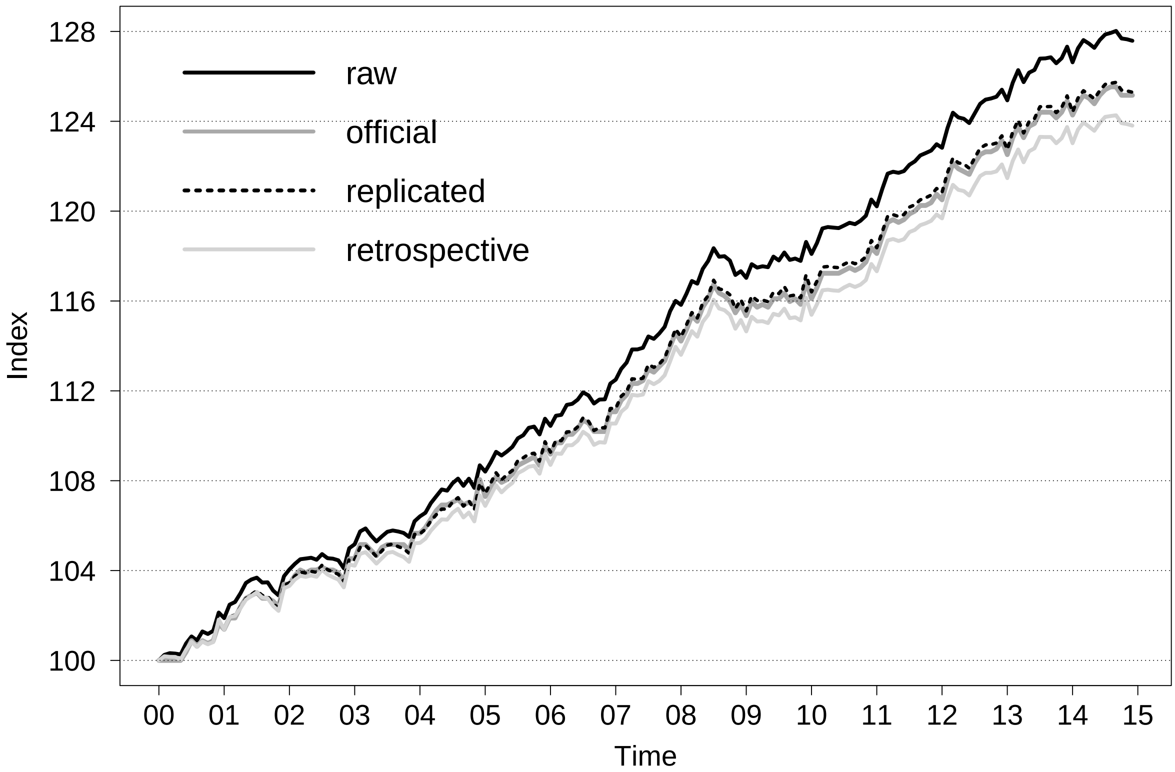
When in 2003 the expenditures of the year 2000 became available, Destatis recalculated the index numbers of the comparison periods starting in January 2000. The revised Laspeyres indices use the year 2000 as the price and expenditure reference period. These index calculations were continued until in 2008 the expenditure weights of the year 2005 became available. This new information prompted a retrospective revision of the Laspeyres index numbers starting in January 2005. Analogous retrospective revisions occurred in 2013 when the expenditures of the year 2010 became available.

In total, these compilations generate a sequence of three consecutive time series of monthly Laspeyres indices, each covering a five-year period. These three series are chained, such that a consistent time series starting in January 2000 is obtained. The details of the official CPI compilation procedure are documented in Appendix A. The official source is Statistisches Bundesamt (2018, pp. 6-7, 15-19).[3] The official time series of monthly price levels is depicted in Figure 1 (dark gray line labeled as "official").

We cannot use the official time series as a benchmark because we do not have access

[3] Additional descriptions are provided by Egner (2003), Elbel and Egner (2008), Egner (2013), and Egner (2019).

Figure 1: German consumer price index (January 2000 = 100) for January 2000 to December 2014. Source: Own calculations based on data of Destatis.



to the same data set. Using the data set available to us, we replicate the compilation procedure of Destatis. The resulting time series will serve as a benchmark for the time series derived from the retrospective index formula. The benchmark time series is depicted in Figure 1 by the black dotted line labeled as “replicated”. The differences between the officially published Laspeyres CPI index and our replicated Laspeyres index are hardly visible. In the final comparison period (December 2014), the deviation between the official and the replicated index number are merely 0.14 percentage points. This is a remarkable result because the information accessible for our research purposes is not as granular as the information processed by Destatis. In the accessible data set, the consumption basket is decomposed into 102 classes of products (third level of Classification of Individual Consumption according to Purpose, COICOP). The original decomposition of Destatis is much finer and some of the original classes are missing in the accessible data set.

Now, if Destatis had ruled out retrospective revisions, in 2003 it would have continued with the old price and expenditure reference period (1995) until January 2005, the month scheduled for the introduction of regular methodological modifications and other updates of the index. Thus, this would have been the time to switch from the old (1995) to the new (2000) price and expenditure reference period. Subsequently, an analogous changeover would have been conducted in January 2010.

Based on the data set available to us, we simulate this non-revisionary compilation procedure. The details are documented in Appendix A. In Figure 1, the black line labeled as "raw" depicts the resulting time series of unrevised monthly price levels. It covers the time interval January 2000 to December 2014 (the index numbers of earlier months would require the expenditure information of the year 1990 which we do not have). This time series is obtained from Laspeyres index numbers based on weights that are outdated by five to ten years. As a result, it is prone to substantial upward substitution bias.

The comparison of the two time series "raw" and "replicated" reveals that the retrospective revisions of Destatis successfully curbed the long-run upward substitution bias. Did these revisions even eliminate the bias? To answer this question, a time series of index numbers is required that can be considered as free of long-run substitution bias.

Therefore, we compile a time series of retrospective Walsh indices and compare it to the time series of replicated Laspeyres indices. The Walsh-based series applies the correction approach. The comparison between the Laspeyres- and Walsh-based series gives us clues about the existence and the extent of the remaining distortion in the long-run time series of the official CPI. The retrospective Walsh index numbers of the time interval 2015 to 2020 would require the expenditure weights of the year 2020. These will become available not before 2023. Therefore, we decided to restrict the analysis to the interval January 2000 to December 2014. The formal details of the compilation procedure of the retrospective Walsh index are explained in Appendix B.

The result of this procedure is the light gray line in Figure 1 labeled as "retrospective". The graph confirms the theoretical predictions. The retrospective Walsh index runs below the replicated Laspeyres index, indicating that the official index still suffers from upward substitution bias. The deviation accumulates over time and in December 2014 it exceeds 1.3 percentage points. This deviation represents only the distortion that can be attributed to the upper level aggregation, that is, the aggregation of the 102 classes into the overall CPI. Since the subclasses at the lower level aggregation exhibit a larger substitutability than the classes at the upper level aggregation, it is quite likely that the actual upward bias is larger than the 1.3 percentage points observed here.

5 Decomposition of the Bias

It would be valuable to know which groups of items are responsible for the upward substitution bias of the Laspeyres index. Usually the CPI is formed by major expenditure categories where each category comprises several items. More formally, the set of items, A, can be partitioned into the subsets A_k where $k \in K$ and K is the set of (major expenditure) categories.

We know from formulas (1) and (6) that both, the Laspeyres index and the Walsh index can be expressed in the additive form

$$P = \sum_{i \in A} z_i \frac{p_i^T}{p_i^0} \ , \tag{12}$$

where p_i^T/p_i^0 is the price ratio of item i and, therefore, the *primary attribute* of the index and z_i is the *secondary attribute.* It can be interpreted as the weight of the primary attribute. In this section, we drop the superscript "$0 \to$T" at the index P. Since $\sum_{i \in A} = \sum_{k \in K} \sum_{i \in A_k}$, index formula (12) can also be written in the form

$$P = \sum_{k \in K} Z_k P_k \ , \tag{13}$$

where $P_k = \sum_{i \in A_k} (z_i/Z_k)(p_i^T/(p_i^0)$ is the price index computed for category k and $Z_k =$

$\sum_{i\in A_k} z_i$ is the weight of category k, with

$$z_i = z_{i,\mathrm{L}} = s_i^0 \qquad \text{(Laspeyres index)} \tag{14}$$

$$z_i = z_{i,\mathrm{W}} = \frac{\sqrt{s_i^0 s_i^T \left(p_i^T/p_i^0\right)^{-1}}}{\sum \sqrt{s_j^0 s_j^T \left(p_j^T/p_j^0\right)^{-1}}} \qquad \text{(Walsh index)}\ . \tag{15}$$

Equations (12) and (13) imply that the same index number is obtained, regardless of whether the index is computed in a single stage over all items in set A or in two stages, where, on the first stage, for each subset A_k the price index P_k and the aggregated weight Z_k are computed and, on the second stage, these values are used to compute the overall result P. In that second stage, the same index formula is used as in the first stage and the single stage computation. Z_k and P_k simply replace z_i and p_i^t/p_i^0, respectively. Therefore, von Auer and Wengenroth (2021) denote the Laspeyres and the Walsh index as *consistent in aggregation with respect to the secondary attributes* (14) and (15), respectively. The authors also show that all other indices listed in Section 2 of the present study are consistent in aggregation with respect to some appropriately defined secondary attribute (e.g., Törnqvist index, Fisher index).

Usually, the property of consistency in aggregation would be used to decompose the overall price change into the contributions of the individual categories. However, this property also allows for the decomposition of the deviation between two different index formulas into the contributions of the individual categories. Since we interpret the deviation between the Laspeyres index and the Walsh index as the upward substitution bias of the Laspeyres index, we obtain a decomposition of that bias.

The categories' Z_k-values can be considered as the weight of the category in the overall index computation. Definitions (14) and (15) imply that the notation must distinguish between the Z_k-values of the Laspeyres index, $Z_{k,\mathrm{L}} = \sum_{i\in A_k} z_{i,\mathrm{L}}$, and the Z_k-values of the Walsh index, $Z_{k,\mathrm{W}} = \sum_{i\in A_k} z_{i,\mathrm{W}}$. Noting that $\sum_{k\in K} Z_{k,\mathrm{L}} = \sum_{k\in K} Z_{k,\mathrm{W}} = 1$ and that the Laspeyres index and Walsh index are consistent in aggregation, we propose the following decomposition of the overall bias:

$$P_{\mathrm{L}} - P_{\mathrm{W}} = \sum_{k \in K} \left[Z_{k,\mathrm{L}} \left(P_{k,\mathrm{L}} - 1 \right) - Z_{k,\mathrm{W}} \left(P_{k,\mathrm{W}} - 1 \right) \right] \ , \tag{16}$$

where

$$P_{k,\mathrm{L}} = \sum_{i \in A_k} z_{i,\mathrm{L}} \frac{p_i^T}{p_i^0} \qquad \text{and} \qquad P_{k,\mathrm{W}} = \sum_{i \in A_k} z_{i,\mathrm{W}} \frac{p_i^T}{p_i^0} \ .$$

6 Decomposition of the Bias in the German CPI

The German CPI is formed by twelve major expenditure categories, $K = [1, \ldots, 12]$. The names of these twelve categories are listed in the first column of Table 1. The decomposition (16) allows us to examine whether the bias in the Laspeyres index is concentrated in only a few of these categories. We start with the comparison period December 2014 ($t = 12/14$) and the price reference period January 2010 ($t = 1/10$). In the notation, we drop the superscript "1/10→12/14" at the price index. For the retrospective computation of the Walsh index, the imputation approach is applied. The computational details are described in Appendix C. The same type of analysis is repeated for the time intervals January 2000 to December 2004 as well as January 2005 to December 2009.

The bottom line of Table 1 shows the all-items index numbers for the price reference period January 2010 and the comparison period December 2014. The all-items Laspeyres index (1) gives the number $P_{\mathrm{L}} = 107.75$, whereas the retrospectively computed Walsh index yields $P_{\mathrm{W}} = 107.34$. Thus, the upward bias of the all-items Laspeyres index is 0.410. The index numbers of each category are also listed in Table 1. The last column decomposes the all-items bias (0.410) into the contributions of the twelve individual categories. To this end, we use the decomposition approach presented in formula (16), where

$$P_{k,\mathrm{L}} = \sum_{i \in A} s_i^{10} \frac{p_i^{12/14}}{p_i^{1/10}} \qquad \text{and} \qquad P_{k,\mathrm{W}} = \sum_{i \in A} \frac{\sqrt{s_i^{10} s_i^{15} \left(p_i^{15}/p_i^{10}\right)^{-1}}}{\sum \sqrt{s_j^{10} s_j^{15} \left(p_j^{15}/p_j^{10}\right)^{-1}}} \frac{p_i^{12/14}}{p_i^{1/10}}$$

$$Z_{k,\mathrm{L}} = \sum_{i \in A_k} s_i^{10} \qquad \text{and} \qquad Z_{k,\mathrm{W}} = \sum_{i \in A_k} \frac{\sqrt{s_i^{10} s_i^{15} \left(p_i^{15}/p_i^{10}\right)^{-1}}}{\sum \sqrt{s_j^{10} s_j^{15} \left(p_j^{15}/p_j^{10}\right)^{-1}}} \ .$$

Table 1: Laspeyres index numbers and Walsh index numbers for December 2014 ($P_{k,\mathrm{L}}$ and $P_{k,\mathrm{W}}$; January 2010 = 100), weights ($Z_{k,\mathrm{L}}$ and $Z_{k,\mathrm{W}}$; in percent), and contributions to the bias of the twelve major expenditure categories of the German CPI.

	Laspeyres		Walsh		Contrib.
Expenditure Category	$P_{k,\mathrm{L}}$	$Z_{k,\mathrm{L}}$	$P_{k,\mathrm{W}}$	$Z_{k,\mathrm{W}}$	to Bias
Food, non-alcoholic beverages	111.87	10.45	111.74	10.09	0.056
Alcoholic beverages, tobacco	111.78	3.81	111.70	3.78	0.007
Clothing, footwear	109.89	4.49	109.84	4.51	0.000
Housing, water, electricity, gas, other fuels	109.04	32.28	108.88	31.92	0.085
Furniture, other household equipment	102.83	5.06	102.80	5.24	−0.003
Health	102.58	4.52	101.61	4.80	0.039
Transport	105.91	13.71	105.78	13.65	0.021
Communication	90.67	3.06	90.04	3.19	0.032
Recreation, entertainment, culture	110.88	11.69	109.36	11.61	0.186
Education	70.29	0.40	70.29	0.37	−0.008
Restaurant, accommodation services	109.62	4.54	109.63	4.66	−0.012
Miscellaneous goods and services	105.42	5.97	105.13	6.17	0.007
Total (Jan. 2010 - Dec. 2014)	107.75	100.00	107.34	100.00	0.410

For example, the contribution of the category "Education" is negative (−0.008), even though the price indices of that category are identical ($P_{10,\mathrm{L}} = P_{10,\mathrm{W}} = 70.29$). The negative contribution says that the category "Education" *reduces* the upward bias inherent in the overall Laspeyres index. The cause of that reduction is the difference in the weights. In the Laspeyres index, the price decline receives the weight $Z_{10,\mathrm{L}} = 0.40$, while in the Walsh index the weight is only $Z_{10,\text{W-im}} = 0.37$. The category "Education" reveals that a comparison of the Laspeyres index and Walsh index of some given category is not sufficient to evaluate that category's contribution to the overall bias. Also the weights must be considered. Besides "Education", two other categories mitigate the bias. The strongest positive contribution to the bias comes from the category "Recreation, entertainment and culture".

The decomposition of the bias can be conducted also for the intervals January 2000 to December 2004 as well as January 2005 to December 2009. The results are documented in Appendix D. The overall bias for January 2000 to December 2004 and for January 2005 to December 2009 are 0.683 and 0.573, respectively. Looking over all three time intervals, one can say that roughly half of the bias can be attributed to the category "Recreation, entertainment, culture". For the compilers of the German CPI, this is a valuable insight. It may prompt a closer inspection of the computational procedures applied in this category. Even though the category "Housing, water, electricity, gas, other fuels" has by far the highest weight, its contribution to the bias is small. The contributions of the categories "Clothing, footwear", "Education", "Restaurant, accomodation services", and "Miscellaneous goods and services" are negligible.

7 Concluding Remarks

The expenditure information for the weighting of a national Consumer Price Index (CPI) is usually outdated by 13 to 60 months. The longer this implementation lag, the larger the risk of upward substitution bias in the CPI. The national statistical offices (NSOs) have addressed this issue in different ways. To keep the implementation lag short, some NSOs collect and process the expenditure information every year. Some NSOs allow for retrospective revisions of their headline CPI or some supplementary CPI. However, even such revisions may not completely solve the problem.

The German CPI is a case in point. Without any retrospective revision, the long-run upward substitution bias during the time interval January 2000 and December 2014 would have amounted to almost 0.22 percentage points per year. The official retrospective revisions of the German CPI reduced that yearly bias to 0.09 percentage points. This is a conservative estimate, since it does not include the upward bias attributable to the lower level of aggregation.

This study has introduced a procedure that can effectively address the source of the bias without requiring any additional information. This procedure replaces the Laspeyres or Laspeyres type index (typically a Lowe or a Young index) by a retrospective price index. Such a retrospective price index can be computed in different ways. Here we discussed the correction approach and the imputation approach and presented various options for implementing these approaches. Both approaches rely on some symmetric price index formula. Suitable candidates include the Walsh, Törnqvist, Fisher, Sato-Vartia, or Marshall-Edgeworth index.

The imputation approach allows for a decomposition of the bias into the contributions of individual groups of items. It was shown that half of the bias in the German CPI can be attributed to a single group of items even though the expenditure share of that group is below 12 percent.

The proposed revision process is not only adoptable to most national CPIs, but also to other price level measures such as the producer price index or the import and export price indices.

Appendix A

First, this appendix describes the compilation process of the officially revised and published German CPI. The resulting index numbers were depicted in Figure 1 by the dark gray line labeled as "official" and the index numbers derived by the replication of that process by the dotted black line labeled as "replicated". Only afterwards, the compilation process that abstains from any retrospective revisions is presented. The resulting index numbers were represented in Figure 1 by the black line labeled as "raw".

When in 2003 the yearly expenditure weights of the year 2000 become available, Destatis calculates a series of monthly Laspeyres CPIs. The price and expenditure reference period of this series is the complete year 2000 (not January 2000!) and the first comparison period is January 2000:

$$P_{\mathrm{L}}^{00 \to t} = \sum_{i=1}^{N} s_i^{00} \frac{p_i^t}{p_i^{00}} , \qquad t = 1/00\,,\, 2/00\,, \ldots\,, \tag{17}$$

where $s_i^{00} = v_i^{00} / \sum_{j=1}^{N} v_j^{00}$ is the expenditure share of item i in 2000, with $v_i^{00} = p_i^{00} x_i^{00}$ denoting total expenditure on item i during the year 2000, x_i^{00} being the item's total consumed quantity, and p_i^{00} representing the item's average price.

Destatis compiles the yearly expenditure weights for every fifth year. When the expenditure information of the year 2005 becomes available, Destatis compiles a time series of Laspeyres indices with the year 2005 serving as price and expenditure reference period:

$$P_{\mathrm{L}}^{05 \to t} = \sum_{i=1}^{N} s_i^{05} \frac{p_i^t}{p_i^{05}} , \qquad t = 1/05\,,\, 2/05\,, \ldots\,. \tag{18}$$

The same procedure is applied when the expenditure weights of the year 2010 become available.

How are the three time series merged into one consistent time series of monthly price levels covering the interval January 2000 to December 2014? Consider the two time series compiled by formulas (17) and (18). Because in the year 2005 the expenditure weights of the expenditure reference period 2000 are outdated, the index number $P_{\mathrm{L}}^{00 \to 1/05}$ and all subsequent index numbers produced by formula (17) are likely to suffer from considerable substitution bias. Therefore, as soon as the expenditure shares of the year 2005 become available, Destatis replaces them by results obtained from formula (18).

Obviously, simple chaining of $P_{\mathrm{L}}^{00 \to 1/05}$ and $P_{\mathrm{L}}^{05 \to t}$ is not feasible because the comparison period of the first index differs from the price reference period of the second index. Therefore, chaining requires an additional chain link that compares the price level of the year 2005 to the price level of January 2005. To this end, Destatis uses a simple Paasche index with the year 2005 as comparison period and January 2005 as price reference period:

$$P_{\mathrm{P}}^{1/05 \to 05} = \left(\sum_{i=1}^{N} s_i^{05} \left(\frac{p_i^{05}}{p_i^{1/05}} \right)^{-1} \right)^{-1} = \frac{1}{P_{\mathrm{L}}^{05 \to 1/05}} \,. \tag{19}$$

Therefore, the chain index is

$$\tilde{P}^{00 \to t} = P_{\mathrm{L}}^{00 \to 1/05} \cdot P_{\mathrm{P}}^{1/05 \to 05} \cdot P_{\mathrm{L}}^{05 \to t} \,.$$

Applying this chaining principle to the comparison period January 2005 and exploiting the second equality of (19), yields

$$\tilde{P}^{00\to 1/05} = P_{\mathrm{L}}^{00\to 1/05} \cdot P_{\mathrm{P}}^{1/05\to 05} \cdot P_{\mathrm{L}}^{05\to 1/05} = P_{\mathrm{L}}^{00\to 1/05} \, .$$

This coincidence between the Laspeyres index $P_{\mathrm{L}}^{00\to 1/05}$ and the chain index $\tilde{P}^{00\to 1/05}$ confirms that the index numbers before and after January 2005 are consistently connected.

Once the expenditure weights of the year 2010 become available to Destatis, the new Laspeyres index numbers (that is, $P_{\mathrm{L}}^{10\to 1/05}, P_{\mathrm{L}}^{10\to 2/05}, \ldots$) must be linked to the index number $\tilde{P}^{00\to 1/10}$. This is achieved by the same approach as before:

$$\tilde{P}^{00\to t} = \tilde{P}^{00\to 1/10} \cdot P_{\mathrm{P}}^{1/10\to 10} \cdot P_{\mathrm{L}}^{10\to t} \, .$$

In sum, Destatis calculates the long-run time series for the time interval January 2000 to December 2014 in the following way:

$$t = 1/00\,, \ldots, 1/05: \qquad \tilde{P}^{00\to t} = P_{\mathrm{L}}^{00\to t} \, , \tag{20}$$

$$t = 1/05\,, \ldots, 1/10: \qquad \tilde{P}^{00\to t} = \tilde{P}^{00\to 1/05} \cdot P_{\mathrm{P}}^{1/05\to 05} \cdot P_{\mathrm{L}}^{05\to t} \, , \tag{21}$$

$$t = 1/10\,, \ldots, 12/14: \qquad \tilde{P}^{00\to t} = \tilde{P}^{00\to 1/10} \cdot P_{\mathrm{P}}^{1/10\to 10} \cdot P_{\mathrm{L}}^{10\to t} \, . \tag{22}$$

Expression (20) defines an index that covers a time span of 61 months (a five-year period plus the first month of the following five-year period). The same applies to expression (21), while expression (22), being the last five-year period, covers only 60 months. For the replication of the official CPI numbers of Destatis the same system of formulas is used.

Now, to estimate the price trend that would arise if no retrospective revisions were admissible (depicted in Figure 1 by the black line labeled as "raw"), the expenditure reference period precedes the price reference period by five years. Therefore, the system of equations (20) to (22) must be slightly adjusted:

$$\begin{aligned} t &= 1/00\,, \ldots, 1/05: & \tilde{P}^{00\to t} &= \tilde{P}^{00\to 1/00} \cdot P_{\mathrm{P}}^{1/00\to 95} \cdot P_{\mathrm{L}}^{95\to t} \, , \\ t &= 1/05\,, \ldots, 1/10: & \tilde{P}^{00\to t} &= \tilde{P}^{00\to 1/05} \cdot P_{\mathrm{P}}^{1/05\to 00} \cdot P_{\mathrm{L}}^{00\to t} \, , \\ t &= 1/10\,, \ldots, 12/14: & \tilde{P}^{00\to t} &= \tilde{P}^{00\to 1/10} \cdot P_{\mathrm{P}}^{1/10\to 05} \cdot P_{\mathrm{L}}^{05\to t} \, , \end{aligned} \tag{23}$$

where $\tilde{P}^{00\to1/00} = P_{\mathrm{L}}^{95\to1/00}/[(1/12)\sum_{r=1}^{12} P_{\mathrm{L}}^{95\to r/00}]$. Therefore, formula (23) simplifies to $\tilde{P}^{00\to t} = P_{\mathrm{L}}^{95\to t}/[(1/12)\sum_{r=1}^{12} P_{\mathrm{L}}^{95\to r/00}]$.

Appendix B

In Section 4, the correction approach is applied to compute the long-run time series of retrospective Walsh indices. The expenditure reference periods are not single months but complete years. Therefore, the weighting parameter λ_t in formula (28) must be modified. Instead of $\lambda_t = t/T$, one has to set $\lambda_t = 0$ for the first twelve months of the five-year period. These twelve months are indexed by $m = 1, \ldots, 12$. For the subsequent months of that five-year period up to and including the first month of the next five-year period ($m = 13, \ldots, 61$), the value of λ_t is defined by $\lambda_t = (m-12)/(61-12) = (m-12)/49$. For example, consider the first five-year period (2000-2005). January to December 2000 ($t = 1/00, \ldots, 12/00$) give $m = 1, \ldots, 12$ and $\lambda_{1/00} = \lambda_{2/00} = \ldots = \lambda_{12/00} = 0$. In January 2001 ($t = 1/01$) one gets $m = 13$ and $\lambda_{1/01} = 1/49$. Finally, in January 2005 ($t = 1/05$) the values are $m = 61$ and $\lambda_{1/05} = 49/49 = 1$.

The long-run time series of retrospective Walsh indices uses this weighting parameter, λ_t, and a modified version of the system of formulas (20) to (22). In this modified system, the Laspeyres indices $P_{\mathrm{L}}^{00\to t}$, $P_{\mathrm{L}}^{05\to t}$, and $P_{\mathrm{L}}^{10\to t}$ are replaced by the retrospective Walsh indices $P_{\text{W-co}}^{00\to t}$, $P_{\text{W-co}}^{05\to t}$, and $P_{\text{W-co}}^{10\to t}$, where "co" stands for "correction approach". The index $P_{\text{W-co}}^{00\to t}$ is defined by

$$P_{\text{W-co}}^{00\to t} = \left(P_{\mathrm{L}}^{00\to t}\right) \cdot \left(\frac{P_{\mathrm{W}}^{00\to 05}}{P_{\mathrm{L}}^{00\to 1/05} P_{\mathrm{P}}^{1/05\to 05}}\right)^{\lambda_t} , \qquad t = 1/00, \ldots, 1/05 ,$$

with

$$P_{\mathrm{W}}^{00\to 05} = \sum \frac{\sqrt{s_i^{00} s_i^{05} \left(p_i^{05}/p_i^{00}\right)^{-1}}}{\sum \sqrt{s_j^{00} s_j^{05} \left(p_j^{05}/p_j^{00}\right)^{-1}}} \frac{p_i^{05}}{p_i^{00}} .$$

For $t = 1/05$, this index gives $\lambda_{1/05} = 1$ and $P_{\text{W-co}}^{00\to 1/05} = P_{\mathrm{W}}^{00\to 05}/P_{\mathrm{P}}^{1/05\to 05}$. The indices $P_{\text{W-co}}^{05\to t}$ and $P_{\text{W-co}}^{10\to t}$ are defined analogously.

Then, the system for the time series of retrospective Walsh indices can be written in the following form:

$$t = 1/00\,, \ldots\,,\ 1/05: \qquad \tilde{P}^{00\to t}_{\text{W-co}} = P^{00\to t}_{\text{W-co}}\,, \tag{24}$$

$$t = 1/05\,, \ldots\,,\ 1/10: \qquad \tilde{P}^{00\to t}_{\text{W-co}} = \tilde{P}^{00\to 1/05}_{\text{W-co}} \cdot P^{1/05\to 05}_{\text{P}} \cdot P^{05\to t}_{\text{W-co}} = P^{00\to 05}_{\text{W}} \cdot P^{05\to t}_{\text{W-co}}\,, \tag{25}$$

$$t = 1/10\,, \ldots\,,\ 12/14: \qquad \tilde{P}^{00\to t}_{\text{W-co}} = \tilde{P}^{00\to 1/10}_{\text{W-co}} \cdot P^{1/05\to 05}_{\text{P}} \cdot P^{10\to t}_{\text{W-co}} = P^{00\to 05}_{\text{W}} \cdot P^{05\to 10}_{\text{W}} \cdot P^{10\to t}_{\text{W-co}}\,. \tag{26}$$

This system of formulas is consistent in the sense that, for January 2005 ($t = 1/05$), formulas (24) and (25) generate the same index number and, for January 2010 ($t = 1/10$), formulas (25) and (26) generate the same index number.

Appendix C

In Section 6, the imputation approach is applied to compute a series of retrospective Walsh indices with the price reference period January 2010 ($t = 1/10$) and comparison periods that begin in January 2010 and end in December 2014. For this purpose, the Walsh formula (5) is generalized in the following way:

$$P^{1/10\to t}_{\text{W-im}} = \sum \frac{\left(v_i^{10}\right)^{1-\lambda_t}\left(v_i^{15}(p_i^{10}/p_i^{15})\right)^{\lambda_t}}{\sum \left(v_j^{10}\right)^{1-\lambda_t}\left(v_j^{15}(p_j^{10}/p_j^{15})\right)^{\lambda_t}}\ \frac{p_i^t}{p_i^{1/10}}\,, \tag{27}$$

where "im" stands for "imputation approach" and λ_t is defined as in $P^{1/10\to t}_{\text{W-co}}$. Since the accessible data set contains the expenditure shares s_i^{10} and s_i^{15} instead of the expenditures v_i^{10} and v_i^{15}, the numerator and denominator of the first quotient in formula (27) is divided by the factor $\left[\left(\sum v_j^{10}\right)^{(1-\lambda_t)}\left(\sum v_j^{15}\right)^{\lambda_t}\right]$. As a result,

$$P^{1/10\to t}_{\text{W-im}} = \sum \frac{\left(s_i^{10}\right)^{1-\lambda_t}\left(s_i^{15}(p_i^{10}/p_i^{15})\right)^{\lambda_t}}{\sum \left(s_j^{10}\right)^{1-\lambda_t}\left(s_j^{15}(p_j^{10}/p_j^{15})\right)^{\lambda_t}}\ \frac{p_i^t}{p_i^{1/10}}\,. \tag{28}$$

The retrospective Walsh indices $P^{1/00\to t}_{\text{W-im}}$ and $P^{1/05\to t}_{\text{W-im}}$ are defined analogously.

Appendix D

Table 2: Laspeyres index numbers and Walsh index numbers for December 2004 ($P_{k,\mathrm{L}}$ and $P_{k,\mathrm{W}}$; January 2000 = 100), weights ($Z_{k,\mathrm{L}}$ and $Z_{k,\mathrm{W}}$; in percent), and contributions to the bias of the twelve major expenditure categories of the German CPI.

	Laspeyres		Walsh		Contrib.
Expenditure Category	$P_{k,\mathrm{L}}$	$Z_{k,\mathrm{L}}$	$P_{k,\mathrm{W}}$	$Z_{k,\mathrm{W}}$	to Bias
Food, non-alcoholic beverages	104.46	10.5	104.37	10.68	0.001
Alcoholic beverages, tobacco	127.82	3.73	126.76	3.55	0.088
Clothing, footwear	99.96	5.58	99.92	5.47	0.002
Housing, water, electricity, gas, other fuels	108.99	30.74	109.06	30.83	−0.030
Furniture, other household equipment	101.87	6.96	101.79	6.47	0.014
Health	124.02	3.60	123.52	3.58	0.024
Transport	110.72	14.08	110.52	13.53	0.086
Communication	85.33	2.56	84.81	3.19	0.109
Recreation, entertainment, culture	106.29	11.26	102.88	11.86	0.366
Education	120.75	0.17	120.75	0.18	−0.001
Restaurant, accommodation services	113.5	4.73	113.75	4.47	0.024
Miscellaneous goods and services	107.77	6.09	107.65	6.19	0.000
Total (Jan. 2000 - Dec. 2004)	108.25	100.00	107.57	100.00	0.683

Table 3: Laspeyres index numbers and Walsh index numbers for December 2009 ($P_{k,\mathrm{L}}$ and $P_{k,\mathrm{W}}$; January 2005 = 100), weights ($Z_{k,\mathrm{L}}$ and $Z_{k,\mathrm{W}}$; in percent), and contributions to the bias of the twelve major expenditure categories of the German CPI.

	Laspeyres		Walsh		Contrib.
Expenditure Category	$P_{k,\mathrm{L}}$	$Z_{k,\mathrm{L}}$	$P_{k,\mathrm{W}}$	$Z_{k,\mathrm{W}}$	to Bias
Food, non-alcoholic beverages	110.3	10.54	110.28	10.43	0.014
Alcoholic beverages, tobacco	114.3	3.96	114.07	3.80	0.032
Clothing, footwear	105.63	4.97	105.63	4.80	0.009
Housing, water, electricity, gas, other fuels	110.37	31.34	110.16	31.59	0.041
Furniture, other household equipment	104.37	5.69	104.23	5.45	0.018
Health	104.90	4.10	104.78	4.38	−0.008
Transport	113.47	13.42	113.54	13.26	0.013
Communication	87.99	3.15	86.69	3.50	0.087
Recreation, entertainment, culture	106.25	11.77	103.67	12.06	0.293
Education	227.00	0.20	227.00	0.20	0.010
Restaurant, accommodation services	114.73	4.48	114.21	4.40	0.034
Miscellaneous goods and services	109.12	6.38	108.99	6.14	0.030
Total (Jan. 2005 - Dec. 2009)	109.30	100.00	108.72	100.00	0.573

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