Aristotle's geometrical model of distributive justice

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Abstract: In the *Nicomachean Ethics,* Aristotle discusses "Distributive justice in accordance with geometrical proportion." (Book V, Ch.3). This suggests that Aristotle used a geometrical model in this context. But the original drawings did not survive and the exact nature of the corresponding model is much disputed. Aristotle claims *(ibid)*: "awards should be 'according to merit'; for all men agree that what is just in distribution must be according to merit in some sense". His basic analytical problem is thus how to treat justice in the context of unequal distribution.

The present article briefly establishes Aristotle’s argumentative context of “distributive justice” *vs.* “corrective justice”. Then a “positive” geometrical model is presented, describing Aristotelian concepts under the assumption of *given* levels of relative merit. It is then shown that once distribution can be seen as following Aristotle’s *positive* principle of respecting merit, there is a subsequent question implied in this model: how far should society go in permitting differentiation according to merit. This question arises because differentiation by merit *justifies* differentiation of distribution of material goods and thus differentiation in the possibility to satisfy the respective needs of the citizen. The paper shows that even if the more 'meritorious' class could dictatorially determine the size of its share in distribution by defining merit in such a way that they gain as much as status and goods as they want, self-interest should keep this class from excessive use of such a power. There is a level of privileges after which further inequality would hurt that class's own interest since their own level of well-being would decrease. Thus our model supplies a utilitarian rationale for Aristotelian praise for self-restraint.

An appendix elaborates the algebra involved in the geometrical model and briefly contrasts the present interpretation with some alternative comments concerning Aristotle’s model of distributive justice.

Keywords: justice, inequality, Aristotle, exchange

“Greece founded geometry. It was a mad undertaking:
we are still arguing about the *possibility* of such a folly.”
Paul Valéry, “The Crisis of the Mind” (1919)

“Have you read the *Ethics* of that superb Aristotle? ...
There never was such good sense talked – before or since.”
John Maynard Keynes (Skidelsky 1983 p.167)

“The genius of ancient Greek geometry not only stands the test of time
(Pythagoras's theorem is as valid now as when it was first proved); its discoveries
can suddenly find new applications in the 21st century.” (Economist 2008)

1 Introduction

There is much reverence for Aristotle's writings on economics, but also much confusion about its significance. Scott **Meikle** (1995, p.1) - in a very circumspect book on *Aristotle's economics thought* - describes the situation in the following way:

> The influence of Aristotle's economic writing has been incalculably great.. .It is usually held to be the first analytical contribution to economics, and histories of economic thought usually begin with it... and most schools of modern economic thought have had claims of Aristotelian paternity made on their behalf, including Jevonian utility theory, mathematical economics, neo-classical economics, and Marxism.
>
> The main texts are NE, 5. 5[2] and Pol., 1. 8-10.[3] The interpretation of them is now in a chaotic state.

As **Theocarakis** (2006, p.10) recently pointed out, one of the main problems for economists is that in the relevant passages Aristotle does not really deal primarily with economics but rather with justice and society - or, as the titles of Aristotle’s respective works plainly state: with Ethics and with Politics. But Aristotle's passages in question are full of references to economically highly charged terms like: "money", "exchange", "need" - he even alludes to a geometrical model of exchange. It is therefore not surprising that modern economists do believe that such passages might be of relevance for the history of their subject. But there is surprisingly much dispute among modern readers about the context in which Aristotle’s economic terms have to be received in order to make sense of them.

In the present paper I intend to develop an extended analytical context in which Aristotle's much disputed geometrical model might be seen. His model refers expressly just to “distributive justice”. Thus the context of permissible interpretations might appear to be very limited – far to much so as to warrant the wide ranging and seemingly contradictory claims which Scott **Meikle** (1995, p.1) mentioned above. It will be seen, however, that all the seemingly contradictory

2 *Nicomachean Ethics,* book V. See fn.4 for further details

3 *Politica,* reference not relevant for the present context.

catchwords mentioned by Meikle in that context – mathematical economics, utility theory, labor values, neo-classical marginalism – appear below in a unified framework, although in a way quite different from the previous Aristotelian literature.

2 Aristotle's concept of distributive justice

In the *Nicomachean Ethics,* Book V, Aristotle starts out by discerning two types of justice: "general justice" which is concerned with respecting laws, and "specific" or "particular" justice which is concerned with what we may call "fairness" in some sense, the sense depending on the particular situation under consideration. In his subsequent text, it is aspects of the latter type of justice which Aristotle mainly investigates. Within this "specific" or "particular" type of justice he makes a further distinction: distributive *versus* corrective (some translate: rectificatory) justice. The latter "supplies a corrective principle in private transactions [1131a 1]" [4]. This wording suggests that corrective justice is meant to be understood normatively, that the legal system is expected to correct a situation in which a particular type of unfairness is diagnosed.

Under the heading of corrective justice we have to discern two sub-cases, "corresponding to the two classes of private transactions, those which are *voluntary* and those which are *involuntary*" (*ibid.,* emphasis added). Examples of voluntary transactions which have to be treated under the heading of corrective justice are "selling, buying, lending at interest, pledging, lending without interest, depositing, letting for hire" *(ibid.).* The aim of corrective justice is to re-establish equality in a particular sense between two individuals. Its algebraic measure is the average between two extremes. An example for a relevant situation in an "involuntary transaction" is theft. In such a case one party has involuntarily "too little" – the former owner has his lawful possessions *minus* the object stolen. The other party, the thief, has too much, namely the object taken unlawfully. Corrective justice re-establishes an initial situation. In cases of this type of justice the standard of reference is the initial distribution of material goods between individuals – but there are also cases where immaterial goods have to be considered under this heading. In any such case, Aristotle's idea seems to be that the judge or the analyzing philosopher has to register whether there are deviations from the lawful standard. If that is the case, then corrective justice is applicable. In the example of theft just considered, the situation may be "corrected" by taking the stolen good from the thief and restoring it to the rightful owner, thereby re-establishing the "normal" lawful situation.[5]

[4] This quote after **Rackham** (1994). The bracketed numbers refer to the respective frames in the internet edition. Further Latin numbering in some of the following quotes relate the original subdivisions of the quoted text.

[5] Some commentators see particular problems in Aristotle's arithmetic in this context, see: **Judson** (1997; p.151): "In the case of rectificatory justice, Aristotle says that an *arithmetical* proportion is involved: that is, one in which

Aristotle's discussion of corrective justice might appear as very limited in perspective. The type of correction which he proposes concerns only the reversal of a misallocation of goods, not the change of the underlying misbehavior of persons.[6] In his parlance, such misbehavior requires a different type of corrective action, namely "forcible correction". Aristotle mentions this other type of correction very briefly (1132b30) with reference to a case in which a citizen aught to be punished for hitting a policeman doing his duty. In the Athenian society at the time of Aristotle's lecturing, such "forcible correction" surely was also applied in cases of theft, robbery, fraud, rape, and many other cases which Aristotle mentions only under the heading of "corrective justice". But "forcible correction" of misbehavior is not part of the taxonomy to which Aristotle wants to get in this context. This aspect of "correction" is not further discussed in the passages here under consideration. An explanation for this strange omission of an important aspect of justice might be that Aristotle just made a clean cut at the beginning of Book V by distinguishing "general justice" from "particular justice". The former deals with general law-abiding or law-offending behavior. In the passages following this distinction, Aristotle concentrates on "particular justice" and this obviously leaves out considerations of "forcible correction". Thus it is not that corrective justice is unimportant. It is just in a different drawer and this drawer is not under consideration in the present context.

There is a further taxonomic problem in dealing with "specific justice". Some commentators believe that in these passages Aristotle introduces a third – but unnamed by him – type of specific justice. Paraphrasing that reasoning we observe that he begins chapter 5 of book V where the case of the beaten policeman occurs, by stating: "Some people think that it is in fact the reciprocal that is unqualifiedly just" (**Rowe and Broadie** 2002, p.165)[7] But Aristotle continues: "However, the [unqualified !, GMA] reciprocal does not fit the case either of the just in the distributive sense or of that in the rectificatory (corrective, GMA) …". Thus Aristotle mentions simple 'unqualified reciprocal justice' and stresses that it fits neither of his two categories of particular justice. Must it therefore not be clear that there has to be a third category of particular justice which takes up this unnamed and unfitting case?

This question suggests that the unfitting case is one which must be accommodated for, that it is a relevant one. But Aristotle criticizes the belief in such a seemingly new type of simple

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the terms on each side of the equation are added to or subtracted from each other.... (The sense in which this is true is by no means straightforward; but I cannot pursue this issue here.)" Contrary to this quoted view, we considered the additions and subtractions involved as straightforward: If individual *B* steals from individual *A* the amount of *x* coins, then rectificatory justice should mean –*x* coins for *B* and +*x* coins for *A*. The average change of coins between *B* and *A* is zero and the initial lawful distribution of coins among the two individuals is re-established.

[6] Spengler (1955, 383) put it quite succinctly "Rectificatory justice… is that of the civil court, not that of the criminal court…"

'unqualified reciprocal justice' as being wrong and as missing the essential point: "For it is reciprocal action *governed by proportion* that keeps the city together" (*ibid*, my emphasis). So, one may well read his discussion as saying: 'Forget about simple *unqualified* reciprocity! But if you think of the seemingly new case of reciprocity as being governed by *proportionality* which "keeps the city together", then it is indeed an extremely important type of justice. But then it falls in fact under the old heading of distributive justice because this type of proportionality is the main characteristic of distributive justice'. This issue of proportionality will be elaborated below in greater detail.

The taxonomic problem just mentioned, namely whether there is a third category of justice which Aristotle might have implied but not named, might seem to be trivial. It is important, however, for the economic understanding of his writings because some writers infer a specific economic sense in this context. The last quote, clarifying the difference between the unfitting, because simple and unqualified, reciprocity and the fitting, because proportional, reciprocity is preceded by Aristotle's statement: "In commercial associations, however, the parties are bound together by a form of the just that is like this, i.e. what is reciprocal in *proportional* terms…" (*ibid*, my emphasis). In our reading, the sense of this passage is quite clear: by the logic just used, this form is an important and clear case of distributive justice because it is ruled by reciprocal proportionality. But this sentence stands near Aristotle's passages about what some commentators consider to be a "third case of justice". It is therefore maybe not surprising that some readers claim that this passage gives an indication that by mentioning commercial association in this place, Aristotle intended to propose *this* case as an additional, third, category of justice, namely "justice in exchange".[8] But since Aristotle insists in this context that this type of reciprocity is "in proportional terms" we stick with his previous counting that there are just two main cases of particular justice. Other cases are treated by Aristotle as being either irrelevant or trivial or they might be discussed under the heading of general justice which is a different topic altogether.

Following Aristotle, we discussed the term "distributive justice" until now just with regard to its taxonomic distinctions. We must now get to more substantial aspects covered by this term. It is introduced in chapter 3 of Book V, NE. The substantive context there is the discussion of a state of society in which there is no quarrel among its members about the distribution of goods and benefits. This state of affairs depends on respecting the social status of the potential contestants and on their receiving the "appropriate" share in whatever their society can offer (**Rackham** 1994; frame [1131a1]; compare **Rowe** and **Broadie** 2002, p.162, [1131a20-1131a30]):

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[7] The corresponding internet frame in **Reckham's** 1994 translation is [1132b 1]

[8] **Judson** (1997; p.149): "We should not, therefore, try to see justice in exchange as a subspecies of either distributive or rectificatory justice, but as a distinct, third form of justice." He follows in this Spengler (1955, 382). and Soudek (1952, 49).

> ... if the persons are not equal, they will not have equal shares; it is when equals possess or are allotted unequal shares, or persons not equal shares, that quarrels and complaints arise.
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> III.[7] This is also clear from the principle of 'assignment by desert [merit, GMA].' All are agreed that justice in distributions must be based on desert of some sort, although they do not all mean the same sort of desert; democrats make the criterion free birth; those of oligarchical sympathies wealth, or in other cases birth; upholders of aristocracy make it virtue.
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> III.[8] Justice [in this (!) context, i.e. in distribution, GMA] is therefore a sort of proportion; for proportion is not a property of numerical quantity only, but of quantity in general, proportion being equality of ratios, and involving four terms at least.

Proportionality thus is the essence of distributive justice and it is merit – some translate: "desert" – which is the basis of this proportionality. It is therefore important to know what determines merit.

Aristotle claims in this quote that he has no own idea of the determinants of merit in a specific society (see section III.[7] in the quote). He rather relates that there are quite different conceptions of this entity, depending on the society under consideration. Hence one may conclude that he describes here certain principles which hold when value systems are given and *fixed.* In a later section of this article we will ask about the characteristics of distributive justice when relative merit is *not* fixed. But in the context of this quote we may conclude that this type of justice is more of a positive description and not so much one of normative proposals, whereas the aforementioned "corrective justice" by its very name has a normative connotation to it.

Since we interpreted "specific" or "particular" justice in terms of fairness at the beginning of this section, we might return to this characterization in trying to bring out the specificity of "distributive justice". In terms of "fairness" one could address two sets of issues: a) It is unfair to diminish your neighbor of goods which he/she obtained legally in the given distributive context of a given society. The system of justice of that society must correct this unfairness. b) What about the original distributive system of society? Was it really fair to begin with? If the answer in this context is "no", then the guardians of justice (judges, philosophers) might have a problem with the fairness under a). Aristotle himself does not pose the question quite in this way. He never suggests, for example, that theft might be justified in a situation where material goods are distributed in an unjust way to begin with. His concern with distributive justice is more general, that it should "keep society together".

In real life the above issues a) and b) are intertwined. But they must be separated analytically, if they obey different principles. The principle ruling corrective justice seems to be straightforward: return the object to the rightful owner. It is predominantly with distributive justice that Aristotle's readers have problems.

In discussing distributive justice, Aristotle makes some seemingly strange quantitative statements, like the following one (**Rackham** 1994; frame [1131b 1]):

> III. [10] Thus the just also involves four terms at least, and the ratio between the first pair of terms is the same as that between the second pair. For the two lines representing the persons and shares are similarly divided.

At the end of this quote the translator H. **Rackham** (1994) adds a footnote stating “Here was another diagram ...”. But since none of Aristotle's original diagrams has come to our time, there is much guessing among posterity about the exact – or even the approximate – meaning of such a statement and about the shape of Aristotle's original diagrams.

One thing is clear from the text, however, although the original diagrams are missing. Although the quoted passages are in part concerned with equals, the specific interest in distributive justice concerns dealings among members of society which are *not* equal. In accordance with the guiding question just stated, the principles of distributive justice are thought to bring order into such inequality. Thus, if we consider a band of robbers, we may observe: If robber "A" is of superior status in the robber band in comparison to robber "B" so that A>B, then their common spoils - say a sack of gold coins - is *not* divided into two heaps "C" (going to A) and "D" (going to B) just so that A-s share is “somehow” larger than B-s share (C > D). Aristotle claims we can be more exact than that: there will be a "just" distribution among the two robbers so that C:D has the same value as A:B . Since he mentions “lines representing the persons and shares” as just quoted, Aristotle suggested that the status of the persons should be translated into respective lengths of lines and the shares which are to be distributed are to be represented by corresponding lengths of lines or line segments.

Aristotle does not discuss how this distributive outcome is achieved in real life through interaction among the parties concerned. Indeed, he wants to find a static state in which there is *no* interaction in form of quarrel, as quoted above. We take this as a further indication that this particular type of reflection about justice is of a descriptive, positive, nature and thus differs from the case of corrective justice. The important thing about distributive justice is that once valuation of the members of a society is given with regard to their relative worth, distributive justice comes into play by some geometrical model which, however, is described rudimentarily but rather unsatisfactorily in the surviving text.

Table 1 below gives an overview of some aspects of the taxonomy discussed in this section.

Aristotelian Particular Justice			
Distributive Justice “diagonal conjunction” positive		**Corrective Justice** “average formation” normative	
unequals physician / farmer	equals farmer / shoemaker	voluntary buying ; selling	involuntary robbery ; adultery

Table 1: Distributive and corrective justice

It is remarkable that in this scheme buying and selling and all the other economic activities of an exchange economy do *not* fall under the heading of distributive justice, although Aristotle's model of exchange is an important part of distributive justice as was seen above. There is thus a seeming contradiction: economic exchange through buying, selling, lending, renting, etc. is the essence of exchange but by express statement of Aristotle himself these activities are not the subject matter of Aristotle's *model* of exchange in the context of distributive justice. This should alert the reader that it is possibly not just conventional economic exchange which is at issue when Aristotle debates distributive justice. We mentioned above that we do not subscribe to the view that Aristotle introduced a third category of particular justice. It is here our aim to see how far we can get in understanding his intended message on the basis of what has actually survived of his writings over the millennia. There have been losses from the original lectures or manuscripts – e.g. the already mentioned illustrations – and there are certainly some corruptions of the text. If that is the case, the scholars who want to reconstruct Aristotle’s intended meaning should not so much be glued to single words or sentences. They should rather try to reconstruct the paradigms which he tried to convey. In our view, the ideal result of interpreting Aristotle anew would be to re-create the formal model of distributive justice which he obviously did present in his lectures but which did not survive. An important requirement for the acceptability of such a re-invented model would be that it does in fact cover the paradigmatic substance of Aristotle’s surviving passages.

3 Aristotle's paradigms of distributive context

Instances where distributive justice is of relevance are robbery, as seen above, and the distribution of spoils of war among the victorious warriors. But these are exotic examples which Aristotle mentions just briefly for occasional simple illustrations of a rather complex principle. A unifying characteristic of all the examples is that they pertain to common enterprises which presuppose an action by a group. But what is a *relevant* group once we step beyond the company of robbers and conquerors?

Some readers of Aristotle occasionally believe that for his analytical purposes a paradigmatically relevant group is a business partnership (e.g. Judson 1997, 150, n 10 – similarly, but not as detailed: p.168,n.28 , 174, n.53):

> In the case of both distributive justice and justice in exchange[9] Aristotle appeals to geometrical proportions relating the two parties *(A* and *B)* and the things they *receive* (R^A and R^B respectively). Aristotle speaks a little loosely here, since strictly speaking it is always some *feature* of the parties which is involved in the proportion – e.g. how much money each has contributed to a business partnership (1131b29-31)· To mark this I shall use 'F^{A}' and 'F^{B}'. Distributive justice involves a standard geometrical proportion: that is, the ratios of the shares that it is just for *A* and *B* to receive is equal to the ratio of the amounts of feature *F* which *A* and *B* possess (so if *A* contributed twice as much money to the partnership as *B* did, she ought—*ceteris paribus*—to receive twice as much of the profits). As an equation, this is:
>
> $$F^A/F^B = R^A/R^B$$

As a principle for distributing profits the message of this quote seems to be indeed totally fair: if A and B are business partners and *A* put in 1000$, *B* put in 100$, then after a year net profits of 110$ should be divided up as 100:10 or ten to one since *A* put in ten times as much as *B*. If both put in equal amounts, they should split 55:55 or one to one etc. But we may ask whether such calculations are really Aristotle's main concern. Is this type of problem really a challenging one for an Aristotelian mind? These are rhetorical questions, however, because it will be seen below that in fact the paradigm with which we are presented by Aristotle is far more complex than this.

The substantive reasons for not accepting this case as being a paradigmatic one can be appreciated only after we have gone into more details of distributive justice below. But before that we can deal here already with some philological grounds for questioning that the example just discussed should be taken as representative for Aristotle's intentions. The interpretation of the above quote is based on a rather isolated passage which – in the translation by **Rowe** and **Broadie** (2002 , 163) of [1131b29-31] – has some words mentioning "the just in the distribution of things belonging to the community" and which deals with "public funds". Since the ancient city-state of Athens did have public enterprises which were commercially exploited, e.g. the famous silver mines of Laurion, a commercial connection might indeed be interpreted into this context. But in these passages it is by no means obvious that private investors are of main interest. The quoted catch-words "public funds", "things belonging to the community" point to a different direction,

[9] Concerning this supposedly "third justice" – "justice in exchange" – see the previous footnote and our text in that context. One could make the point that the case discussed in the present quote is not identical with Aristotle's geometrical proportionality in the sense to be discussed later in this text. The latter is constrained in a particular way whereas the proportionality here discussed is unconstrained. It could be 1:1 as well as 1000000:1, depending on the capital shares in the business. But it is not plausible to call this a case of "exchange". It is a case of distributing profits plain and simple. So if one wants to give it a proper name involving the term justice, call it "justice in the sharing of capitalists' profits". But there is no textual evidence that Aristotle wanted to make a special case with such a name. The more general case of his distributive justice goes far beyond any such unnamed special case as

namely some aspects of communal activity. In addition, the textual context of this quote is chapter 4 of Book V of *NE*. This chapter is meant by Aristotle to clarify the “remaining form” of *rectificatory* or corrective justice as he states at its beginning. Therefore distributive justice is only a side issue in this context and we should not look for its paradigmatic treatment in a chapter dealing with a different subject. The main ideas of exchange under conditions of *distributive* justice are developed in the preceding chapter 3 where this concept is presented and from where we quoted already above in our section 2. After Aristotle’s two hitherto mentioned chapters 3 and 4 dealt with definitions and distinctions of the two variants of particular justice, the subsequent chapter 5 of Book V offers an in-depth discussion of the more challenging aspects of the concept of distributive justice. It is in this chapter which we may name: “distributive justice further explained” that we should seek the most extensive clarification. In this latter context we will read now that Aristotle refers to shoemakers, builders, farmers, physicians and their products but not to profits of business investors. The paradigm offered in this context is quite different from mere profit sharing and far more challenging intellectually. Indeed, some readers doubt whether the text is understandable at all.[10]

Central passages of Aristotle’s original discussion of exchange and distributive justice in chapter 5, Book V read (**Rackham** 1994 frame [1133a 1]; compare e.g. **Rowe** and **Broadie** 2002, p.165, [1133a5-1133a10] for a slightly different wording of the translation.)

> [8] … Now proportionate requital is effected by diagonal conjunction. For example, let A be a builder, B a shoemaker, C a house, and D a shoe. It is required that the builder shall receive from the shoemaker a portion of the product of his labor, and give him a portion of the product of his own [labor ! GMA]. Now if proportionate equality between the products be first established, and then reciprocation take place, the requirement indicated will have been achieved; but if this is not done, the bargain is not equal, and intercourse does not continue….
>
> [9] … For an association for interchange of services is not formed between two physicians, but between a physician and a farmer, and generally between persons who are different, and who may be unequal...
>
> [10]…As therefore a builder is to a shoemaker, so must such and such a number of shoes be to a house.

Theocarakis (2006, p. 15) commented the last phrase in this quote that it “has created many problems of interpretation. This is the prerequisite for exchange and for association”. Let this expression of exasperation stand for a myriad similar ones by previous commentators. But there seems to be agreement: “This is the heart of justice in exchange” as **Judson** (1997, 161) commented this passage. So it is *here* where we have to search for the Aristotelian paradigm of distributive justice, although for many readers this search was often an extremely frustrating

will be demonstrated presently.

[10] Cf. **Theocarakis** (2004, p.7) who refers to some of Aristotle’s exercises in accounting in this chapter and then reports: “For **Rothbard** (1995, 16) this passage is “a prime example of descent into gibberish” and “this particular

experience.[11]

4 Aristotle's concept of need

In section 2 we quoted Aristotle's dictum that "…it is reciprocal action governed by proportion that keeps the city together". In section 3 we then elaborated that his "proportionate requital" by "diagonal action" appears as being the essence of this doctrine. It is one of the many puzzles of Aristotle's doctrine of distributive justice that as soon as he made these claims he proceeded to claim that society is held together not just by distributive justice but also by "need" (*chreia*)[12]:

> Everything, in that case, must be measured by some one thing, as was said before. In truth this one thing is need, which holds everything together, for if people did not need things, or if they do not need them to the same extent, then either there will be no exchange, or the exchange will be a different one.

By itself the idea expressed in these lines is clear: everybody experiences need for things. That need is satisfied through exchange of things. In so far as a particular society (polis) can enable its members to pursue successfully their satisfaction of need, "everything" will be held together. Remarkable in this quote is the use of the word "thing": once in singular ("one thing") then in a plural sense (people "need things"). Need is treated by Aristotle as a measuring rod, as "one thing" by which measurement takes place – or rather: which stands behind measurement. His statement implies that there must be some sort of transformation of a multitude of things - shoes, houses, etc. - into this "one thing". There is no satisfactory suggestion in Aristotle how that should happen. He does take money as proxy for need in the singular. The text in a number of passages, which for brevity's sake we abstain from quoting here, is quite clear about that. The underlying idea seems to be: if one person spends 10 drachmae on 3 pairs of shoes and 1000 drachmae on one house, then total spending is $10 \times 3 + 1000 \times 1 = 1030$ drachmae. If a different person can spend only 515 drachmae during the same time, then the observer of such a comparison might comment that the former person is in a position to satisfy his need better than

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exercise should be dismissed as an unfortunate example of Pythagorean quantophrenia"."

[11] **Judson** (1996; p.147 n.1) gives an interesting collection of other commentators' unhappy comments to this particular passage. His section III (ibid., p.160-168) gives an in-depth critical overview of a number of interpretations. We agree with his critical comments about the previous literature but we offer an interpretation which is totally different from the one he subsequently offers.

[12] This quote from NE, 1133a25 uses the translation by **Rowe** and **Broadie** (2002, p.166). This translation differs from that of Rackham by using the term "need" instead of "demand". **Meikle** (1991; p.161 n.6) explains quite well the problem involved in the choice of words. "… I have left *chreia* (need) in place of the translation "demand" which, together with "supply," is now a theory-laden term carrying a weight of suggestion that cannot be attributed to a Greek author. The use of "demand" might also suggest that ways might be found for attributing to Aristotle a modern subjective theory of value. He held nothing of the kind …" All of this is quite reasonable. But one should not go to the extreme of thinking that Aristotle used this term without any analytical intentions of relevance for us moderns. In the above text it is argued that Aristotle mentioned the satisfaction of need because his model implicitly contains a measure of the standard of life based on "need". See also Judson (1997; p.159) "*chreia* does not mean 'demand', either in a technical economic sense or in the sense of

the latter. But Aristotle insists that this comparison in terms of money values is only a very indirect evaluation of need satisfaction. The advantage of the money expression is that it is just one sum (1030 or 515 in our example) whereas to the left of the equation sign above we have several products (shoes and houses in this case, but many more goods under every-day circumstances). It is simpler to compare the one term on the right side of the equation sign for different people than to compare the many terms on the left-hand side for different people.

In the parlance of modern linear algebra one could say that money value is a single scalar value which is based on a vector of a multitude of goods and a vector of a multitude of respective prices of goods. It should go without much further comment that the elements in each of these two vectors are conceptually and dimensionally totally different. The *quantity* of one apple, element of the goods vector, can be eaten or thrown around or used for cider-making, etc.. A *price* for one apple, element of the other vector, can be registered in a statistic, can be moaned about because it is considered as being far too high etc.. The elements of each of these two vectors are in principle many – their combination is one. In other words: the multiplication of these two conceptually and practically totally different vectors containing a multitude of elements gives a single scalar value, namely *one single* money value of the collection of goods and prices under consideration. It is a severe conceptual problem that such a scalar value combines into a single number such totally different elements: goods, which do enter our satisfaction of specific needs or frivolities, and prices which are of no direct relevance for well being. Indirectly, they are of course of relevance because high prices mean that the purchasing power of a budget given in terms of money is lower than it would be if prices were low. Money values are thus inherently complex with regard to welfare measurement. They are explicitly addressed by Aristotle - but as just stated, for him they are only a proxy for (the satisfaction of) “need” which is the “true” measuring rod of relevance for the economic community which he discusses in the context of distributive justice. We therefore must return once more to the question: how do collections of goods satisfying a multitude of specific *needs* (in the plural !) translate into the satisfaction of “need” as a single measure of well-being comparable to – or rather: imperfectly represented by – a single money value?

Still today, the problem which lies behind such a question is an important, relevant and not satisfactorily solved one. Indeed, in modern social accounting it is an old debate that money values – or even mathematically constructed index values – of national product are by no means a reliable indicator of the satisfaction of need in the respective countries. Therefore there are many suggestions to use more or less complicated alternative indicators of social well-being which do not

'desire': it means need.”

rely just on money values and prices.[13]

A further approach to the problem of translating goods baskets at people's disposal into welfare indicators is to use "utility functions". Modern economists are accustomed to transforming specific goods into "utils" via utility functions, say, of "CES"-type or of "Cobb-Douglas"-type. For later reference we may state a typical specimen of the latter type. It might read for example

$$U_A = \sqrt[3]{C_A}\left(\sqrt[3]{D_A}\right)^2 = (C_A)^{1/3}(D_A)^{2/3} \qquad (0)$$

where U_A measures the level of satisfaction ("utility") for individual A and where C_A and D_A are quantities of two goods which are fit for generating A-s satisfaction. In constructing such functions the modern economists are, of course, not restricted to just two goods. Thus, here too, we have the mental possibility that a multitude of quantities of very diverse goods is translated into one single unit, the "*U*" as dependent variable of the utility function of equ.(0). Such "modern" functions are required to have certain "plausible" characteristics like "decreasing returns" in the sense that with respect to variations of the inputs C_A or D_A they have first derivatives that are positive and second derivatives that are negative etc. . This is why they have an algebraic appearance like the one of our specimen. It is, of course, totally anachronistic to bring Aristotle in connection with such functions. But if he did have a formal model in which the satisfaction of need played some role, one might well ask about the analogous characteristics of an Aristotelian variant of modeling the satisfaction of need.

The sad fact is, however, that in trying to represent Aristotle's *single* measuring rod "need" we are left clueless by his written text. Nevertheless, since in the context of his debate of distributive justice Aristotle does go into some detail about the "truth" of this measure, I suppose one can well have the feeling that some aspects of this "measuring rod" should appear somewhere in his geometrical model of exchange. We will return to this issue once we have familiarized ourselves with the relevant model.

5 The geometrical model of equal exchange

The above passage [1133a 1] [8] about "proportionate requital" is translated by **Grant** (1973, p. 118) as: "Now the joining of the diagonal of a square gives us proportionate return". For the translation of Aristotle's argument into its intended geometrical version this might be considered as being more helpful than the former version, since it stresses that a "square" is the starting point of the exposition.[14] **Jackson** (1973, p.95) criticizes this translation on philological grounds. But on

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13 The seminal paper in this line of research was the one by **Nordhaus and Tobin** (1971). The authors there make an attempt to replace GDP-measurement of welfare by a new MEW – "Measure of Economic Welfare".

14 Compare **Theocarakis** (2004; p.4) "The standard figure that is present in most old editions and commentaries of NE is a square and its diagonals...". There is some debate whether the letters used by Aristotle signify points or lines. It would go too far to document that debate here anew. We support here those views which attribute lines and line segments to Aristotle's letters. The reference to some sort of construction using diagonals is also preserved in the translation by **Judson** (1997; p.160): "What produces proportionate exchange is *diagonal linkage*." (his emphasis).

argumentative grounds it can indeed be said that a geometrical construction is needed which can re-create the proportionality doctrine which was so much emphasized by Aristotle in this context.

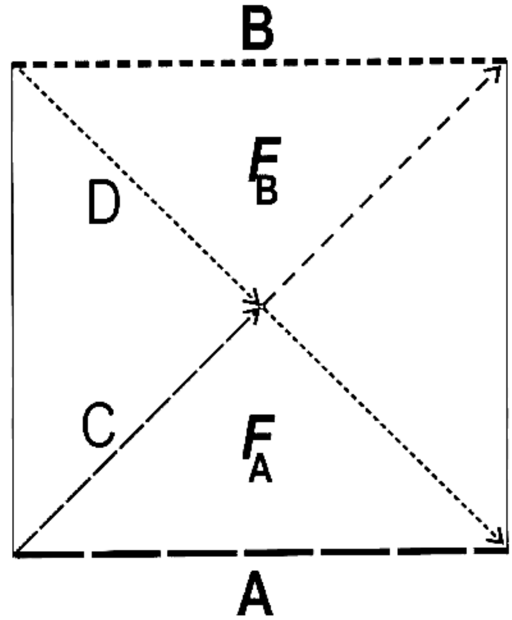

Figure 1 : Exchange among equals

When person A resp. person B are of equal status, then proportionality becomes equality, of course. This means that their respective yearly produce C resp. D must be evenly divided so that A gives to B half of his yearly product C/2 and receives half of B-s yearly product D/2. With the remaining quantity C/2, person A can satisfy his need for this product. Measure the satisfaction of need resulting from good C-consumption by individual A by the area of a triangle with basis C/2 and the other sides being A/2 and the perpendicular line running from the intersection of C and D to line A. Analogously measure the satisfaction of need given by the consumption of D/2 by the analogous triangular area above the basis D/2 with the other half A/2 and the perpendicular as above. Taking the two partial expressions of the satisfaction of need together, we get the area F_A as a measure of the satisfaction of need which is experienced by person A. Analogously, F_B is a measure of the satisfaction of need which is experienced by person B.

In interpreting Aristotle's doctrine of distributive justice, we have not only reproduced a variant of what some authors consider to be the "standard figure that is present in most old editions and

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Jaffé (1974; p.385) discusses the appropriate figure as well and quotes the translation by D.P. Case of this passage which reads "Now acts of mutual giving in due proportion may be represented by the diameters of a parallelogram, ….". This, too, is in agreement with the interpretation given below since a square is valid only in the case of equality. According to some definitions of geometrical figures, the trapezoids of our figures 2 and 3 are indeed parallelograms since they have two parallel sides.

commentaries of NE"[15]. We went slightly beyond that standard procedure and we claimed that in that “standard figure” there is contained a measure of the satisfaction of need.

6 The case of unequal exchange

The representation of the case of equal proportions is rather banal. It does not contain the interesting case of dealing with determining the material shares going to members of society in a case of inequality. But if it comes to unequal proportions, there are a number of challenges. Aristotle [1133a 1] addresses the problem that the products exchanged might be of unequal "worth":

> [8]... For it may happen that the product of one of the parties is worth more than that of the other, and in that case therefore they have to be equalized.
>
> [9] ... For an association for interchange of services is not formed between two physicians, but between a physician and a farmer, and generally between persons who are different, and who may be unequal, though in that case they have to be equalized.

Thus we have here two types of inequality: one between the worth of products, the other between the worth of persons. But the different worth of persons is the very root of the theory of distributive justice. Therefore the equalization mentioned in Aristotle’s text cannot mean to make persons equal. Equalization must refer to making goods equal in some sense. But in which way should equalization take place? What is the analytical problem which Aristotle might have had in mind?
It seems to be clear that the basic idea of the model is that a part of one year's product of person A should exchange against some part of one year's product of person B. The equality involved in this analysis is the time of production, say, one year. The obvious inequality is the payment for the different products or services. It seems that Aristotle proposed to “equalize” in such a way that it is clear that the analytical time involved is the same for any member of society wo worked there for, say, one year. The difference in "worth" as far as the goods are concerned should therefore be attributed to the persons compared, while their working time is treated as equal. C and D should therefore be equal for persons A and B - no matter whether they are equals or not equals.

[15] See the previous footnote

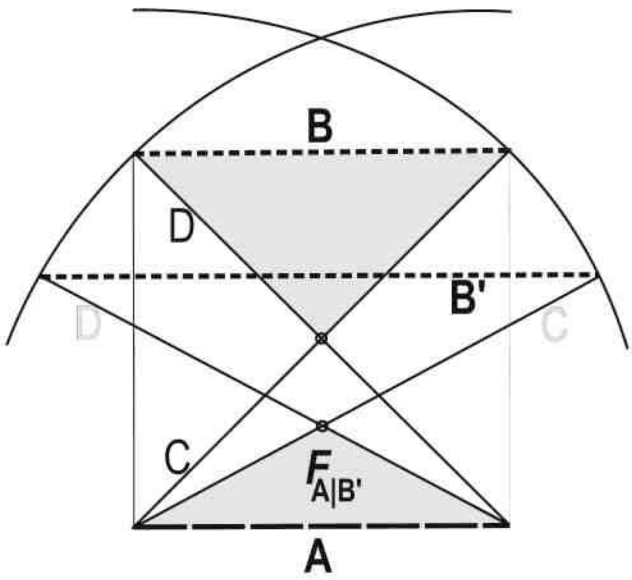

Figure 2: Equalization and exchange

Now suppose that we have a new case in which subject B is replaced by subject $B^{'}$ > B. Therefore $B^{'}$> A. The diagonals C and C light resp. D and D light have the same length since they measure the time of a - say - yearly production, which is one year in each case. They are thus contained in two respective circles as partly drawn in fig.2. But since B' >B, the distributive shares cannot be represented as before. The square of fig.l changes now to the trapezoid of fig.3. The diagonals are now divided unevenly. This signifies: Denote two sections of the diagonal marked with C light in fig.3 with C_A and C_B, C_A being the side of the new triangle on the basis A. Do likewise for diagonal marked D light. C_A resp. D_A now measure the amount of the yearly product of C resting with A resp. the amount of yearly production of D received by A. Both measures are obviously smaller than C_B resp. D_B which are the sides of the isosceles triangle on the basis B'. This means: person A now can keep less of his own product C for his own satisfaction. In exchange for the very large amount C_B which he gives up to B' he now receives the small amount D_A in return. Thus he gives up much and receives little.

This is distributive justice according to Aristotle! This follows from the fact that

$$\frac{C_A}{A}=\frac{C_{B'}}{B'} \quad \text{resp.} \quad \frac{D_A}{A}=\frac{D_{B'}}{B'} \quad \text{or} \quad \frac{B'}{A}=\frac{C_{B'}}{C_A}=\frac{D_{B'}}{D_A} \qquad (1)$$

holds. These ratios express that products C and D are divided up according to the "worth" or merit of the persons involved. Geometrically, this type of proportional division follows from the fact that C and D are diagonals in a trapezoid with the two parallel lines A and B'.[16] Since these lines now describe triangles of unequal volume, namely F_B and $F_{A|B'}$ it follows that the two

[16] The proportionality characteristic stated in the text follows from the second intercept theorem (see, e.g., **Harris** and **Stocker** 1998, p.65).

persons experience quite different levels of “satisfaction of need”.

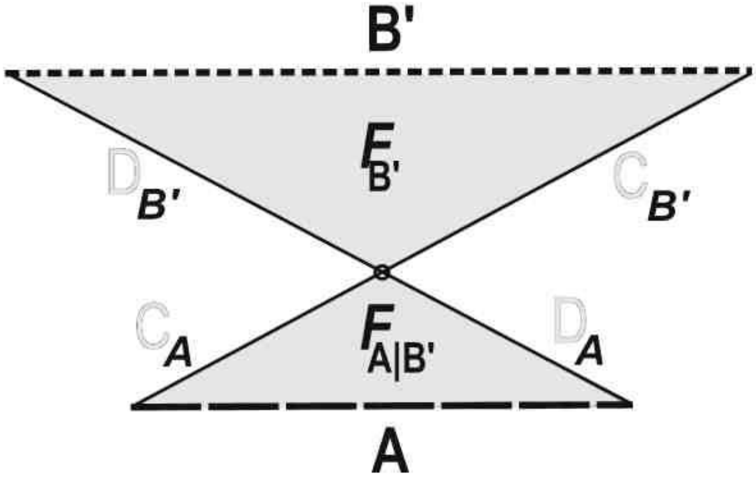

Figure 3: Distributive justice

Thus the triangles over the bases A and B resp. A and B' may be seen as indicative of the way in which the need of the two parties are covered. They are indicative of the standard of life which the persons enjoy. It comes, of course, not as a great surprise that that person who gets a larger proportion of the goods produced in a society during one year enjoys a higher standard of life. But since the area is also affected by the length of the base of the respective triangles, the implication is that the status also contributes as such to well being. The visual expression of these implications of our interpretation of distributive justice is the difference in the triangular areas $F_{B'}$ and $F_{A|B'}$.

7 Satisfaction of need in the Aristotelian model

We may now return to the above discussion of need in the Aristotelian context which we left for the exposition of his formal geometrical model of distributive justice. Let us re-state a rather obvious aspect of the present analysis: If distributive justice rules, then the “worthy” members of society get comparatively more of *all* the (two) goods produced in that society. But goods are used to satisfy specific needs. Therefore the worthy ones can satisfy all their needs better than the other ones. This is, *nota bene*, not a specific result of the present model but the essence of differences in remuneration. The privileged members of society can lead a materially better life. This is a fact of life. One of Aristotle’s specific questions in this context is: can this difference be quantified in a single measure of satisfied “need”? One could say: just look at the basket of goods which they have at their disposal (here: C_A and D_A for party A; $C_{B'}$ and $D_{B'}$ for party B’) and you see that the

B'-party has more. But Aristotle wants to have a measure of need as "one thing", as one single measuring rod for the implication of material differences. It was mentioned above that the reader might think that the obvious measurement of material difference is in terms of money value of the possessions of A and B' so that money values might become this measuring rod for Aristotle. There are some passages in his text which might be interpreted in such a way. But in the present context of his exchange model money values do not show up: goods C and D are measured so that differences in money values are "equalized out" as Aristotle explicitly states (for the method of doing this see the appendix below).

Could money values show up in some other context? They could be contained in the "worthiness" of citizen but for Aristotle the extent of this influence on worthiness depends on the society analyzed (Aristotle mentions "oligarchy" as a type of society where "wealth" is paramount in determining social worth, as quoted above). Since there are societies where this aspect is of comparatively little relevance (e.g. "aristocracy"), the money worth of individuals is no reliable measure for their societal worth. Here now *chreia* – need – comes in once more. What really should be measured when we want to compare the relative well-being of citizen is the alternative satisfaction of need. As stated above, for Aristotle the satisfaction of need *is* the ultimate measuring rod for the well-being of members of society. Holding money and spending it certainly has something to do with the satisfaction of need in the present and in the future, as Aristotle writes in this context. But it is an arbitrary measure for the more basic issue of "need". Money is just *nominal* – and Aristotle explicitly goes into the philology of this aspect of money. [1133b 1]

> [14]…Though therefore it is impossible for things so different to become commensurable in the strict sense, our demand furnishes a sufficiently accurate common measure for practical purposes.
>
> [15] There must therefore be some one standard, and this accepted by agreement (which is why it is called *nomisma*, customary currency); for such a standard makes all things commensurable, since all things can be measured by money.

But in the geometrical model the satisfaction of need cannot be measured by money as a proxy since money just does not appear there. If we want to represent the satisfaction of need in this model we require some "thing" different from money. We claimed that this new proxy is given by triangular areas in the Aristotelian square or trapezoid or parallelogram, depending on the case and on the definition which you have in mind. This is now a claim for which we have absolutely no direct textual basis. In a way, this is not surprising: since the once existing illustrations did not survive the ages. Therefore detailed aspects of these illustrations had even less chance to survive. The text tells us only that Aristotle insisted that money is "not real" in the above sense and that "need" is the one "thing" which is the true measuring rod.

The way of testing the appropriateness of our proposed interpretation is to investigate its implications and then to see whether such implications are in accordance with Aristotle's intended messages.

8 Aristotle and the protohistory of the Edgeworth Box

We began this article by quoting that "most schools of modern economic thought have had claims of Aristotelian paternity". An interesting case in point is neoclassical contract theory, exemplified by the so-called "Edgeworth Box".[17] This is a geometrical construction representing exchange between two individuals who have an initial basket of two goods with each good entering their respective utility functions (e.g. of equ.(0)-type). In this framework a "Contract Curve" is derived. That is a geometrical *locus* of equilibria of isolated barter between the two trading partners.

William Jaffé (1974, 382) claims:

> It has not been noticed … that over two millennia before the appearance of the *Mathematical Psychics* [by F.Y. Edgeworth, 1881, GMA] Aristotle had divined certain aspects of Edgeworth's Contract Curve theory…

Jaffé (1974, 391) concludes:

> …Aristotle's theory of exchange … must be accounted a clearly identifiable figure in the protohistory of the Edgeworth Contract Curve.

The unconvinced reader could comment such statements by saying that "tradition, like beauty, lies in the eyes of the beholder" and turn to other questions. But the problem here is that if we relate Aristotle too much to modern contract theory we loose sight of the specificity of his approach. He is *not* interested in isolated exchange as such but in the cohesiveness of society. Therefore it is not exchange itself which is in the center of Aristotle's analysis but the "just" distribution of goods between the members of society.

We want to make clear that our interpretation contradicts a number of other ones like that of Soudek p.46 who maintain that Aristotle

> . . .was preoccupied with the isolated exchange between individuals and not with the exchange of goods by many sellers and buyers competing with each other.

This is not the appropriate juxtaposition in our view. In Aristotle it is not the question of isolated *barter* vs. atomistic *market*. It is rather: *isolated* barter vs. *societal* exchange in a setting which aims at keeping the state together in the face of potentially challenged survival. In this latter context it will become clear that Aristotle was *not* interested in isolated barter and did not treat the process of barter as such. This point will be elaborated in the next section by showing that the geometrical model has some interesting prescriptive implications for society.

Let us briefly add, however, that in our view Joseph Spengler (1955, 386) comes nearer to Aristotle's concern when, in the context of contract curves, he claims:

> Had Aristotle had the concept of a contract locus at his disposal, he would probably have said that justice in exchange was to be found in the middle region of the locus, provided that this was compatible with receipt by the exchangers of rewards which correctly reflected the difference between their respective skills.

This statement is motivated by the fact that on such a contract curve there is a multitude of possible equilibria with each having quite different welfare implications for the participants of the exchange under consideration. In the quote just given, Spengler emphasizes the viewpoint of justice in the distribution of goods. He answers the question for justice by advocating a middle position in which it is presumed that welfare is divided up evenly between the partners. But the really interesting question is: what justifies *uneven* distribution of welfare and how far should such inequality go? It is in this field that Aristotle seems to have been beyond the standard analysis of modern economics. His model has important implications how far inequality can go and what would be the "best" inequality as will be shown in the next section.

9 Prescriptive aspects of the Aristotelian model

The quest for distributive justice has (at least) two aspects. Until now, we have treated only one of these: the question about the distribution of goods and welfare in a society in which relative worth was treated as given and where the rules of distributive justice were obeyed. But these rules are variable. Aristotle was quoted above as having been well aware that in different societies the members have different "worth" and therefore also different "just" shares obeying Aristotelian "proportionate requital". The next question is now: if the relative worth is variable, how will such variation affect the welfare of the different classes and of society as a whole? Let us remember that Aristotle repeatedly asked in this context about the cohesion of society, about the mutual satisfaction of need. With such a perspective in mind, the next question seems to be evident: are there arrangements of relative worth which lead to an "unjust" situation in the sense that the *polis* would suffer even if the rules of distributive justice are adhered to? An extreme case could be a

[17] For a demonstration of this analytical instrument and its history see Humphrey (1996)

tyrannical ruler who can dictate the laws of “his” society. Could he proclaim that he is worth a billion times the worth of the rest of society? Would distributive justice then require that he had the billion-fold share of the others?

It is an interesting feature of the present model that it excludes such a case as not being covered by distributive justice. In order to see this, turn to fig.2 and extend line B' more and more while leaving line A constant, i.e. extend the "worth" of B' very much in relation to A.

It is, of course, not possible to extend B' billion fold. Since C and D are "equalized" by Aristotle - say, to one year's production - it follows from fig.2 that B' must be contained in the two radiuses drawn. At the maximum B' can extend to the left of A by the line segment (D-A). To the right of A it can extend by the line segment (C-A). The total maximal length is then

$$B'_{max} = (D-A) + A + (C-A) = 2 - A \quad (2)$$

if we have “equalization” in the sense of C = D =1. But if B' has this maximal length, it lies exactly over A. Then the entire area between the two lines must be zero and hence the two triangular areas measuring satisfaction of need are also zero. In this case the measure of well-being for both A *and* B' is zero. An index value of zero for the own standard of life can not be in the interest of the supposedly privileged class or person, however. Even if the privileged ones were not interested in the total welfare of society, they cannot be interested to drive the situation to such an extreme that the own wellbeing would become zero. Thus equ.(2) and the associated argumentation gives a “natural” limit to social differentiation. But this upper limit has such an appallingly bad welfare implication that it is not a realistic value.

This result does not mean that this model implies that it is futile to strive for higher honors or privileges. Quite to the contrary. We did see from fig.3 that increasing the worth of B to that of a slightly larger B' does indeed generate a larger "area of wellbeing" for that person ($F_{B'}$) at the expense of the wellbeing of the other person A. Thus it is clear from these considerations: if the 'worth' of B' becomes more and more unequal to that of A, the standard of life of the privileged first increases, then decreases and eventually reaches zero.

What has just been stated follows from plain graphical discussion. These are results of elementary geometric manipulations which can be produced by anybody who can use the intercept theorem in order to illustrate distributive justice. It is quite plausible that Aristotle did use this theorem and that he used it in some sort of quadrangular construction in which “equalization” of the value of goods played a particular role. If Aristotle did have in his mind constructions like the present figures 1 to 3 when he wrote about distributive justice, then he might also have been aware that there are two quite different types of proportionality. Proportionality in the quadrangular case is a proportionality constrained by equ.(2). Proportionality in the sense of our section 3 above is a totally unconstrained proportionality. The claim made in the context of section 3 was that Aristotle meant that business profits should be shared according to the formula $F^A/F^B = R^A/R^B$. If this type of proportionality rules, then it is possible to say that if $F^A =$ 1 billion$\times F^B$ and hence R^A should be 1 billion$\times R^B$. But in the context of our geometrical argumentation such a claim would have been utter madness, as equ.(2) shows.

The essential difference between these two types of proportionality is, of course, not a manner of expression but an implicit assumption. Our geometrical model uses an implicit assumption of equality – that the trading partners use and share through exchange one yearly production each. This "equalization" which was indeed explicitly demanded by Aristotle for the purpose of analyzing distributive justice precludes the type of difference which the algebraic proportionality permitted in bringing a $-billionaire and a 1-$ pauper into a common business venture.

In the context of distributive justice the citizen get different shares in spite of the fact that they each interchange on the basis of one years' work. Because of their differing status they participate in different ways in their common pool of yearly production. With these remarks one is reminded of some passages in Adam Smith's *Wealth of Nations* – that it is not gold and jewels which constitutes the wealth of a nation but the yearly production of goods and services; that men live better than dogs because men contribute to a "common pool" from which they profit through exchange contracts and dogs cannot establish exchange and communality etc. But this is one of the many possible links between Aristotle and more modern economics which it would be too distracting to follow up in the present context.

Here we should rather return to the equivalence of algebraic and geometric expressions of distributive justice. In spite of this equivalence the algebraic mode of expression sometimes has the advantage of making quantitative relations more explicit. For this purpose consider fig.4.[18]

Define the ratio of B' to A as

$$B'/A \equiv \alpha \qquad \text{with} \qquad \alpha_{max} = 2\sqrt{2} - 1 \tag{3}$$

The value of α_{max} in equ.(3) follows directly from equ.(2) after elementary manipulations when $A=\sqrt{1/2}$ (see below, fig.5 and equ.(4) in the appendix). When $\alpha = 1$ we have equality of worth and hence the case of fig.l with the respective indicators of needs satisfaction or well-being given by the areas F_A and F_B. When $\alpha > 1$ we have the case of fig.3 with $B' > A$. The associated areas of well-being are now $F_{B'} > F_B$ and $F_{A|B'} < F_A$. We express now these alternative areas as functions of α. Write $f(\alpha)$ for alternative values of $F_{B'}$ and $g(\alpha)$ for alternative

[18] I gratefully acknowledge that I owe the drawing of fig.4 to Prof. Jürgen Müller, mathematician at Trier University.

values of $F_{A|B'}$. Letting α increase from the values 1 - equality - to increasing values of inequality, the functions just defined have the shapes as depicted in fig.4. From this figure we see that the "second class" person A will certainly be reduced in his standard of living by increased status levels and thus increased privileges of the "first class" citizen B', since the value of $g(\alpha)$ is continuously decreasing the more the relative "worth" of B' increases. But the converse does not hold. There is no continuous increase in the standard of living experienced by B' if he can increase his status continuously. There is a level beyond which a further increase of B'-s privileges as expressed by the value of α is detrimental for the standard of life of B' *himself* as appears from looking at the shape of function $f(\alpha)$.

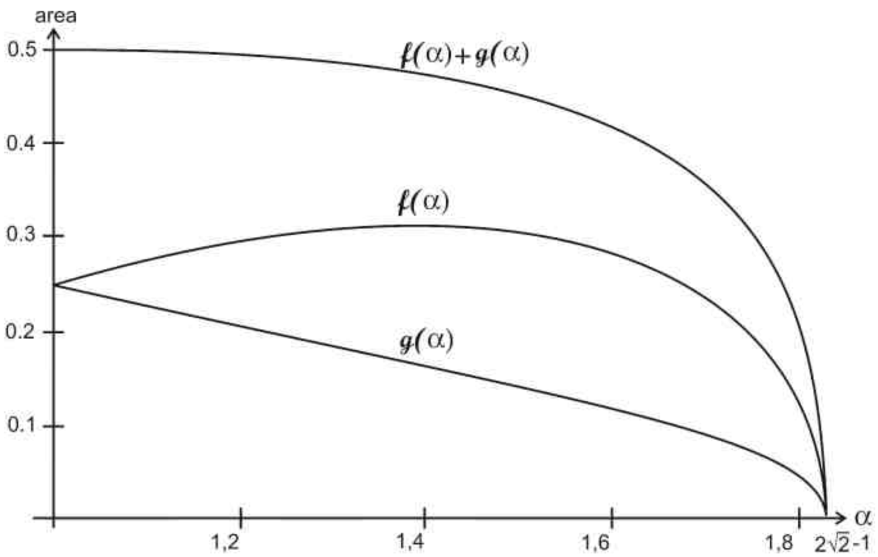

Figure 4: Well-being and relative worth (α)

A number of further observations come to mind when regarding fig.4. It is remarkable that total welfare as given by $f(\alpha) + g(\alpha)$ is not seriously affected for moderate changes in status. But when the B'-class overdoes its upward valuation, rapid decline of total welfare results. This decline sets in because there are diminishing returns to upward status manipulation for class B'.

It would be, of course, totally anachronistic to attribute marginal analysis to Aristotle. But the mere geometrical manipulation of the trapezoidal model of distributive justice of fig.3 shows that the total area covered by this figure would eventually decrease dramatically when the length - the worth of - B' is continually increased. The limiting case given by equ.(2) is obvious. It would be astonishing if an author so keen on geometrical argumentation as Aristotle would not have engaged in some reflections about this aspect of his model. With these possible implications in mind, one may wonder whether Aristotle did not use this (type of) model in order to give his philosophical advice about the "appropriate" application of distributive justice. But I must admit that I find little direct evidence in the passages here under discussion that Aristotle in fact engaged in such advice.

One might also ask in this context: why does society permit significant differences in the status of its members? We saw above that Aristotle professed to be agnostic about the reasons for different valuations of different members of society - apart from saying that there are different societies. There just are these differences and in a first round of argumentation one might be contented to ask about their implications.

As soon as one departs from this rather limited scope of enquiry, many complications are conceivable, e.g.: could it not be that particular members of society have a high status because their presence in society has high positive external effects for the society as a whole? Thus, a good doctor who can save lives is not only important in case of individual illness but also for the preservation of human capital in a society. Likewise, good professors or philosophers might be esteemed as generators of superior human capital among their listeners. If they left the community because they were honored more and had a more comfortable life abroad, this might be a loss which could be avoided by offering them sufficient amenities so that they stay. But all of these considerations go beyond the scope of this enquiry which was concerned with finding a model of distributive justice which can be considered as covering the basic ideas which Aristotle formulated in this context.

10 Some wider perspectives

For a fuller assessment of Aristotle's geometrical model of exchange it might be helpful to consider it under a perspective which is somewhat wider than the purely formal reconstruction. A rather broad new perspective would mean to place the formalisms of this model into the context of philosophical argumentation at the time of Aristotle's authorship. A narrower perspective might lead us to ask for the immediate interconnections of this model with specific overarching issues of Aristotelian thinking. We will briefly cover a sample of each of these two in the following section.

Concerning the topicality of "geometrical equality" at the time when Aristotle authored the *Nicomachean Ethic*, there have been some indications among commentators (**Gordon** 2007, 121) that this was indeed a well established topic for his contemporaries. In the dialogue *Gorgias*, Aristotle's teacher Plato traces this topic to his own teacher Socrates in the following passage (**Lamb** 1967, 508a):

> **Socrates**: … gods and men are held together by communion and friendship, by orderliness, temperance, and justice; and that is the reason, my friend, why they call the whole of this world by the name of order, not of disorder or dissoluteness. Now

> you, as it seems to me, do not give proper attention to this, for all your cleverness, but have failed to observe the great power of geometrical equality amongst both gods and men: you hold that self-advantage is what one ought to practice, because you neglect geometry.

Thus probably for at least two philosophical generations before Aristotle the basic idea of the present model – "geometrical equality" – was treated as a "great power", as a prerequisite for a proper understanding of an orderly life in society. The quote considers this concept as a challenge even for a man characterized by "cleverness".

If such wide-ranging claims could have plausibly been made by Platon's Socrates for the study of geometrical equality, its contemporary discussion cannot have consisted in just looking at the crossing of some lines or in writing down a banal division of some letters. Especially if the Socrates of this dialogue claimed that the knowledge of geometric justice would restrain the student from an excessive belief in self-advantage, there might well have been some additional pedagogical elements in this construction which went beyond merely demonstrating proportionality. The geometrical model referred to by Socrates in this quote must have stood in some substantial connection to what he described as the characteristics of a stable society: orderliness and temperance. How could such concepts enter the minds of people looking at a square or at a trapezoid as constructed above?

In the last section we tried to convey a few important implication of the geometrical model: one is the destructiveness of unbridled strife for self-advantage through privileges, the other is the idea of an optimum level in privileges. The reader might object: the ancients had no idea of optimization because they had no differential calculus. This mathematical technique was indeed invented only in the 17th century by Isaac Newton and by G.F. Leibnitz. Therefore the ancients could not have solved optimization problems by means of differential calculus. To such criticism it can be answered: even if the modern instruments of differential calculus were not available to Socrates, Plato, and Aristotle, the ancient thinkers cannot have overlooked the general properties of the model of geometrical proportionality in an isosceles trapezoid. Although it was not possible for them to derive algebraically the maximum of the $f(\alpha)$-function of the last section, mere visual observation of the underlying geometrical model shows that such a maximum *must* exist somewhere between the extreme right and extreme left positions, between self-annihilation through an excessive strife for self-advantage on the one side and total egalitarianism on the other side. If an algebraic solution to such an observation was not available, this does not mean that the search for some solution was not a constant challenge.

The main pedagogical point of studying the $F_{B'}$-area as indicator of well-being of the privileged class was that beyond an optimum level, there should be restraint in claiming privileges. Its important message is then: self-restraint is good for *enlightened* self-advantage. It seems that this maxim goes well with what Socrates wanted to convey in the above quote.

The observation that the $F_{B'}$-area must have a maximum between the extreme right and the extreme left leads to a further topic which was of great importance for Plato and then again for Aristotle: the doctrine of the mean. Oates (1936) gives a thorough treatment to this topic. As far as Aristotle is concerned, he stresses (ibid,. p.389) that "at certain points, for example when he expounds his definition of virtue, the mean becomes under his hand an hypostatization or an entity, or, so to speak, a mathematical or geometrical location". In his interpretation, this "location" is an apex. Oates (1936, 390) actually draws a diagram in which the mean is not just the middle, but the apex in a semicircle, in other words: the mean is a maximum. He stresses in that context (*ibid.*) that we should note: "That the mean is of the nature of an extreme." In a footnote he gives textual evidence for that view, quoting *Nic. Eth.* 1107a and adding: "Failure to take it into account has led to a general misunderstanding of the ethical doctrine. Living according to the mean connotes to many people, in my experience, going through life 'carefully avoiding both good and evil', as some wit has put it." This paraphrase was, of course, meant ironically and should stress once more the extreme value characteristic of the Aristotelian mean. In Rowe and Broadie **(**2002, 117, 1107a1-1107a10) the quoted passage reads:

> Excellence, then, is … determined by rational prescription… And it is inter-mediacy between two bad states, one involving excess, the other involving deficiency; … Hence excellence, in terms of its essence ... is intermediacy, but in terms what is best, and good practice, it is extremity.

Oates (1936, 391n) remarks that similar to himself the Nicolai Hartmann school of ethics has virtually the same geometrical model of an extreme value on a semicircle when it interprets Aristotle's doctrine of the mean but that they have the wrong philosophical reasons for this.

We cannot here go into the details of that debate. But its existence is of interest because it shows some consensus concerning the extreme value aspect of Aristotle's doctrine of the mean, even if there was controversy concerning the ontological and axiological aspects of that doctrine.

We are here concerned with the formal and in particular with the geometrical aspects of Aristotle's model of distributive justice. We have noted some of its extreme value characteristics. Since justice is one of the virtues – if not the highest one – the apex approach just discussed should fit well into our debate of distributive justice. It links it to the concept of a mean in the Aristotelian sense. But in our case of fig.4, there are three apices: two on the far left – for total and for A-s well-being – and one in the approximate middle – for B'-s well-being.

Both of these positions seem to be eligible for Aristotle's "mean". On the far left we have a mean in the algebraic sense: both members of society receive the same quantities of goods, thus differences in distribution are evened out. It can be shown (appendix) that in this case – with equal distribution of goods given – the optimal distribution of merit is also one of equality from the individual point. This is a variant of Aristotle's dictum that equal shares should be given in conditions of equal merit. But the fact of live is that neither merit nor goods are equally distributed. If we look for a mean between the unrealistic and therefore probably unworkable total equality of distribution on the extreme left and the totally destructive extreme on the right side, we are led to consider the "middle" apex. A detailed choice theoretic analysis would show that this position is not just "somewhere in the middle" on the B-axis. It is also an equilibrium between positive and negative incentives. The positive ones come from the better satisfaction due to enhanced goods consumption which remuneration by merit makes possible if the meritorious status increases. The negative ones can be interpreted as burdens of the higher status. That there is such an element in Aristotle follows from the fact that for B'_{max} or α_{max} the area of well-being for B' is zero. Thus B'-s increase in status eventually must become detrimental for society *and* for himself. A comparable negative effect cannot come from enhanced goods consumption as such, however. This follows from regarding e.g. fig.1: The F_A or F_B area will definitely increase if the sides of the respective triangles are extended *cet.par.*. But when the bases of the respective triangles are increased *cet.par.*, then the areas of the triangles are first increased but eventually will be reduced to zero. The optimum is where the positive and the negative influences just balance. These influences can best be isolated with mathematical analysis (see appendix). But the substance of the argument may also follow from geometrical considerations.

Thus the search for extreme value solutions in the context of distributive justice seems to be well in accordance with Aristotle's writings on the mean as well as in accordance with a certain tradition of the secondary literature on "Aristotle's mean". The type of geometrical analysis proposed here opens the interpretation of Aristotle's writings to modern choice theoretic analysis. Although it must be clear that methods of differential calculus were in no way available to Aristotle, it should be equally clear that Aristotle did mention extreme positions as optimal ones in his analysis of the mean. We hope to have indicated successfully that such considerations can be well discussed also in terms of rather simple geometrical models the usage of which can be inferred from Aristotle's extant writings.

11 Concluding remarks

The graphical model here presented is an *interpretation* of Aristotle's writings. We cannot claim that it is the one and only "true" rendering of distributive justice. But we can safely stress that the model gives a comparatively simple interpretation for a number of utterances which have vexed and frustrated readers of Aristotle's *Nicomachean Ethics* for a long time. The puzzles we deal with here are in particular: (i) the proportionality relation of status and distribution of goods exchanged and (ii) the "equalization" of goods of unequal money value.

There cannot be any doubt that Aristotle did use graphical figures in his original presentation of his ideas. The modern translations and commentaries sometimes indicate the exact place in the text where they must have appeared in his original lectures.[19] The figures themselves are lost, however. But Aristotle's surviving text does describe a number of features which the present model is attempting to re-create.

Models, once stated, have a "life", i.e. a number of implications, of their own. Sometimes the creator of the model was not quite aware of all its implications. One particular implication which we stressed here was the idea that there is an "area of satisfaction of needs" which is associated with distributive justice once one starts to relate economic exchange to the satisfaction of needs. Although the idea of an *area* was not stated in the Aristotelian text, the line of thought which Aristotle certainly did follow in the passages here under review was that the satisfaction of needs is essential for societal cohesion and for exchange. Our model extends this idea in suggesting a measure for this satisfaction. This is only a slight aberration from the original text because Aristotle associated money-values with the satisfaction of needs. Thus he had a quantitative, measurable, conception of the satisfaction of needs, not just a qualitative philosophical one. Money is only a proxy for the satisfaction of needs. Aristotle mentions this in his text. We have an alternative proxy for the satisfaction of needs - the triangular area depicting the standard of living in the trapezoidal model. This area can be larger and smaller, depending of the relative merits of the actors of exchange. By itself this idea is almost obvious: if you get a lot through distributive justice you have a higher standard of living than if you got little.

Our model builds on this obvious observation but goes beyond the merely obvious. It gives a quantitative idea concerning the satisfaction of needs. Our approach is similar to the statement: "Person A has less money than person B'. Therefore A can satisfy his needs less well". Aristotle did take money as proxy for the satisfaction of needs. But since in his model of exchange money values do not appear - he expressly "equalizes out" any difference in money values of goods – one cannot take money as proxy for the satisfaction of needs. Therefore we used a geometrical measure for this purpose, namely the triangular area formed by the available goods and the worth of the person considered.

The present model suggests some self-restraint in manipulating higher honors for one's own advantage. One message of the model is: "Do not overdo societal differentiation, even if you have the upper hand." In this it follows one of Aristotle's basic tenets, namely the call for wise self-restraint for the sake of one's own good. Virtue lies in the middle of extremes. One extreme for attributing worth is total equality ($\alpha = 1$). The other extreme is $\alpha_{\max}$. Figure 4 shows that a wise value for α might be somewhere between these extremes ($\alpha \approx 1.4$)

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19 See, e.g., above our quote from **Rackham** (1994, frame 1131b 1) in section 2: “Here was another diagram…”.

Apart from showing that growing inequality eventually harms the very persons or groups who are the beneficiaries of the privileges, the model states that the best level of societal well-being would be achieved if there were equality of status among all. We did not linger on this probably obvious point because there was no textual basis for such a discussion. Aristotle just stated that there are differences in merits attributed to members of a society, depending on the constitution of the particular society. He seems to have been realistic enough to realize that the prescription of an egalitarian distribution would have been illusionary. The model is thus one of "second best". Its implication is: in an imperfect world do not strive for an extreme solution. But do try to give advice – or try to give models which convey the message - that "superior" people and classes should abstain from foolishly overplaying their hands. Although this is not quite the letter of Aristotle's passages on distributive justice, it seems to me to be the spirit of his philosophy. It is a message which some of us might like to convey to the "big shots" of our age as well. It is an important implication of Aristotle's model of distributive justice in the version here proposed. We therefore would not support the idea sometimes uttered in comments about Aristotle's doctrine of distributive justice that it is of interest just in the slave society of his time. As long as social differentiation does exist, this doctrine is of relevance for thinking about its material implication and for restraining excessive greed - not for the sake of the underprivileged ones but for the sake of the self-interest of the privileged ones.

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12 Appendix

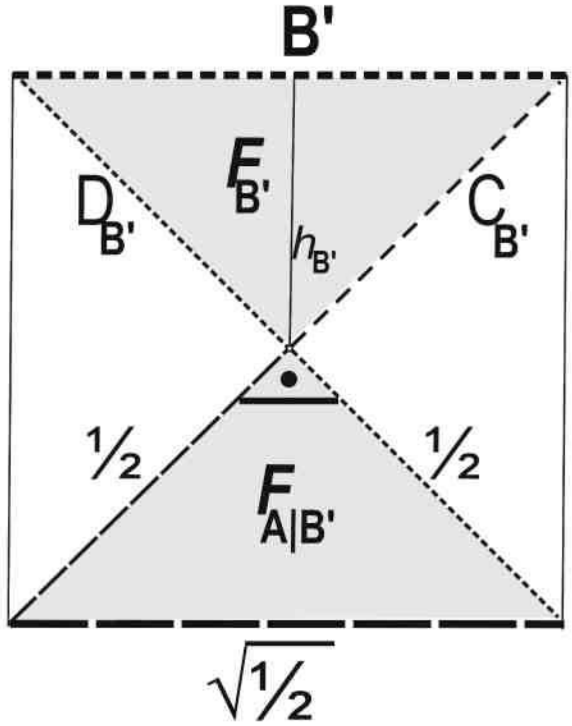

Figure 5: Quantitative relations in the Aristotelian model

Some precisions on equal exchange: Figure 5 should clarify a number of quantitative relations which are contained in our interpretation of the Aristotelian model. The lower half represents the case of equal exchange where α =1. The upper part has the more general symbols which stand for magnitudes in the case of unequal exchange where α >1.

When α =1, the exchange model generates a square, as in fig. 1. The diagonals of a square generate four right triangles. For us, the grey shaded lower and the upper ones are of interest. Since C and D always have unit value (see text), the sides of the triangles have value ½ in this particular case. According to the Pythagorean theorem base A has then the value

$$A = \sqrt{\left(\left(\tfrac{1}{2}\right)^2 + \left(\tfrac{1}{2}\right)^2\right)} = \sqrt{\tfrac{2}{4}} = \sqrt{\tfrac{1}{2}} \qquad (4)$$

as marked at the lower line of fig.5. This value of A is given and constant since the relative “worth” of B’ is with reference to a given worth of A. This constant reference value of worth can be stated numerically as shown by equ.(4).

Because of equ.(3) we have $B' = \alpha$ A. The upper line in fig.5 represents B=A for α =1 resp., in a more general notation, $B' = \alpha$ A for $\alpha \geq 1$.

Some algebra concerning unequal exchange: From equ.(l) and equ.(3) it is known that (5 a) holds. From the unit length condition for the diagonals it is known that (5 b) holds.

$$\text{a)}\ C_{B'} = \frac{B'}{A} C_A = \alpha C_A \qquad \text{b)}\ C_A + C_{B'} = C = 1 \tag{5}$$

Therefore we can eliminate $C_A = 1 - C_{B'}$ in equ.(5 a), thus obtaining

$$C_{B'} = \frac{\alpha}{1+\alpha} \quad \text{with}\ C'_{B'} > 0\ , \qquad C''_{B'} < 0 \tag{6}$$

where $C'_{B'}$, is the first derivative with respect to α and $C''_{B'}$ is the second derivative. Thus the model depicts decreasing returns of goods – as represented by line segment $C_{B'}$ – to augmenting the status of B', i.e. its relative length.

Our measure for B'-s standard of life is given by the triangular area

$$F_{B'} = h_{B'} \frac{B'}{2} \quad \text{where} \quad h_{B'} = \sqrt{\left((C_{B'})^2 - \left(B'/2 \right)^2 \right)} \tag{7}$$

We just saw that B'-s share of C-goods, $C_{B'}$, and B' relative worth may be expressed in terms of α . Making the respective replacements in equ.(7) and rearranging terms gives

$$F_{B'} = \sqrt{\left(\left(\frac{\alpha}{1+\alpha} \right)^2 - \left(\frac{\alpha}{2} \sqrt{1/2} \right)^2 \right)} \times \left(\frac{\alpha}{2} \sqrt{1/2} \right) = \sqrt{\left(\frac{1}{(1+\alpha)^2} - \frac{1}{8} \right)} \times \left(\frac{\alpha^2}{2} \sqrt{1/2} \right) = f(\alpha) \tag{8}$$

This function is depicted in fig.4. It has a maximum at α =1.382975767, in other words: near $\alpha \approx 1.4$. The derivative of equ.(8) generating this result is a rather involved expression which will not be reproduced here. It was shown in the text, however, that the existence of such a maximum is plausible: Since area $F_{B'}$ first is equal to F_A when α =1, then increases along with α and eventually reaches zero at $\alpha_{\max} = 2\sqrt{2} - 1$, there should be a maximum between α =1 and $\alpha_{\max}$. When $\alpha = 1$, equ.(8) gives the value of ¼ = 0.25, the left starting point of the $f(\alpha)$ and $g(\alpha)$-functions of figure 4.

A procedure analogous to the one which gave the $f(\alpha)$-function just discussed gives the $g(\alpha)$-function corresponding to the respective curve in figure 4. The area of well-being of subject A is given by:

$$F_{A|B'} = h_{A|B'}\frac{A}{2} \quad \text{where} \quad h_{A|B'} = \sqrt{(C_A)^2 - (A/2)^2} \qquad (9)$$

(In fig.5 the reader should replace "1/2" on the left side of the lower triangle by symbol C_A, the square root $\sqrt{1/2}$ at the base by A. Draw also the height $h_{A|B'}$ in the lower triangle in analogy to h_B, in the upper triangle). From equ.(5 b) and equ.(6) we know that

$$C_A = 1 - C_{B'} = \frac{1}{1+\alpha} \qquad (10)$$

holds. We also know from the above that A = $\sqrt{1/2}$. We therefore get for equ.(9) the new expression in terms of α :

$$F_{A|B'} = \sqrt{\left(\frac{1}{1+\alpha}\right)^2 - \left(\frac{1}{2}\sqrt{1/2}\right)^2} \times \left(\frac{1}{2}\sqrt{1/2}\right) = g(\alpha) \qquad (11)$$

This function is monotonously falling when α increases from the unit value to $\alpha_{\max}$, as shown in fig.4.

The Aristotelian equalization of money values: One of the puzzles of Aristotle's writings on exchange is that he claims that in the case of unequal money values of goods, the money values should be "equalized" before the analysis of his model is set in motion. It is not clear how this should happen. Who should do the correction and how should it be done? Is corrective justice involved or no justice at all?

We propose the following interpretation: Suppose the money value of good C is VC drachmae and the money value of good D is VD drachmae. If now the money value of B'-s product D is relatively high so that VD > VC holds, it might be expected that this implies that party B has accordingly relatively high yearly earnings of EB drachmae in comparison to A whose good is sold for less money value. Denote A-s yearly earnings with EA < EB. Apply now the formula for distributive justice so that

$$\frac{EB}{EA} = \frac{VD}{VC} \quad \text{whence} \quad \frac{VC}{EA} = \frac{VD}{EB} \quad \text{and} \qquad C = D = 1 \qquad (12)$$

as will be explained in the following: The left-hand side equation says: money values in exchange are proportional to yearly incomes of the producers of the respective goods. The middle equation results from a trivial manipulation of the first equation. The right hand equations then claim the equality of the ensuing measure of the two goods C and D.

The reasoning behind this line of thought is: the money value of a stock of goods has the dimension [*money*]. The earnings have the dimension [*money*]/[*year*]. The middle equation in equ.(12) reads therefore in terms of dimensions

$$\frac{[money]}{[money]/[year]} = \frac{[money]}{[money]/[year]} \quad \text{whence} \quad [year] = [year] \qquad (13)$$

The manipulation thus transforms money values into year-"values". By choosing appropriate money values of earnings EA and EB, the required equality may be established. The equalization involved is thus an accounting measure which translates differences in money values into differences in earnings while ensuring that the "year-values" are equal - say, of unit value.[20]

An example can show that the underlying idea is very elementary: suppose that the net value of carpenter B-s beds (net of all input costs except of B-s labor) is 1200 drachmae (=VD). The net value of shoemaker A-s shoes is 1000 drachmae (=VC). Obviously, the money value of good C and the money value of good D are not equal. “Equalization” requires a statement of the money worth of individuals A and B. The net values of the products are the incomes of B and A from one year of their respective efforts, thus they are their yearly incomes EB=1200 and EA= 1000, respectively. Applying equ.(12), middle term, we thus have:

$$\frac{1000(=VC)}{1000(=EA)} = \frac{1200(=VD)}{1200(=EB)} \quad ; \quad 1 = 1 \qquad (14)$$

The result of this manipulation is, of course, that the economic analyst of an exchange economy focuses on differences in income. In income terms we have, e.g., $EB = 1.2 \times EA$ meaning: “in terms of yearly income, carpenter B is 1.2 times the worth of shoemaker A”. This is, of course, what we would say today as well: If Mrs. Smith can sell her products with a net value twice as high as Mr. Miller can sell his products, then she is money-wise more worth, as far as income from the sale of goods is concerned. Such a comparison is no problem in the Aristotelian context since differences in "value" of persons are a starting point of that very analysis. Thus we have here a further element for differentiating between parties of an exchange economy: their relative worth might not only be based on merit or on esteem in ancient Greek societies. Social differentiation might also be based on money earnings as well. This is totally in keeping with the gist of the argument of the model: the larger (richer) party B' is relative to party A, the better is their standard of living – at least within reasonable limits of social differentiation. The relative size could be determined by esteem of the personality due to “virtue”, as in an aristocratic society. But Aristotle mentions explicitly the possibility that “wealth”, and thus the size of yearly earnings, might be a legitimate measure for differentiating among members of the community, the latter case belonging to “oligarchy”. As stated in the text, Aristotle seems to be very eclectic about the

[20] This is contradicted by Soudek (1952, 60): “ `Labor’ in the abstract, the labor-time of Political Economy, is alien to Aristotle’s thinking; he never mentions it or implies it in his deductions.” In fact, labor-time is the very basis of his “equalization”.

determinants of relative merit. But money income might well be included in his list.

There is a further indication that Aristotle saw a connection between personal income and personal worth in the passage [1133b1] (Broadie and Rowe 2002, 166):

> But one should not introduce them [the money values of the products, GMA] as terms in a figure of proportion when they are already making the exchange (since otherwise one of the two terms at the extremes will have both of the excess amounts).

In our reading this means: do not take just the money worth of the trading partners (EA and EB) and of their products (VC and VD) for making the "figure of proportion" – i.e. the trapezoid. Because then, in the above example, EB = 1.2×EA *and* VD = 1.2×VC. If you now would construct something like the above figure 3, then not only the upper side of the trapezoid would be 1.2 times of the lower side, but the D-diagonal which should have the value 1 would now have the value 1.2. The trapezoid of fig.3 would be distorted so that the upper left corner now protrudes upwards. Not only would the construction be ruined but also a distribution according to diagonal segments would give B' an unfair advantage. B' would not only profit from "his" side being longer than A-s but also from having "his" D-diagonal longer than the C-diagonal. The rich one would profit in such a construction from both "excesses" – from excess worth and from an excess diagonal. Therefore do not use such a "figure of proportion". Equalize the diagonals first as described by equ.(14) above. Only after that you may construct the figure of proportion. If this is a fair reading of the quoted passage, then one of the "excess amounts" would be the money worth of B'. Since "equalization" refers to goods and not to persons, the excess amount of B' due to his higher money worth may be preserved for distributive considerations.

Whether Aristotle did indeed think along the lines her proposed cannot be verified beyond any imaginable doubt. But his text is clear in conveying that he asked for an "equalization" of divergent money values of goods through some manipulation. Admittedly, it is not so clear how that "equalization" should be brought about. But our proposal as stated in equ.(12) is "Aristotelian" in spirit in that it alludes formally to his concept of distributive justice. There seems to be agreement in the literature commenting these passages that the distributive justice formula should be applied in this context, but there seems to be disagreement what it should reasonably contain (see the "remarks on interpretation" below).

An approach reminiscent of the present division of money values by earnings was proposed by Adam Smith in his *Wealth of Nations.* He wanted value of goods to be counted by the amount of labor which the respective amount of money-value could buy. This procedure is tantamount to dividing money values of goods by wage rates of labor. The resulting magnitude also had the dimension of "labor" - similarly to what was stated above. Still later J.M. Keynes in his *General Theory* proposed accounting in what he called "wage units" - a procedure which was quite similar to that of Adam Smith.[21] These conceptual interconnections between Keynes and Adam Smith were discussed by the present author at some length (**Ambrosi** 1988). Although both Adam Smith and J.M. Keynes knew "their" Aristotle extremely well, I would not go so far, however, as to claim that they got the inspiration for their value-accounting straight from Aristotle. Nevertheless, it might be reassuring for the reader that the implicit accounting method of "value" which we attributed here to Aristotle is not so exotic as not to have had its place in later economic literature.

Utilitarian implications of the Aristotelian geometric model. The triangular areas as given algebraically by equs (7) and (9) are geometrical indicators of the satisfaction of need. They are variants of what modern economics considers to be utility functions. It might be of some interest to compare the utility functions implied by the Aristotelian geometry to the more conventional ones as exemplified by equ.(0) above. The two triangles obviously stand for the utility functions

$$U_A = U_A(C_A, D_A, A) = \frac{A}{2}\sqrt{C_A \times D_A - (\frac{A}{2})^2} \qquad (15)$$

$$U_B = U_B(C_B, D_B, B) = \frac{B}{2}\sqrt{C_B \times D_B - (\frac{B}{2})^2} \qquad (16)$$

where C_B^2 of equ.(9) is replaced in equ.(16) by $C_B \times D_B$ since the relevant triangle is an isosceles one, therefore $C_B = D_B$ always holds. An analogous replacement holds for equ.(15). If we compare these expressions to a standard utility function like equ.(0), we notice that the present ones use goods inputs similar to the former one, namely C_i , *i= A,B* and D_i , *i= A,B*.

In the relevant range, say, for A=B=$\sqrt{1/2}$, $C_B = 1/2$ they have the shapes as given by fig. 6. The general algebraic expression are in this case given by, e.g., equs.(16) and (17)

$$\frac{\partial U_B}{\partial D_B} = \frac{BC_B}{2\sqrt{4C_B D_B - B^2}} \qquad (17)$$

The characteristics of these utility function and marginal utility functions with regard to changes in goods consumption are quite similar to conventional utility theory.

What is special in the Aristotelian model is, of course, its treatment of “merit” as expressed by A and B. The relevant partial derivative reads, e.g.:

$$\frac{\partial U_B}{\partial B} = \frac{2C_B D_B - B^2}{2\sqrt{4C_B D_B - B^2}} \qquad (18)$$

Equ.(18) shows that for small B, i.e. for small merit levels, the marginal utility will be positive in the relevant realm but for large ones it will rapidly become negative. From equ.(18) it can be seen

[21] The important difference between the Smith-Keynes approach to valuation and the one which we attribute here to Aristotle is that the former approach uses division by a uniform wage rate whereas the Aristotelian approach uses division by a remuneration EA $\neq$ EB which is *not* uniform.

that for the case $C_B = D_B = \frac{1}{2}$, an extreme value (maximum) is given with B = $\sqrt{\frac{1}{2}}$ which is thus equal to the value of A. We thus have the remarkable result that with an egalitarian distribution of goods being given, the optimal level of merit is A=B. It is exactly that one which is consistent with that distribution of goods. The result is remarkable not because it is surprising, but because it is so consistent with what Aristotle wrote: when equal shares are handed out, it is best that equal status prevails. Otherwise there might be discontentment and strife in society. It is remarkable that this rule which is formulated with regard to society is also a maximum with regard to individual partial analytic utility maximization.

It should be noted, however, that the present analysis is not directly comparable to the previous one discussed in the text.

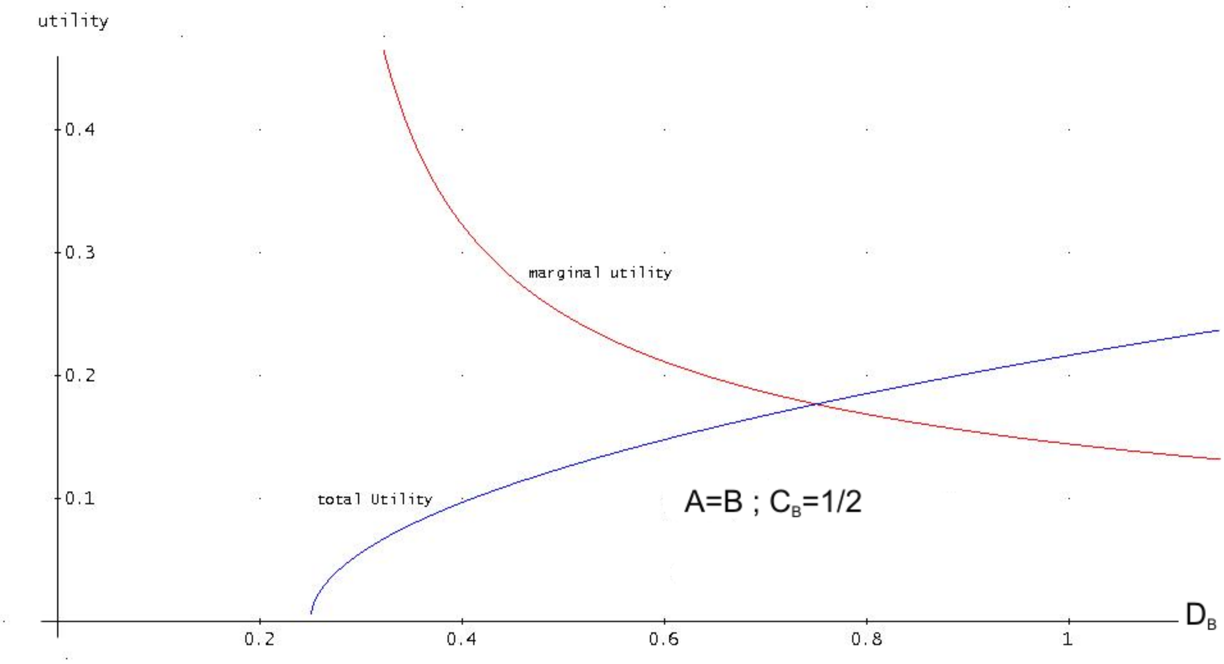

Figure 6: Total and marginal utilities for individual *B* with respect to *partial* variations in D_B

The utility considerations discussed in this paragraph are partial analytic ones. They are different from the thought experiments associated with the $f(\alpha)$-function. The question posed in that context is based on the total derivative

$$\frac{dU_B}{dB} = \frac{\partial U_B}{\partial C_B}\frac{\partial C_B}{\partial B} + \frac{\partial U_B}{\partial D_B}\frac{\partial D_B}{\partial B} + \frac{\partial U_B}{\partial B} \qquad (19)$$

The maximum of the $f(\alpha)$-function obtains where equ.(19) is zero, subject to C=1, D=1 and $A=\sqrt{\frac{1}{2}}$. The solution is a balance between positive marginal utility out of status increase dB and negative marginal utility due to the same dB as shown in figure 7. The maximum utility is obtained for B= 0.9779115436. (The value obtained here is different from the one for α because we have here absolute worth, not relative worth as in the case of α). This equilibrium is seen to be obtained where the positive marginal utility from goods consumption balances the negative marginal utility from status enhancement. The implicit idea thus is that status, i.e. "merit" is enjoyable only in a very narrow range beyond which "merit" is rather a burden, if considered by itself. Greek culture is full of myths and anecdotes which exemplify this claim. Sometimes this paradigm is presented in a rather grotesque way as in Aesop's fable about the frog that wants to be as big as an ox and explodes in the attempt of pumping itself to that size. Sometimes the context is tragic like in the case of the mortal mother Niobe who brags about her many healthy children and who believes to compare favorably with Goddess Hera – only to see all her children killed by the envious Gods. Sometimes the context is practical and political as in the Athenian cult celebrating the murderers of tyrants. The tyrant's death may be seen as the negative reward for his daring to unlawfully assume a higher status than is becoming for a lawful citizen who respects the democratic constitution.

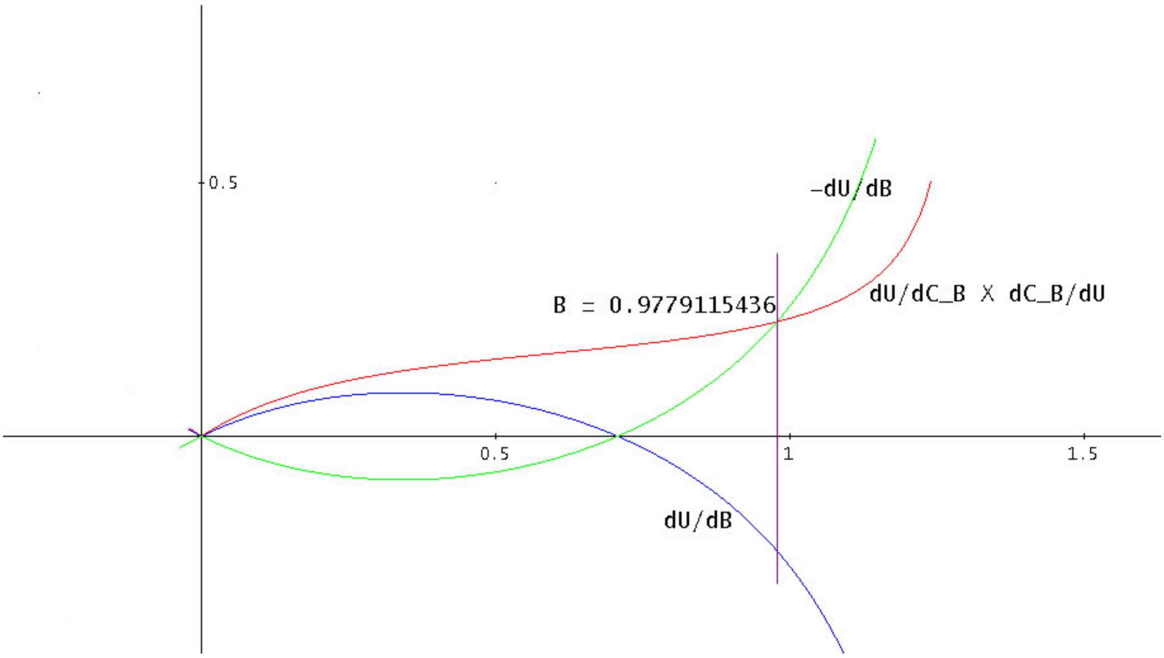

Figure 7: Balancing positive and negative marginal utilities for status optimization

Returning to our Aristotelian model we may emphasize: At the optimal merit level of social differentiation, the increased satisfaction due to the additional goods consumption which goes with higher status under conditions of "distributive justice" only compensates for the increased "burden of merit". The unwise person may see only the positive side of wealth. The wise will consent not to go beyond the reasonable limit of B= 0.9779115436.

Stated in this way, the utilitarian restatement of the Aristotelian model sounds like an over precise overstatement. But although Aristotle certainly would have abstained from such an overstatement, it is clear that with his doctrine of distributive justice as "geometrical proportionality" he did want to go beyond mere qualitative comparison and intended to reach precise statements of quantitative proportion.

Some remarks on alternative interpretations of the Aristotelian equalization: In the above I tried to avoid discussing alternative interpretations at considerable length. In view of the hundreds of existing commentaries any particular selection must appear as arbitrary. But I am tempted to quote a passage from the *Cambridge Companion to Aristotle* (Barnes, 1995). The title of this fairly recent publication suggests particular authority since "Cambridge", UK stands for centuries of research in ancient Greek literature and philosophy. In a chapter on Aristotle's *Ethics*, **Hutchison** (1995, 223 n.9) states in that publication (bracketed terms are my additions):

> "Aristotle thinks it illuminating to represent the equality before the exchange as a proportionate equality:
>
> $$\frac{the\_carpenter\_\{money\_income;\_EB\}}{the\_shoemaker\_\{money\_income;\_EA\}} = \frac{a\_bed\_\{money\_value;\_VD\}}{a\_pair\_of\_shoes\_\{money\_value;\_VC\}}$$
>
> the same sort of proportion as in distributive justice. But I can't see how this illuminates the situation, nor can other scholars, whose interpretations of this passage differ widely."

Hutchison refers here to the problem of "equalization" and finds Aristotle to be incomprehensible. His problem stems probably from the fact that "the carpenter" and "the shoemaker" are taken too literally. The issue under Aristotle's discussion in this context is, however, how to deal with money values. Everything is made "commensurable" by stating it in terms of money values. It therefore is not the persons or their moral worth which should appear in the distributive formula but their money worth – an alteration which I included in brackets at the corresponding places of the quote. Equally, it is not just the products "bed" or "shoes" which are under discussion in this context but their money values. If interpreted in this way, it will be seen that Hutchison's ratio is nothing else but the left-hand term of equ.(12) above. But that term does indeed "illuminate the situation", since on this basis we derived that C=D=1 may hold – and this is the lynchpin of the entire model.

There is an interpretation of this passage which comes somewhat nearer to the present approach in that it mentions "value of work", albeit in a rather inconsistent manner. **Heath** (1949, 274) characterizes Aristotle's model of exchange in the *Nicomachean Ethic* in the following way:

> As in the earlier case *A* and *B* represented the 'worth' of two parties, so here we may suppose A and B to represent what the builder and the shoemaker are 'good for'. It might be measured, say, by the value of the work [money value of the salary? GMA] that they could respectively produce [money value of products? GMA] in the same time, say an hour, or a week. But in any case, before there can be barter between the builder and the shoemaker it is necessary, in the first place, to find a relation between the values [in terms of money? of time? of need? GMA] of the things produced by the two respectively, namely, a house and a pair of shoes. As Aristotle says, [1133a12-14], 'There is nothing to prevent the work of one from being superior to the work of the other; they must therefore be equalized' … ; [1133a22-4] 'hence, as a builder is to a shoemaker, so must so many shoes be to a house (otherwise there will be no exchange and no *κοινωνία)'*.

We have here an array of unrelated ideas suggesting that in this context some sort of valuation must come in. The second sentence in the quoted passage refers to “value of work that they can produce”. This is a most infelicitous expression. For the “builder” and the “shoemaker” of the quote, work is an *input* of production, not an *output,* not a ‘product’. The next sentence is also somewhat problematic. Why does “barter” – a voluntary direct exchange of one thing against another thing – require a “relation between the values” as claimed in the quote? The essence of barter is the *inconsistency* of subjective valuation between the parties of the exchange: builder A wants to get rid of a house (C) in order to get shoes (D). A-s valuation is C<D. Shoemaker B wants to get rid of shoes (D) in order to get a house (C). B-s valuation is D<C. For voluntary exchange to take place, the offering party giving up a product *must* attribute to his product comparatively little value. Otherwise that party would stay with the basket of goods which they had to begin with and no barter would take place. I think that in spite of Heath’s contrary opinion, this aspect of barter has nothing to do with Aristotle’s passages at hand. But if this mutual *diversity* of subjective valuation is what Heath did mean by “relation” in the context of barter, then one must ask why do we have to “find” this relation? It is just the other side of voluntary barter. What does Heath’s “relation” explain in connection with Aristotle’s subsequently quoted ‘equalization’? We find in the quote and in the related passages no suggestion for a solution to Aristotle’s problem that the monetary turnover value of the products made by A and B might be of different *market worth* once we have *monetary* exchange. The fact that within one economy, be it now ancient Athens or modern age Britain, one year’s labour in one sector or industry, as David Ricardo would have put it, can change only against one year’s labour of another industry is a matter quite different from personal evaluation. It is not helpful that Heath seems to bring all these things together without any formal scheme. All this verbose circumlocution we find in a book which is entitled: *Mathematics in Aristotle.*

If one considers all the alternative commentaries to Aristotle’s “equalization” one might well find that the textually most consistent interpretation is the one offered here, especially in the context of equs.(12)-(14).