The Keynesian reception of classical analysis:

Pigou´s Theory of Unemployment

and it´s critique in Keynes' General Theory

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1. Introduction

The orthodox view of the Keynes-Classics controversy based on the famous
interpretation by Hicks (1937) led to a number of interpretatorial legends. Eventually
Leijonhufvud (1968) exposed some of them as what they were. Nevertheless textbooks
still abound with various versions of Keynes-Classics juxtapositions. Some modern
authors like Malinvaud (1977;p.32,n.) tend to be conscious of the lack of authenticity of
their interpretation of Keynes, however.

The problem of authenticity is not new to the Keynes-Classics debate. But from the
perspective of orthodox interpretations, it was seen in a peculiar way. At the time of the
Hicks-inspired Keynes-Classics debate it referred not so much to Keynes but rather to
the Classics. They were his professed antagonists. But since prior to Keynes no
economist elegible for the epithet 'classical' ever employed the type of IS-LM analysis
proposed by Hicks (1937), the impression arose that the controversy which Keynes
claimed to have initiated was to a considerable extent one of his own invention.

Writing in the era of orthodox Keynesianism, Blaug (1968;p.622) saw a particular
difficulty in the fact that

> "almost no economist after 1870 considered the type of macroeconomic
> problem with which Keynes was concerned...".

But in this quote it is very important whether the stress is put on the first or on the
second word at the beginning. In the latter case the authentic characterization of
classical economics must appear as virtually impossible. Undoubtedly this is what Blaug
himself believed in that context.

It was only in comparatively recent years that economists realized that the 'Classics' of
the General Theory are not just strawmen posing as convenient targets for Keynes'
polemical attacks as even an anti-orthodox author like Leijonhufvud (1968) still
believed. We have now the quite different judgement of Corry's (1978:p.8) to rely on
according to which

> "... the classical model presented and rejected in Keynes' General Theory
> is entirely that of the first three sections of Pigou's Theory of
> Unemployment "

This view was later seconded by Solow (1980;p.3) who insisted that

> "Pigou, besides being a great economist, was in particular the embodiment of the Marshallian tradition, the leading figure in the 'classical economics' that the Keynesian revolution was explicitly intended to overthrow."

But, as noted above, such utterances are of comparatively recent date. It appeared only slowly to economists that when discussing Keynes and the Classics, it is very well possible to give "chapter and verse" for the classical views - and that these views were those of A.C.Pigou.

In the meanwhile much of Pigovian analysis remained dormant as far as mainstream economic discussions were concerned. This led to a very unbalanced perception of the development of economic thought: Whereas over the last decades an enormous literature developed which was concerned with supplying and debating microfoundations of Keynesian analysis, little of comparable effort has gone into an explication of Pigovian analysis. Even quite recent macroeconomic analysis of 'new classical' brand seems curiously remote from the writings of Pigou, as Solow (1980;p.7) observed.

But if we are concerned with the authentic Keynes-Classics debate, we cannot content ourselves with continuing to disregard the Pigovian analysis, especially not since the old orthodox Hicksian interpretation of the Keynes-Classics issue has fallen in disrepute - even with Hicks (1974) himself.

In the following we propose to return to the hitherto neglected beginning of the Keynes-Classics debate, namely to Pigou's (1933) Theory of Unemployment. This is, after all, "the only detailed account of the classical theory of employment which exists" according to Keynes (1936;p.7). But the corresponding economic model is not self explanatory so that a detailed reconstruction of the Pigovian theory seems to be called for. The test which we propose in order to find out whether the reconstructed model is really authentic as far as Keynes' conception of the classical theory of employment is concerned will be the generation of the list of classical employment political measures as given by Keynes (1936;p.7). Only if we can reassure ourselves that the classical model presented is really by and large the one which Keynes had in mind when discussing this type of theory does it seem sensible to proceed to a detailed inspection of Keynes' criticism of the classics.

The aim of such an enquiry may be seen in an attempt to gain insight into the general style of analysis prevalent in the 'Cambridge mind' at the time of the General Theory and into the differentia specifica of the Keynesian analysis - a much debated topic which still has not found its definitive treatment.

2. A reconstruction of the Pigovian model of the "real demand for labour".

a) Sectoral disaggregation and the Pigovian approach

Solow's (1980) just quoted call of attention for Pigou (1933) was a rather cursory statement and did not go much beyond the quoted passages. It could not have been expected to be otherwise in the framework of a presidential address in which it was presented. An important aspect which therefore did not survive in that presentation was the essentially disaggregated, yet macroeconomic nature of Pigou's theory. In the passages leading to the ones which are central for the later Keynesian conception of the classical model, Pigou (1933; II,ch.VI) discusses at considerable length the particular problems resulting from the existence of several "different centres of production".

Solow (1980;p.4) refers this discussion to labour market segmentation in the sense "that 'labor' is not a well-defined homogeneous factor of production". But Pigou's main concern in this context is different. It deals with the problem of inferring changes of total labour demand from changes of industry-specific real wages. Industrial differences need not rely on an inhomogeneity of qualifications of workers but could result simply from technological differences. If such differences exist, then the demand for even a homogeneous work force could - in principle - not be seen as depending on some representative real wage, since changes in the labour demand in one sector might be counterbalanced by offsetting changes in another sector.

We find here a theme which was later discussed in considerable detail by some younger Cambridge authors. The most concise presentation of such a discussion is maybe Champernowne's (1935) demonstration of the interplay of the elasticity of substitution of inputs on the one hand and the price elasticity of product demand on the other hand in determining the comparative statics of labour demand on the industry level. Pigou´s (1933) main concern is to reach the negative result "that the concept of a number of independent demand functions for labour... cannot be used as an instrument for analysing the factors that determine aggregate labour demand"(op.cit.p.71).

Pigou is thus clearly aware of the fundamental differences between aggregate analysis and one conducted at a lower level of resolution of economic phenomena.

b) The basic assumptions of the Pigovian employment model

a) The textual basis

Having established the problematic nature of a simplistic application of real-wage

analysis of labour demand, Pigou (1933;p.89f) proceeds to a presentation of the positive aspects of his theory in explicating a number of "basic" assumptions:

"The position from which we start may be set out broadly as follows

[i] There are engaged in making wage-goods ... x men.

[ii] The output in value of wage-goods of these men we call $F(x)$:

[iii] and the general rate of wage is $F'(x)$.

[iv] There are also engaged in other industries y further wages-earners, the wage payments to whom amounts, of course, to $yF'(x)$.

[v] There is thus a total wage payment $(x+y)F'(x)$: and there is left over, as so to speak, a trading surplus to non-wage-earners in the wage-good ... industries, $\{F(x)-(x+y)F'(x)\}$ value of wage-goods ...

[vi] Given the surrounding conditions, we are entiteled to write $(x+y)=\varphi(x)$ "

In the last step, marked as [vi] by us, Pigou draws the consequence from his negative results concerning the real wage analysis of aggregate labour demand. Pigou here postulates a reliable relationship $\varphi(x)$ between total employment (x+y) as dependent variable of sectoral employment x. This enables him to break up the analysis of aggregate labour demand into two steps, namely a $F'(x)$-theory of sectoral labour demand and a $\varphi(x)$-theory of aggregate labour demand. But why and under which conditions are we entiteled to formulate this function in the first place?

Pigou himself quickly proceeds to a detailed discussion of the characteristics of his newly invented concept of a $\varphi(x)$-function without giving hints for its theoretical generation. Supposedly he regards it as plausible within the argumentative context which he seems to presuppose. This analytical context is, of course, that of the marginal analytical English School as developed in the writings of Jevons up to Marshall. When the attempt is made to explicate the analytical foundations of the Pigovian $\varphi(x)$-function then it should therefore be referred to the canonical assumptions of the English School. After restating this analytical basis we will, in the following, proceed to a reformulation of the Pigovian concept of a $\varphi(x)$-function by presenting the analytical steps which are implicit in its conception and the characteristic parameters which may be used in order to discuss further the implications of this novel macroeconomic tool.

b) A reductionist view of the Pigovian model

Having in mind the canonical assumptions of the English School we may disentangle the Pigovian macroeconomic approach by formulating the following set of postulates which may be seen as standing behind his model economy:

A.1.   There are two sectors of production with sectoral employment being given by x and y[1], and labour being the only variable factor of production.[2]

A.2.   The production functions are normal in the sense of the marginal analytical theory of production.[3]

A.3.   Labour is remunerated by its marainal product.[4] Markets and competition are perfect and entrepreneurs maximize their profits.

A.4.   Workers' households have the choice between the derivation of utility either from consuming wage goods or from consuming leisure[5]. As far as they are concerned, the real wage reflects the marginal calculus resulting from utility maximization in this framework.[6]

A.5.   Workers' income is spent entirely on wage goods. Non-workers have a choice between wage goods and non-wage goods. Typically they demand both.[7]

A.6.   The demand for goods by households other than workers' households follows the equimarginal rule.[8]

We will refrain from giving further comments on these assumptions beyond the ones contained in the footnotes.

c) The microfoundations of the Pigovian model

If rebuilt from the canonical assumptions as just presented, the microfoundations of the Pigovian model are given by the following set of equations:

On the basis of the above A.1 and A.2 we may state sectoral production functions. In the case of wage goods we have

(1)   Q1 = F(x)   Q1 = wage goods .

For non-wage goods we have correspondingly

(2)   Q2 = G(y)   Q2 = non-wage goods.

The first classical postulate of A.3 gives the real wage in the wage goods sector as

$$\frac{w}{p_1} = F'(x)$$
(3)

where p1= price of wage goods , w= nominal wage.

The relative price of goods reflects the relative real wages:

$$\frac{p_2}{p_1} = \frac{F'(x)}{G'(y)}$$
(4)   ; p2 = price of non-wage goods.

The second classical postulate of A.4 requires

$$\frac{w}{p_1} = \frac{-U_n}{U_{Q_{1l}}}$$
(5)

where $U_{Q_{1l}}$ = marginal utility of wage goods experienced by workers and

$-U_n$ = marginal disutility of labour with $-U_n = U_{\bar{t}-n}$; $(\bar{t} - n)$ being leisure.

The consumption of wage goods by workers (index l) and that of non-workers

(index k) is represented by

(6) $Q1 = Q1l + Q1k$

The quantity of consumption of wage goods by non-labourers is given by

$$Q_{1k} = Q_1 - \frac{w}{p_1} n$$
(7)

according to Pigou's concept of a trading surplus as quoted in [v] above where

(8) $n = x + y$

is total employment.

Finally, the equimarginal rule invoked under A.6 above gives

$$\frac{V_{Q_{1k}}}{p_1} = \frac{V_{Q_2}}{p_2}$$
(9) $V_{Q_{1k}}$ = marginal utility of Q1k-goods

$$V_{Q_2} = \text{marginal utility of Q2-goods.}$$

From this system of nine equations we may now simultaneously determine the nine endogenous variables

$$Q1, \quad x, \quad y, \quad n, \quad Q1l, \quad Qlk, \quad Q2, \quad \frac{w}{p_1} \quad \text{and} \quad \frac{p_2}{p_1}$$

and we may discuss their simultaneous changes in the well-known comparative static way. Pigou's original procedure was a somewhat different one, however, in combining some of these canonical relationships into analytical subsystems, one of them being his j(x)-function. Directly proceeding from the microfoundations to the comparative statics of the model would therefore blurr an important aspect of Pigovian method.

The first step in discussing the workings of the Pigovian model must therefore be to reconstruct his j(x)-function of aggregate labour demand.

3. The workings of the Pigovian model: A reductionist interpretation of the elasticity of the real demand for labour

a) The elasticity of the real demand for labour as centerpiece of the Pigovian model

The basic approach of the Pigovian theory of employment was characterized above as being one which aims at establishing a functional relationship between macroeconomic employment on the one hand and a sectoral or microeconomic real wage on the other without resorting to the type of analogical theorizing which may be found in much of more modern macroeconomic model building. Pigou's procedure manifests itself in the postulate of the existence of an elasticity

$$(10) \qquad E_r \equiv \frac{\varphi'(x)}{\varphi(x)} \frac{F'(x)}{F''(x)}$$

which represents labour demand via the novel concept of a j(x)-function, the

reconstruction of which on the basis of a microfoundation being the object of our present reformulation. The discussion of the "Elasticity of the Real Demand for Labour" as expressed by Er of equ.(10) takes up considerable space in Pigou's theory of employment. His procedure is reflected in the fact that Keynes (1936;p.273), in his detailed critical review of the Pigovian approach, also treats this elasticity as the main quaesitum of the Pigovian theory of employment. In Keynes' treatment of this concept it is remarkable that he sees "no significant difference between this and my own modes of expression". Keynes (1936;p.273) concludes in this context:

> Thus Professor Pigou's 'elasticity of the real demand for labour in the aggregate' is a concoction similar to some of my own, depending partly on the physical and technical conditions in industry... and partly on the propensity to consume wage-goods...".

A detailed reconstruction of this elasticity will therefore make more apparent not only an important step in the Pigovian argumentation but also the nature of the later critical reference to it by Keynes. From Keynes' quoted reaction it may also be inferred that Keynes' method of macroeconomic model building does not differ significantly from such a mode of expression. Keynes seems to anticipate recent comments about his theory that it involves 'concoctions' of demand and supply conditions and he readily concedes the patent of priority for such a brew to Pigou (1933). This observation suggests that from reconstructing the Pigovian j(x)-function we may also gain some knowledge about the Keynesian method of analysis.

b) A reductionist view of the real demand for labour concept: The Pigovian employment function.

In the attempt to investigate the exact way in which the Pigovian j(x)-function may be seen as a conglomerate of demand and supply conditions we may first turn to the demand conditions as represented by the equimarginal rule of equ.(9).

Simple reformulations will generate from this equation[9] the elasticity condition for equilibria of the non-wokers´ households

(11) $$\frac{\gamma_1}{p_1 Q_{1k}} = \frac{\gamma_2}{p_2 Q_2} \quad \text{resp.} \quad \frac{p_2}{p_1} = \frac{\gamma_2 Q_{1k}}{\gamma_1 Q_2}$$

where

$$\text{(12)} \quad \gamma_i \equiv \frac{Q_i V_{Q_i}}{V}, \qquad i = 1k, 2$$

is the elasticity of utility of the two goods under consideration as perceived by the non-workers.

As far as workers consumption is concerned it follows from (7) as being given by

$$\text{(13)} \quad Q_1 - Q_{1k} = \frac{w}{p_1} n \qquad \text{or by} \qquad F(x) - Q_{1k} = nF'(x).$$

From this expression we obtain an alternative formulation for Q1k in (11). Eliminating this variable via (13) and replacing Q2 by its production function yields then from (11) the first of the following equations

$$\text{(14)} \quad \frac{p_2}{p_1} = \frac{\gamma_2}{\gamma_1} \frac{F(x) - nF'(x)}{G(y)} = \frac{F'(x)}{G'(y)}$$

the last term in (14) being just a restatement of equ.(4). The last fraction thus gives the assumption of equilibrium supply whereas the preceding one represents demand equilibrium in the goods market. Equ.(14) therefore represents a generalization of the equilibrium assumption, but not yet the condition for general equilibrium since the labour market is still missing.

From (14) we see that this "generalized" equilibrium condition gives a relationship between n,x, and y. The latter may be eliminated via equ.(8) so that we finally reach a relationship between just x and n - but this is exactly what the Pigovian j(x)-function was supposed to express.

For a definite formulation of this function along the lines just sketched, assume the partial elasticities of production

$$(15) \quad \alpha_1 \equiv \frac{xF'(x)}{F(x)} \quad \text{and} \quad \alpha_2 \equiv \frac{yG'(y)}{G(y)}$$

to be constant. The marginal productivities in the equilibrium conditions of equ.(14) may then be replaced by

$$F'(x) = \frac{\alpha_1 F(x)}{x} \quad ; \quad G'(y) = \frac{\alpha_2 G(y)}{y}$$

After this substitution and some rearrangement the last equation in (14) gives then

$$(16) \quad \frac{\gamma_2}{\gamma_1} \frac{1 - n\alpha_1}{x} = \frac{\alpha_1}{x} \frac{y}{\alpha_2}$$

Via equ.(8) we may finally eliminate y in order to obtain after some rearrangement

$$(17) \quad n = \frac{\alpha_1\gamma_1 + \alpha_2\gamma_2}{\alpha_1\gamma_1 + \alpha_1\alpha_2\gamma_2} x \quad \text{with} \quad \frac{dn}{d(\frac{\gamma_2}{\gamma_1})} = \alpha_1\alpha_2(1 - \alpha_1) > 0$$

as our reconstructed version of the Pigovian j(x)-function relating aggregate employment n to sectoral employment x.

Summing up the above argument we note that the j(x)-function is the locus of goods market equilibria. Thus the orthodox IS and LM curves are not entirely alien to Pigovian argumentation in that they also are loci of particular market equilibria. As far as the characteristics of the j(x)-function are concerned, our reformulation demonstrated that it is indeed a 'concoction', involving the description of both, demand and supply conditions. For the former we used the elasticities of utility gi for the latter the elasticities of production ai. These elasticities affect the shape of the j(x)-function in such a way that for given sectoral employment:

·   an increase in the relative elasticity of utility of non-wage goods $\left(\dfrac{\gamma_2}{\gamma_1}\right)$ leads to increased total employment

·   an increase in the elasticity of production of labour in the wage good industry (α_1) leads to decreased employment

·   an increase in the elasticity of production of labour in the non-wage goods industry (α_2) leads to increased employment.

Algebraically we thus have the function

$$\text{(18)} \qquad n = \varphi\left(\underset{+}{x} \quad , \quad \underset{+}{\gamma_2 / \gamma_1} \quad , \quad \underset{-}{\alpha_1} \quad , \quad \underset{+}{\alpha_2} \right)$$

where a sign beneath a variable denotes the sign value of the corresponding partial derivative.

In a comparatively simple framework built up from a microeconomic reduction of the Pigovian model we have thus reconstructed a function which may be considered as being the central and novel concept of the Pigovian theory of employment.

c) Some reflections on the reception of the Pigovian analysis of employment

The recent literature on the theory of employment - if it notices the writings of Pigou at all - concentrates mainly on a reception and discussion of the elasticity of "the" real wage of labour. Solow (1980;p.6) is a particularly relevant case in point when he remarks

> "Let me remind you that in the old standby, two factor Cobb-Douglas case, the elasticity of demand for labour with respect to the real wage is the

reciprocal of the share of capital. Every body's back-of-the-envelope puts the capital share at 1/4 and the elasticity of demand for labor at 4. This ist not exactly the way Pigou proceeds, but he reaches the same conclusion...".

In commenting on Pigou (1933), Solow alludes to the now common practice to discuss macroeconomic employment with reference to a single macroeconomic production function. In the Cobb-Douglas case mentioned, macroeconomic output Q is given by

$$(19) \quad Q = AN^{\alpha}K^{1-\alpha} \quad \text{with} \quad \frac{dQ}{dN} = \alpha AN^{\alpha-1}K^{1-\alpha}$$

where A = efficiency parameter, N = aggregate employment, K = capital input. Under the assumption of perfect competition it follows then immediately that

$$(20) \quad E_N^{\left(\frac{w}{p}\right)} = (\alpha - 1) \quad \text{where} \quad \frac{w}{p} = \frac{dQ}{dN}$$

holds, E being the elasticity operator. In this analytical framework changes in employment must appear to depend entirely on the technological conditions as given by a.

To associate Pigou (1933) with this type of argument is rather unfelicitous, however, even if Solow correctly adds a disclaimer concerning the authenticity of such a procedure.

Little can be said against the use of sectoral production functions when discussing the Pigovian model. In deriving the Pigovian employment function of equ.(18) we even went so far as implicitly to assume sectoral Cobb-Douglas functions by setting the ai as constants. But as was seen in that context, a variation in the a-s might have different employment effects depending on the sector where it occurs. A generalizing talk about the consequences of changes in "the" elasticity of production therefore blurrs an important result of the Pigovian approach.

In addition, equ.(18) depicted the employment theoretic importance of demand factors as expressed by $\frac{\gamma_2}{\gamma_1}$. Even if we assume sectoral uniformity in production by setting

$\alpha_1 = \alpha_2$, the influence of the demand parameters will not disappear as may be gathered from equ.(17) after the relevant changes are made.

Thus, once it is admitted that the economy consists of not just one single industry and that there are different goods to choose in the formulation of demand decisions then we cannot return to some simplistic idea about "the" marginal product of labour or "the" economy-wide elasticity of production of labour. In a reception of the Pigovian analysis of employment it is therefore not advisable to abstract entirely from a sectorally disaggregated reconstruction of his j(x)-function, even if it were true that the gist of Pigou's argument may be summarized in the way proposed by Solow in the above quote.

As far as the purely formal side of the argument is concerned, the particular way of our reconstruction of the Pigovian employment function as given by equ.(17) permits us to make a considerable concession to Solow (1980), however. From the definition of the Pigovian elasticity of the real demand for labour as given by equ.(10) it follows that the relationships

$$(21) \qquad E_r \equiv E_{F'(x)}^{\varphi(x)} = E_x^{\varphi(x)} : E_x^{F'(x)} = (\alpha_1 - 1)^{-1}$$

must hold if the elasticity of the employment function j(x) is unity as will follow from equ.(17) where total employment (n) is proportional to sectoral employment (x).

From these considerations the Pigovian E_r might seem indeed to be nothing else but the reciprocal value of an elasticity reminiscent of the one given in equ.(20) above. But the important difference remains that aggregate employment in this analysis is determined by a particular sectoral magnitude - namely the α_1 as defined by equ.(15) - and not by a macroeconomic production function as given by equ.(19).

It should also be emphasized that the simplification just discussed pertains only to the elasticity of the real demand function for labour. The absolute magnitude of aggregate employment might also be of considerable interest so that situations are conceivable in which not just the fact of proportionality between x and n is of interest but the order of proportionality as well. In this case a return to the Pigovian employment function of equ.(17) is called for.

We summarize that in our interpretation an appropriate reconstruction of the Pigovian analysis of employment should take note of the following elements of his theory:

(i) Pigou (1933) regards the theory of employment in the aggregate as being based on analysis pertaining to the sectoral level of an economy. This sectoral disaggregation is not motivated by some conception of inhomogeneous or segmented labour markets. The sectoral differences in Pigovian analysis are rather the outgrowth of specific sectoral production conditions as is the case in, e.g., the pure theory of trade.

(ii) In his explicit discussion of technological conditions Pigou (1933) regards a single production function F(x). But this function pertains not to the economy as a whole, it characterizes rather just the wage goods industry. Consequently the corresponding marginal product F'(x) characterizes no abstract "macroeconomic" real wage but that particular real wage which is relevant for the wages-spending households.

(iii)Pigou (1933) is able to employ a plausible technological analysis on a disaggregated resolution level in his theory of aggregate employment by postulating a functional relationship between sectoral and total employment. This relationship and hence the Pigovian theory of employment depends on demand conditions in an essential way.

4. The reconstruction of the Pigovian model and Keynes' immanent critique of the Classics

a) Keynes, Pigou, and the Marshallian Concept of general equilibrium

It is by now generally accepted that Keynes, like Pigou, must be seen as standing firmly in the Marshallian tradition of economic analysis. But what does this tradition imply for the type of macroeconomic problems to which Keynes addressed himself in the General Theory? It seems that many economists would still subscribe to Robert Clower's (1975;p.4) view that Marshall "never felt the need to model the workings of the economic system as a whole." In the following we want to argue against such a view, however. It leads to the wrong impression that Keynes and Pigou, insofar as they stood in the Marshallian tradition, could not be related to the concept of a general equilibrium of an economy.

This view is wrong because Keynes´ criticism of Pigou was exactly that unless Pigou disassociated himself from the Marshallian tradition, correct theorizing required him to cast his model in a general equilibrium mould. It is in this general equilibrium framework that Keynes (1936;p.7) then derived that meagre but supposedly comprehensive list of classical means to raise employment. Was that procedure really just an unfair trick against Pigou as Corry (1978) and Bleaney (1986) argue? The answer to this question depends crucially on the view one takes concerning the general

equilibrium issue in Marshallian economics.

If we follow Schumpeter (1954) the opposite of Clower's characterization of Marshallian economics seems to be true. Schumpeter (1954;p.836) quotes a passage from Alfred Marshall's Memorials in which Marshall, writing to his famous American collegue J.B. Clark declared emphatically:

> "My whole life has been given and will be given to presenting in realistic form as much as I can of my note 'xxi'."

But this note 'xxi' is concerned with the determinacy of a system of interdependent demand and supply equations. Marshall concludes there that because of an equality of equations and endogenous variables the problem is indeed always "determinate". Clearly the argumentation in that note has essential elements of a general equilibrium theory so that Schumpeter (1954;p.923ri.1) could comment

> "... the contents of 'note xxi' in the Appendix to the Principles constitute the core of Marshall's theoretic analysis. This note blocks out a system of general equilibrium!"

Let us therefore reemphasize: the general equilibrium perspective certainly was deeply entrenched in Marshall's own mind. It can be documented that he worked out the formal prerequisite for such a theory and that he considered his life's aim to be a realistic rendering of that formalism.

It could maybe be argued that Marshall's research program was his private matter which was of no relevance for his disciples, in particular for Keynes and Pigou. But in this regard it is important to note that it was not Marshall himself but Pigou ed.(1925) who published Marshall's confession of being a devotee of the idea of general equilibrium. Furthermore, the preface of that publication reveals that these papers were chosen in close collaboration with none other but J.M.Keynes!

Thus we have strong indications that Keynes and Pigou insofar as they considered themselves as standing in the Marshallian tradition must have been aware that in that tradition the ultimate analytical reference for their theories must be a system of general equilibrium. Certainly it cannot be considered as unfair if Keynes, arguing as a Marshallian, discussed Pigou's (1933) analytical performance under the perspective of such a system.

b) Keynes' perspective of general equilibrium in the critique of Pigou (1933)

Keynes' first critical reaction to Pigou (1933) may indeed be seen as an application of

the main theme of Marshall's 'note xxi'. As just emphasized, this note was written "with the object of making sure that our abstract theory has just as many equations as it has unknowns, neither more nor less." It came to the conclusion that 'the problem', namely the solution for prices and quantities in the Marshallian model, was indeed 'determinate' because this condition was 'always' fulfilled. [Marshall (1920;p.704)].

In reading Pigou (1933) before this background, the question quite likely to be raised would have been whether he supplied a Marshallian model in this sense. If it were so, then it had to give such a determinate solution for employment with reference to Marshall's (1920;p.270) "two opposing sets of forces, those which impel man to economic efforts...and those which hold him back." In the above discussed reduction of the Pigovian model to its basic assumptions these sets of forces were identified as resulting from the maximization of profits and of utility. It was shown that the ensuing model was indeed "determinate" in the Marshallian sense in so far as the endogenous variables were exactly equal to the number of equations. Among the endogenous variables were real wages $\left(\dfrac{w}{p_1}\right)$ and employment (n), of course. Without a change in the parameters or some other exogenous aspect of the model no change would be possible in such a determinate model.

When now Pigou (1933) embarks upon a discussion how real wages and employment vary in an economic system supposedly depicting an entire economy it is strictly in the Marshallian tradition as just elaborated that Keynes (JMK,Xiii,p.310) remarks:

> "He [Pigou (1933)] arbitrarily takes two items, namely employment and real wages, out of a complex, but presumably determinate, system and then treats them, without proof or enquiry, as being analytic functions of one another. But they are not independent variables. If everything is given except real wages and employment, then neither real wages nor employment are capable of more than one value."

That it was really the Marshallian "two opposing sets of forces" which supplied the basis for this criticism becomes quite apparent when we look at the long controversy between Keynes and R.G.Hawtrey after the publication of the General Theory. Hawtrey's defence of Pigou's classical position resorted to the remark that there really is no such thing as a determinate labour supply function in that theory. To this Keynes (JMK,XiV;p.25) replied:

> "When ... you tell me that 'the marginal disutility of labour plays a very restricted part in the works of the great economists' I am again simply staggered. The whole of Pigou's Theory of (Un)Employment ... is based on

the level of employment being the resultant of what he calls the real demand for labour and the supply of labour given by the schedule of the different quantities of labour, the marginal disutility of which is balanced by differing amounts of wage goods. Moreover, the whole of Marshall's theory is based on this and, above all, the whole of Jevons' theory. In fact, there is no other theory that I am acquainted with."

Thus there is ample evidence that Keynes saw the classical theory in the context of Pigou's (1933) writings as a theory of general equilibrium. There seems to be little that could be considered as unfair in such a view by Keynes, given the common Marshallian background of the two authors. The two postulates of the classical theory of employment as stated in ch.2 of the General Theory determine a specific equilibrium combination of real wages and employment. Classical theory of employment is therefore a theory of alternative levels of full employment. Pigou has failed to show where he deviates from this theory. His claim for his theory to be one of Unemployment is, therefore, something of a misnomer [Keynes (1936;p.275)].

c) Keynes' extension of the Pigovian model

a) An outline of the Keynesian critique

In arguing out his critique of Pigou (1933) in the General Theory, Keynes, very much like Marshall in his 'note xxi', resorts to counting the equations of the model concerned. He concludes that in Pigou's model one is 'one equation short' for a theory of unemployment. Since shortly before that statement Keynes claims that the Pigovian theory is a full employment theory and gives the corresponding supply and demand equation of the labour market, he implicitly characterizes a theory of unemployment as one which - from the standpoint of full employment general equilibrium theory - must be overdetermined. A theory of unemployment must therefore generate an additional equation giving a constraint on employment other than labour supply.

Keynes' critique of Pigou has thus two aspects, namely a positive and a negative one. Arguing out the first aspect, Keynes affirms one general-equilibrium-full-employment nature of the Pigovian model and draws the corresponding policy conclusions of the classical model. In the rest of this section we will try to unravel this part of Keynes' argumentation. Later we will then turn also to the other part of his argumentation.

b) A graphical illustration

The positive aspect of Keynes' discussion of the Pigovian model might be demonstrated graphically by first establishing the original Pigovian approach an then referring it to the Keynesian extension.

In fig.1 the second quadrant represents Pigou's (1933) sectoral production function for wage goods as F(x)-curve. His real demand function for labour is represented in quadrant three as j(x)-curve. The real wage in terms of labour is given by tana in quadrant one. The corresponding marginal conditions for entrepreneurs are fulfilled in point I. along F(x), representing the fulfillment of the first classical postulate. The corresponding production of wage goods is given by $\overline{xI}$. and Pigou's 'trading surplus' of entrepreneurs corresponds to the stretch $\overline{xP}$. In Pigou's theory of employment one then regards movements of point I. in the second quadrant in relation to movements along the j(x)-curve in quadrant three. The n-axis identifies the corresponding changes in total employment.

The Keynesian view is, however, that such movements cannot occur freely, since labour demand as derived from the first classical postulate must always meet labour supply as identified by point II. Thus the j(x)-curve in quadrant three must be met by another appropriate curve named c(x) by Keynes (1936;p.274). The intersection of these curves is in point E, determining employment at a level of $\overline{n}$. Total employment n=x+y is then seen in quadrant two to be at such a level as to satisfy the second classical postulate. The marginal rate of substitution along the indifference curve U corresponds then to the real wage rate as given by tana and it corresponds to the marginal productivity of labour as given in point I..

The classical theory of employment, according to Keynes, can now be seen as being not permitted to limit its analytical perspective to movements along F(x). It can analyze changes in employment only with reference to changes in point E. This opens just two possibilities for increasing employment: a shift of the c(x)-curve away from the origin or a shift of the j(x)-curve towards the n-axis.

Fig 1: The modell of Pigouvian Real Demand for Labour

We summarize that so far, the following picture emerges concerning Keynes' reception of the classical theory: Keynes takes up the original analysis of Pigou (1933) and confronts it with the Marshallian perspective of general equilibrium. In the light of this perspective he then extends the Pigovian model showing its general equilibrium features. The final step of this handling of the Pigovian model is then to explicate the type of employment political measures which this analytical framework logically permits.

Before embarking upon this final step it remains to investigate in a more formal way whether, given the basic assumptions of the Pigovian model, Keynes did justice to Pigou in confronting him with the existence of the crucial c(x)-function.

g) A reductionist view of the Keynesian extension: The algebra of labour supply implications in the Pigovian model.

In this section we intend to show in some detail that the c(x)-function which Keynes (1936;p.274) insisted to be implied in the Pigovian analysis of employment is indeed nothing else but the representation of labour supply in a general equilibrium context.

As labour supply function the c(x)-function certainly looks unfamiliar. This particular formulation of labour supply is due to Pigou's (1933) peculiarity of discussing labour demand not in a real wage and employment plane but in the sectoral employment and total employment plane. Therefore Keynes had to make a corresponding transposition of the conventional classical labour supply analysis to Pigou´s - in this narrow regard unconventional framework. The result of this transposition is that the c(x)-curve in fig.1 represents the locus of joint validity of the first classical postulate for entrepeneurs in the wage-goods industry and of the second classical postulate for workers in general.

In a formal discussion of this curve it is therefore to be demonstrated that the c(x)-curve is a graphical representation of the two-equation system

(22) a)
$$F'(x) = \frac{-U_n(t-n, Q_{ll})}{U_{Q_{li}}(t-n, Q_{ll})}$$

b) $Q_{ll} = nF'(x)$

where the left-hand side of the first equation states the validity of the first and the right-hand side that of the second classical postulate and where (22b) restates equ.(5), i.e. that workers spend their entire income on wage goods. Treating x-employment as independent variable, the two equation system of equ.(22) may then determine the corresponding value of total employment supplied and the corresponding expenditure on wage goods as the endogenous variables n resp. Q1l.

The question of labour supply is somewhat complicated by the well-known and much discussed phenomenon of "backward bending" labour supply. Robbins (1930) gave this phenomenon which was discussed by Marshall (1920) already a thorough choice theoretic treatment which must have been known to Keynes. But, abstracting from these complications for reasons one can only speculate about, he believed the classics to adhere to the doctrine of labour supply to increase with increasing real wages. Since the latter increase when x-employment decreases, this should guarantee the negative slope of the c(x)-curve as drawn in fig.1.

Pigou (cf.JMK,XiV,p.54) communicated to Keynes, however, that it was not the

'normal' labour supply curve which he employed, but one with constant equilibrium labour supply. This assumption would give a c(x)-curve vertical to the x-axis. But as Keynes realized, this would not have much influence on his interpretation of the classical model.[10] Both types of curves are easily to be integrated into the standard body of marginal analytic argumentation. This will now be shown in order to match the parametric formulation of the j(x)-curve of equ.(17).

The Pigovian case of constant labour supply is generated when the elasticity of substitution in consuming wage goods resp. leisure is set at unity. The elasticities of utility of these two activities are then constant, being defined as

$$\delta \equiv \frac{Q_{11}U_{Q_{11}}}{U} \qquad \lambda \equiv \frac{-(\bar{t}-n)U_n}{U}$$
(23)   resp.   .

Employing these definitions, the second classical postulate of equ.(5) may be reformulated to give

$$\frac{\delta}{p \ Q_{11}} = \frac{\lambda}{w(\bar{t}-n)} \qquad Q_{11} = \frac{w}{p_1}(\bar{t}-n)\frac{\delta}{\lambda}$$
(24)   or

But Q1l in this expression can be replaced by equ.(22b) after the latter is rewritten as

$$Q_{11} = \frac{nw}{p_1}$$
(25)

giving, after minor reformulations, the c(x)-function in the Pigovian case as a constant value, namely

$$n = \frac{\delta}{\lambda + \delta}\bar{t}$$
(26)

This constant value of labour supply does not mean, however, that Pigou already was negating the validity of the seconed classical postulate. It rather means that he assumed specific utility functions to be at the basis of his theory of labour supply.

A slightly less specific version of the labour supply theory may be generated when the elasticity of substitution is not unity as in the case just discussed but just constant. For completeness' sake a "normal" c(x)-function might therefore be generated by starting from a CES-version of the workers' utility function

$$U^{-\beta} = aQ_{11}^{-\beta} + b(\bar{t}-n)^{-\beta}$$
(27)

The second classical postulate then requires

$$\frac{w}{p_1} = \frac{b}{a}\left(\frac{n}{\bar{t}-n}\right)^{1+\beta} = \frac{-U_n}{U_{Q_{11}}}$$

(28)

to hold. Again replacing ; Q1l by (25) gives

$$\left(\frac{w}{p_1}\right)^{-\beta} = \frac{b}{a}\left(\frac{n}{\bar{t}-n}\right)^{1+\beta} \qquad \text{with} \quad \frac{dn}{d\left(\frac{w}{p_1}\right)} > 0 \qquad \text{for } 0>b>-1.$$

(29)

From this expression the new c(x)-function is now derived by substituting the real wage $\frac{w}{p_1}$ with sectoral employment x as given by a marginal productivity function. The exact shape of the latter depends on the productive conditions of the economy. If they may be characterized by sectoral Cobb-Douglas functions as was conceded above, then a simplified version of such a function may be written as

$$F(x) = B_1 x^{\alpha_1}$$

(30)

where the (fixed) capital stock is contained in the efficiency parameter B1. From the associated marginal productivity function

$$F'(x) = \frac{w}{p_1} = \alpha_1 B_1^{\alpha_1-1}$$

(31)

it follows that replacing $\left(\frac{w}{p_1}\right)$ in (29) by this expression finally gives a c(x) function with the characteristics

$$n = \chi\left(\underset{-}{x}, \underset{-}{\frac{b}{a}}, \underset{+}{B_1}, \underset{+}{\alpha_1}\right); \quad \text{with} \quad n \longrightarrow \infty \text{ for } x \longrightarrow 0$$

(32)

where signs under a variable signify the corresponding sign value of the respective partial derivative. The curve given in fig.1, quadrant three represents this latter version of the c(x)function.

We have thus reached the goal of this section to supply a detailed microfoundation of the c(x)-function which Keynes claimed to be implied in the reasoning of Pigou (1933).

d) Employment policies in the extended Pigovian model

After the technicalities of the last sections we would like to recall briefly the argumentative context of the present section. The main aim of the critical reconstruction of the Pigovian model in the General Theory is not just to show that it is a general equilibrium model. Keynes rather intends a reductio as absurdum by going further and to discuss the very limited employment political measures which follow from this model.

It might be that this two-step procedure was not entirely transparent in the way Keynes presented it. He relegated the first step to the appendix of ch.19,GT., undoubtedly because it involved some boring equation counting and a shuffling of algebraic symbols. Far more readable than this - albeit hardly understandable by itself - is the obscure list of classical employment political measures which Keynes gave in ch.2,GT.. Thus Keynes' reader is confronted with the last step first.

Risking the boredom which Keynes wanted to avoid, we reversed his order of presentation and first discussed the Pigovian model and its Keynesian extension. Due to repeated contestations in the literature, the legitimacy of this extension had also to be discussed in some detail. We concluded that it was indeed in a properly understood Marshallian - and hence for Keynes and Pigou a legitimate - context when Keynes (1936;p.274) proclaimed: It is not permissible in the classical context to discuss employment just with reference to the Pigovian j(x)-function. The complete classical theory of employment is rather contained in the equations

$$(33) \qquad n = \varphi(x) = \chi(x)$$

The legitimacy of this extension established, there remains the question of the analytical correctness of Keynes' claim that there are "only four possible means of increasing employment ... This, to the best of my understanding, is the substance of Professor Pigou's Theory of Unemployment". The task now at hand is therefore to go through this list of employment political measures and to relate it to the Keynesian extension of the Pigovian model.

The item (a) in Keynes' list relates to 'frictional' unemployment. This involves an 'off-

curve' analysis which is not strictly the domain of traditional economics although Hutt (1939) and much later Phelps et al.(1970) have shown that the traditional principles of economic analysis may very well be applied in order to discuss certain aspects of 'frictions' caused by search and by the acquisition of knowledge. In the rather different context here under discussion, this point may be illustrated with reference to the above fig.1 by saying that the equilibrium point E is not really relevant. Instead, point F with the associated relatively low value of employment of nf is claimed as characterizing the economic situation due to frictions in the economy which prohibit the enactment of the frictionless plans as identified by the j(x)- and c(x)-curves. Employment policy could then aim at reducing frictions, thereby shifting the broken lines determining point F in such a way as to come nearer to the frictionless level of employment $\overline{n}$. This measure is represented in fig.1 by the arrow marked (a).

Item (b) in Keynes' list refers to 'voluntary' unemployment, i.e. to the 'problem' that at the ruling real wage workers do not produce sufficient 'effort' . Lowering this type of unemployment involves a decrease in the marginal rate of substitution between the consumption of goods and leasure. In terms of our previous discussion of labour supply this means a decrease in the $\frac{\lambda}{\delta}$ ratio as given by equ.(26) or, in the more general case of equ.(28) a decrease in the $\frac{b}{a}$ ratio - provided the elasticity of substitution in consumption is sufficiently high, so that the order conditions for b as given in equ.(19) are indeed fulfilled. In fig.1 this measure will shift equilibrium point II. in quadrant two away from the origin and the c(x)-curve in quadrant three accordingly as marked there by arrow (b).

Keynes' next item in the list, (c), shifts the focus from utility of workers to their productivity. It may seem strange that he refers in that context just to their marginal physical productivity in the wage-goods industries, leaving the other sector undiscussed. The following interpretation for this omission suggests itself: Suppose we stick to our old concession that the technology of the economy may be characterized by sectoral Cobb-Douglas functions. Suppose further that the elasticities of production are constant. Then changes in the technology can only be associated with changes in the efficiency parameters of the functions. But there was only one such parameter which entered our discussion of the j(x)- and c(x)-curves, namely B1 in equ.(32). We conclude from this fact that the present formulation of the classical model is indeed in considerable accordance with Keynes' perception of this model. His item (c) refers to a shift of the c(x)-function as given in equ.(32). The corresponding shift of the c(x)-curve in fig.1 is marked there by arrow (c).

The last item in Keynes' list is (d), discussing demand shifts by non-wage earners away

from wage-goods to non-wage goods so that the $\left(\dfrac{p_2}{p_1}\right)$ -ratio increases. This measure refers us back to equ.(11). It is seen there that this case is generated by an increase in the $\left(\dfrac{\gamma_2}{\gamma_1}\right)$ -ratio. We noted already in the discussion of the j(x)-function of equ.(17) that this increase does indeed imply an increase in employment as explained by this function. This case is marked by arrow (d) in fig.1.

That this seemingly garbeled list should be indeed "comprehensive" as Keynes (1936;p.7) claimed, might appear as strange. In order to evaluate this claim consider table 1 below where a synopsis is given of all possible parameter shifts in our reconstruction of the classical model.

Table 1: The classical means of increasing employment

	$\dfrac{b}{a}$	a1	B1	a2	$\dfrac{\gamma_2}{\gamma_1}$
n	-	*	+	+	+
	„b)"		„c)"		„d)"

The first line of Table 1 identifies changes in employment which occur in the general equilibrium system of equ.(33) in reaction to increases in the parameters given in the head of the table. From the second line it appears that Keynes' list might seem to be not quite complete since he forgot to discuss changes in the elasticities of production a1 and a2. But table 1 shows that in fact nothing definite can be said about effects of changes in the first of these. As far as the second of these parameters is concerned, it is seen that its increase does have a definite positive effect. But it seems that Keynes considered such changes as not being covered by Pigou's (1933) short period analyses in which even capital equipment was assumed to be given.[11] Keynes' discussion of an increase in the marginal physical productivity of labour (as represented in our B1-colum) is thus already an extension of Pigou's argumentation when strictly interpreted.[12] Further discussions of changes in technology might therefore be dismissed as an overextension of the classical framework, particularly if the results of such changes are indeterminate as in the case of a1

If this reasoning is accepted, then Keynes´ list of classical means of increasing employment is indeed comprehensive in the type of model which we presented above as being a representation of Keynes´ perception of the classical model.

e) A summary of Keynes' positive critique of the classics

In the introduction we proposed a test for an authentic model of Keynes' perception of the classical theory. This test demands that a classical model in the sense of Keynes must generate as comprehensive list exactly those classical measures for increasing employment which Keynes claimed to have derived from that model in ch.2,GT. This test was passed by our candidate, as just noted.

But what, one may now ask, is so critical about this list? Keynes (1936;p.7) does not discuss this question but passes on to questions of involuntary unemployment proper in the sense of the second classical postulate not being fulfilled. Thus he quickly steps out of the framework so far discussed and enters the realm of his negative critique in which he then develops quite a different system of argumentation.

It seems that Keynes thought this list to be so obviously banal and unrealistic in the face of the then pressing problem of involuntary unemployment that he refrained from stating the obvious in ch.2,GT.. But in his more formal and systematic critique of Pigou (1933) he returned to this theme. There he did indeed justify what in the light of the literal words of ch.2, GT, might have seemed to be mere speculation, namely that the purpose of his reconstruction of the Pigovian model was to show its triviality in his eyes: Keynes (1936;p.278) explicitly criticizes Pigou to be "forgetful how narrow a thing" his Real Demand Function for labour is.

This criticism of Pigou is particularly interesting because it anticipates later criticism of Keynes that he overlooked that in Pigou's theory there are not only real wage changes discussed but likewise changes of demand. Keynes was well aware that the casual reader could get the impression that there was no significant difference between him and Pigou as far as "demand" was concerned and it is not without interest to consider his reaction to this impression:

> Finally, when Professor Pigou comes to the 'Causation of Unemployment' he speaks, it is true, of fluctuations in the state of demand, much as I do. But he identifies the state of demand with the Real Demand Function for Labour, forgetful of how narrow a thing the latter is on his definition. ...Yet in Part V of his Theory of Unemployment fluctuations in the state of 'the real demand for labour' are given a position of importance. ... To the reader all this seems, at first, reasonable and familiar. For, unless he goes back to the

definition, 'fluctuations in the real demand for labour' will convey to his mind the same sort of suggestion as I mean to convey by 'fluctuations in the state of aggregate demand'. But if we go back to the definition of the 'real demand for labour', all this loses its plausibility. For ... there is nothing in the world less likely to be subject to sharp short-period swings than this factor.

We quoted these passages from the last two pages of Keynes' appendix (GT,ch.19) on 'Professor Pigou's 'Theory of Unemployment' at some length because they are of particular relevance for the interpretation of Keynes' dealings with the classics. Keynes himself here suggests the 'reductionist' approach which we followed above in reconstructing his classical model by asking the casual reader of Pigou (1933) to go back to his definition of the Real Demand concept as characterized by Er of equ.(10). If we do go back to that definition, we find that there are two analytical elements involved in it, namely, according to equ.(10), the sectoral production function F(x) and the total employment function j(x). Of these two the former must be treated as given in the short period context chosen by Pigou (1933). This leaves us with the j(x)-function as the only relevant analytical concept. In order to evaluate its employment political potential we must again resort to going "back to the definiton", namely to our equ.(18). From there it emerges that if we want to change Er we can only change j(x) and this function in turn can only be changed by a change of the $\left(\dfrac{\gamma_2}{\gamma_1}\right)$ constellation if technology is given. All of this is indeed discussed by Keynes - not in the same detail as we did it but with the same analytical perspective.

We therefore consider it as a misrepresentation when Bleaney (1986;p.4) states that Keynes' reference to Pigou's (1933) lengthy discussion of the numerical value of Er in Part II "is already indicative of Keynes' view that Pigou is more interested in the slope of the [Real] demand function [of Labour] than possible shifts in position." Keynes has no such view. The above quote from the General Theory documented that Keynes acknowledges that Pigou wants to believe that shifts in Er and hence in j(x) explain the business cycle phenomenon. But it follows from Keynes' list of classical means to raise employment that just a single influence works on this magnitude, namely the $\left(\dfrac{\gamma_2}{\gamma_1}\right)$-ratio as just noted. Keynes' criticism of Pigou is not that he did not believe that there are Real Demand shifts. It is rather that Pigou erroneously believed that his particular concept of real demand is fluctuating when the level of economic activity changes.

The gist of Keynes' argument as analyzed so far may now be put briefly thus, using Keynes' own terminology: Pigou did discuss the composition of effective demand in a

'concoction' similar to some to be found in Keynes' own analysis. With given technology such a composition is determined by a particular constellation of marginal propensities to consume. But this does not determine the level of economic activity yet. What Pigou left undiscussed was the volume of effective demand. This analysis is what Keynes thought to be new in his own theory and thus to be the centrepiece of his anti-classical conception.

5. The classical reductionism and Keynes' analytical alternative

a)The framework of the Keynesian perspective

As outlined above, Keynes' attack on the Classics was two-pronge. So far, we reconstructed only one aspect of this attack, namely the elaboration of the limited nature of the analytical pontential of the traditional classical model. The classical limitations are two-fold:

(i)   Insofar as the classical model is one of general eqilibrium, it can not supply a rationale for employment political manipulations, since the actual situation as represented in the model is in accordance with all the plans so the eccnomic subjects.

(ii) But insofar as the classics concede the necessity for employment political action they are incapable to model in the framework of the model just reconstructed the genesis of a situation in which the realization of economic plans is forestalled by a specific mechanism contained in the classical model.

With these observations in mind it seems quite consequential if after reconstructing and discussing the classical model as one of full employment, Keynes mentally proceeded by formulating two anti-theses to these observations:

(i') The factual existence of 'involuntary' unemployment in the strict sense must be the starting point for employment political considerations if measures to increase employment should not be forestalled by a reduced labour supply due to the possibly reduced real wages and hence a possible reduction in the incentive to work. But then the conditions for a general equilibrium model must be considered as obsolete at least with regard to the labour supply decisions. From this thesis springs Keynes' negation of the second classical postulate.

(ii')   Although the level of employment cannot be seen as being determined by an interaction of labour supply and demand when conditions of involuntary unemployment prevail, nevertheless a theory of employment under these conditions must treat employment as being determined. The mechanisms of determining employment under conditions of involuntary unemployment are to be discussed. From this thesis springs Keynes' doctrine of 'effective demand' and its volume, not its composition as being of prime importance for a theory of unemployment proper.

From Keynes' negation of the second classical postulate as expressed in (i') there follows the desirability for full employment policy, fromhis doctrine of effective demand follows ist political feasibility - given the appropriate conditions of the employment theoretical model to be constructed.

We want to stress that when Keynes (GT,ch.2) negates the validity of the second classical postulate it is not the underlying reductionist interpretation of labour supply which he rejects as analytically unsound - maybe in the sense that workers in principle do not behave in a utility maximizing way or that they are incapable of making the marginal calculations which the second classical postulate presupposes. The attainment and the validity of this postulate is rather the measure of success for employment political action. But only insofar as one of the classical postulates can be negated is the classical model able to supply a formal framework which has the necessary degree of freedom for employment political action. Keynes (GT,ch.2) makes it amply clear, however, that he is content to do away with the second classical postulate under preservation of the validity of the first. Thus, a general disequilibrium model in the sense of the new macroeconomics is not what Keynes intended to construct.

b) The classical reductionism and Keynesian 'effective demand'

a) Keynes and the determination of employment through 'effective demand'

In developing his concept of 'effective demand', Keynes seems to break entirely with the main tools of classical analysis. Although in the second chapter of the General Theory Keynes just had stated his acceptance of the first classical postulate, a few pages later in the third chapter he seems to be in a totally different theoretical world. Real wages and marginal products seem to be irrelevant. Instead of these, Keynes discusses economy wide expectations of "proceeds" $[Z=j(N)]$ which on one page he calls "aggregate income" (p.24) and on the next he calls "the aggregate supply price" (emphasis added), thus compounding in his exposition novelty of analysis with semantic confusion. Even his modern editors are perplexed but unable to correct Keynes' "more substantial errors such as the unsatisfactory presentation of aggregate supply and demand on page 25". (GT,p.385)

In addition to a lack of terminological clarity, there are grave questions concerning the model employed in Keynes' discussion of effective demand. The sectoral questions of production just discussed by Keynes under item (c) of his list of classical means of raising employment disappear entirely in his theory of effective demand and give way to sectoral questions of demand as expressed by his D1=c(N)-function which relates consumption demand D1 to total employment N. Keynes then adds D2, signifying investment demand and, rather abruptly, declares on the "page 29" which is so very much in disfavour with his modern editors: "This is the essence of the General Theory of Employment."

Keynes' exposition of the 'principle of effective demand' has puzzled many economists and it has inspired some. Clower's (1965) juxtaposition of notional and effective demand in an economy operating under conditions of unemployment is a case in point for the latter remark. But, as we have just noted, even for his modern editors the communication of Keynes' ideas by himself seems to have been grossly deficient in this context. We will proceed to argue, however, that this deficiency is a seeming one which results from the lack of awareness in many of Keynes' readers of the interrelations and adaptations from classical analysis to that of Keynes.

b) Keynes and the Classics: some analytical correspondence

The seemingly incomprehensible smoothness of Keynes' transition from his immanent critique of the classics in chapter two, GT, to the exposition of his principle of effective demand in chapter three could find an explanation in some common analytical basis which, in Keynes' own mind, was uncontroversial and therefore taken as being generally accepted as common knowledge in economics. A hint in this direction is given in Keynes' (1936; p.273) discussion of the Pigovian analysis in the course of which he declares that:

Pigou's " $\dfrac{F(x)}{F'(x)}$, being the value of the output of the wage-goods industries in terms of the wage-unit, is the same as my CW."

But Keynes' CW is nothing else but his D1 when measured in wage units. Since we have Keynes' word that his CW or D1 contains Pigou's F(x) and thus his first classical postulate, we have from this hint a glimpse of a possible interconnection between the seemingly unrelated chapters.

The correspondence between Keynes and Pigou just noted has found astonishingly little attention in the later Keynes-Classics-debate. But it offers an insight into the significance of Keynes' professed acceptance of the first classical postulate: This postulate supplies a convenient measuring device by which it is possible to make a

transition from Pigou's analysis in terms of wage-goods to one in terms of the Keynesian wage units, thus changing Pigou's analysis in terms of quantities of (wage-) goods to one in terms of Keynes' easily to be aggregated values in terms of wage units.

If we accept the first classical postulate as giver by equ.(3) it seems almost banal to write out the quote just given as

(34)
$$\frac{F(x)}{F'(x)} = \frac{p_1}{w} Q_1 \triangleq C_w = F_1(N_1)$$

where the left hand side of the correspondence restates the Pigovian terminology and the right-hand side gives Keynes' own mode of expression with index 1 standing for the consumption goods sector, Fi(Ni) i=1,2 giving sectoral supply functions in the sense of Keynes.

Utilizing the definitions for ai as given by equ.(15) and extending the correspondence of equ.(34) further, gives then

(35) a)
$$C_w \triangleq \frac{F(x)}{F'(x)} \triangleq \frac{1}{\alpha_1} N_1$$

(35) b)
$$I_w \triangleq \frac{G(y)}{G'(y)} \triangleq \frac{1}{\alpha_2} N_2$$

(35) c)
$$N = N_1 + N_2 \triangleq n = x + y$$

(35) d)
$$C_w = \chi(N) = cN \triangleq \varphi(x)$$

(35) e)
$$Z_w = C_w + I_w = cN + \bar{I}_w = D_w$$

In a) we have a simple restatement of (34) giving a correspondence between the sectoral

value of supply CW and sectoral employment N1. Equ. b) may then be assumed to hold analogously. Equ. c) measures total employment and states just the notational correspondence between Keynes and Pigou. In equ. d) we adopted Keynes' (1936;p.273) simplifying assumption that the propensity to consume corresponds to Pigou's j(x)-function. In fact, the correspondence is a bit more complicated and should give a relationship not between CW and N but between N1 and N. But if we turn back to equ. 35.a), we see that we may substitute according to that equation, thus obtaining

$$(36) \qquad N_1 = \alpha_1 c N \quad \text{with} \quad \frac{dN}{dN_1} = (\alpha_1 c)^{-1} = \varphi'$$

as a Keynesian version of a function between sectoral and total employment.[13] As far as the above equations 35 a) to d) are concerned there is no significant change between Keynes' model and the classical one. It follows then that only the last equation e) can possibly contain the Keynesian differentia specifica. Thus, in a presentation of his analytical alternative Keynes could easyl believe that it is sufficient to conduct his discussion in terms of his novel Z- and D-variables. But without the knowledge of an implicit analytical perspective of considerable correspondece with the classics, Keynes' Z&D-analysis could easily appear as badly presented - and still does so appear as was nosed above.

If, however, it is seen before the background of a considerable correspondence between classical and Keynesian analysis, Keynes' discussion of the principle of effective demand does not appear to be unduely elliptic and neither 'erroneous' nor 'unsatisfactory' but just as economical in exposition. But recent discussion of the Keynesian concept of effective demand might make it advisable do elaborate the classical correspondence to Keynes in more detail, as will be done below.

g) The concept of effective demand: Some recent discussions

After several years of published discussion of Keynes' theory of effective demand, Patinkin (1982 ;p.150) came to the conclusion

> "that the obscurity with which the aggregate supply curve is presented in the General Theory is a sign not of profundity, but of obscurity: not, as some would have us believe, of a deep underlying analytical framework in which everything falls into place,..."

Although a number of authors, in particular Roberts (1978) and Casarosa (1982) contradicted vehemently against Patinkin's repeated attribution of obscurity to Keynes'

exposition of the Principle of effective demand, neither protagonist nor antagonist in this debate gave any consideration to the Keynes-Pigou connection just discussed. Such a development in this debate is most astonishing in view of Patinkin's (1978;p.593) conviction that

> "... a clarification of this issue depends first and foremost on a proper perspective on the nature of the history of ideas in general, and of the history of the development of Keynes' ideas in particular."

If the historical context of Keynes' development of the effective demand concept is invoked with such an insistence it is strange that Patinkin never even explains why he disregards Keynes(1936;p.273) correspondence between his own concepts and those of Pigou.

Patinkin's lack of historical perspective concerning the Keynes-Pigou debate is particularly remarkable when it comes to Patinkin's (1976;p.88) - later discarded - claim that Keynes' aggregate supply curve is a 45° curve. Ironically, this construction is not implicit in Keynes' presentation of aggregate supply but in Pigou's (1937) 'simplified model' devised to refute some 'recent arguments' coming from the Keynesian quarters. In the published literature, Patinkin's interpretation of Keynes' aggregate supply concept was criticized in detail by Roberts (1978). His rejection of Patinkin's attribution of obscurity was based mainly on definitorial investigations. His conclusion was that

> "most of Patinkin's criticism of Keynes' analysis are based on his definitions of Keynes' aggregate supply and demand functions, and these definitions are not consistent with what Keynes did say." [Roberts(1978;p.575)]

Roberts maintains that there is strong textual evidence that Keynes' aggregate supply function may be interpreted along traditional lines of profit maximizing firms, thus also emphasizing important aspects of the present exposition but without explicating the Keynes-Pigou connections which were given prominence in our investigation.

Further criticism came from Casarosa (1982) who gave particular attention to Patinkin's (1976;p.93) daring suggestion that those "words should simply be deleted from the General Theory which state that for a particular value of effective demand "entrepreneurs' expectation of profits will be maximized" (GT,p.25). Casarosa shows, however, as many other authors have done, that Keynes' aggregate supply and expected demand analysis can indeed be seen to be based on the theory of the competitive firm. In this context, Patinkin's criticism against Keynes is indeed groundless.

There is a certain problem involved, however, in invoking situations of perfect competition when discussing the microfoundations of Keynes' aggregate supply

analysis, since the latter relies on expectations of proceeds (GT,p.24). But for a firm operating under conditions of perfect competition the price of the product seems to be given irrespective of its supply so that there is no limit to the proceeds to be expected by the firm other than those imposed by the cost function of the firm itself. Casarosa therefore rightfully insists that in the context of the perfect competition assumption it would be incompatible to assume at the same time that entrepreneurs are working out specificc expectations concerning an expenditure function. Casarosa therefore proposes to see the aggregate expected demand function and the expenditure function as separate and unrelated concepts.

A consequence different from that of Casarosa could be chosen, however. The microfoundations of Keynes' agrregate supply function could very well be based not on the assumption of perfect competition but on, a Cournot-type oligoply model. In a second step the perfect competition case could then be preesented as a limiting case of the oligopolistic one, thus following a similar argument as Cournot himself once did. A detailed exposition of this suggestion would lead too far in the present context, however.

In the light of the controversies just reviewed we would like to close by noting that as far as Keynes himself is concerned, the textual evidence concerning the analytical position chosen is quite clear: Keynes was undoubtedly prepared to relate his own analysis of aggregate and sectoral supply to Pigou's assumption of profit maximization under conditions of perfect competition To claim otherwise would not only set the interpreter of Keynes crossing out a number of then unpalatable passages in the General Theory, vide Patinkin (1976). It would also obscure the essential contents of Keynes' explicit criticism of-Pigovian classicism .

d) The Pigovian contents in Keynesian effective demand: A graphical representation

The reconstruction of the classical model as perceived by Keynes is not only important in order to understand what Keynes wanted to argue against. It is also important for a knowledge of what Keynes argued with, i.e. of what he believed to be a common analytical ground between him and the classics.

In the preceding subsections one of the approaches to this conclusion resorted to "counting equations". It was argued that of the equations a) to e) stated in (35) it was only the ones giver in e) which were genuinely Keynesian. The others all had a classical correspondence. The effective demand doctrine thus appeared as the key-stone of a structure which corresponds strongly to the classical one.

In the following we want now to demonstrate in detail how the separate analytical

elements of the Pigovian classical model may be folded together in order to give the Keynesian argumentative structure which, in the case of effective demand is still very much in dispute as was just documented.

The Keynesian effective demand analysis of equ.(35) e) is represented in the fourth quadrant of the following fig.2. There, the c(N)-curve denotes cN which, together with $\bar{I}_w$ gives the aggregate demand curve f(N). This curve meets aggregate supple ZW= f (N) in point G. This point shows employment to be determined as $\overline{N}$. It is this analysis which must now be related to the Pigovian model of fig.1. This model is reproduced with minor omissions in quadrants two and three of fig.2.

Figure 2: Keynesian effective demand analysis and the Pigovian model

First it should be noted about this representation that a particular level of total employment $\overline{N} \stackrel{\wedge}{=} \overline{n}$ in the third quadrant may be translated into a specific level of sectoral employment $\overline{x} \stackrel{\wedge}{=} \overline{N}_1$ via the Pigovian j(x)-curve. But the corresponding amount of production stated as $F(\overline{x})$ in quadrant two may be evaluated in terms of wage units as

$$\overline{C}_w = \frac{F(\overline{x})}{F'(\overline{x})} = \overline{x}A$$

along the horizontal axis. This follows simply from the fact that

we have

$$\tan\alpha = \frac{w}{p_1} = F'(\overline{x}) = \frac{\overline{x}I}{\overline{x}A} = \frac{F(\overline{x})}{\overline{x}A}$$. Therfore $$\overline{x}A = \frac{F(x)}{F'(x)}$$ holds. A 45°-line in

translates then

$\overline{x}$ A from the horizontal axis as $O\overline{C}_W$ on the perpendicular axis. Points $\overline{x}$ and $\overline{C}_W$ are then the coordinates for a point on a sectoral supply function OCW in quadrant two. But transferring $O\overline{C}_W$, being effective demand in the consumption sector, from the perpendicular axis to the horizontal one by a further 45° construction gives, together with the employment level $\overline{N}$, a point on the consumption function c(N) in quadrant four.

A point on the aggregate supply curve f(N) is found by starting from a given $\overline{N} = \overline{n}$ and $\overline{x}$ constellation as represented by point E in quadrant three. The associated employment in the second sector is then $\overline{y} = \overline{N}_2$. Together with $\overline{I}_W$, $\overline{y}$ describes a point on the sectoral aggregate supply curve OIW in quadrant four, to be constructed in analogy to the OCW-curve in the second quadrant. Adding the value of $O\overline{C}_W$ to that of $O\overline{I}_W$ then gives $\overline{Z}_W$ which, together with $\overline{N}$, describes point G on the aggregate supply curve f(N) in quadrant four. This point represents the determination of employment by the intersection of Keynes' f(N)-curve on the one hand with $f(N) = \chi(N) + \overline{I}_W$ curve. It is easily to be seen that this determination need not violate the first classical postulate which is still valid in point I. of quadrant two. The second classical postulate is irrelevant, however, and not represented since it plays no role in the determination of employment due to the assumption of involuntary unemployment in the sense of Keynes.

As illustrated in fig.2, we may regard Keynes' aggregate supply and effective demand analysis indeed as representing in a very condensed form the disaggregated macro-model of Pigou (1933). In this sense it does stand for a "deep underlying analytical framework in which everything falls into place"[Patinkin (1982)] - and this framework is none other than the classical Pigovian one.

As the argument now stands, the level of employment may be seen either as being classically determined by a given level of real wages in the second quadrant through point I. or, alternatively, as being determined in a Keynesian way through the level of effective demand as given in point G of the fourth quadrant.

Thus, the Keynes-Pigou controversy is still open in our presentation. In order to tilt the scales in Keynes' favour, the logical possibility of classical employment determination through a given sectoral real wage must still be eliminated as being not valid.

c) Keynes' effective demand concept as a tool of critique

A purely formal juxtaposition of Keynesian and classical determination of employment is not satisfactory, of course. The substantive difference as stated by Keynes (1936;p.26) himself in this context was his rejection of Say's law that

> "the aggregate demand price of output as a whole is equal to its aggregate supply price for all volumes of output..."

What Say's law means in historical context is by no means clear as demonstrated by Sowell (1972). In Keynes' exposition of this concept we have again the rather unfortunate lack of distinction between the term "price" on the one hand and the aggregate value of a specific vector of outputs on the other. The latter should better be denoted as "value" in the sense of a price-quantity scalar product.

Incidentally, this lack of distinction is no peculiarity of Keynes'. It is well entrenched in the classical tradition of the English school and recently was demonstrated by Parinello (1985) as to be found in Marshall's (1920) analysis as well. But the Keynesian terminology need not distract from the now firmly established result that the Keynesian aggregate supply function is not just an analogy to microeconomic supply functions but results from an aggregation of sectoral price-quantity values as demonstrated in connection of the discussion of fig.2 above.

In this connection it should be emphasized that Keynes ' rejection of Say's Law does not imply the rejection of a

> "similar-looking proposition which is indubitable, namely that the income derived in the aggregate by all elements in the community concerned in a productive activity necessarily has a value exactly equal to the value of the output." [Keynes (1936;p.20), first two emphases added, GMA.)

Thus Keynes obviously accepted the identity between total income accruing in a specific period of production with the value of total production. Such fanciful constructions as to be found, e.g. in Malinvaud's (1977) reconsideration of Keynesianism according to which profits are not to be considered as income during the period of production under analysis due to deferred payment of dividends have nothing to do with Keynes' original analytical intentions and they are not covered by his rejection of Say's Law.

The problem which Keynes wanted to treat in this context clearly fall exclusively in the realm of a comparison of the value of total production as offered by entrepreneurs (ZW) on the one hand and the value of total demand forthcoming at a specific level of employment (DW) on the other hand. If equating these two values determines a

particular level of employment, then employment can not be seen as being determined by a coincidence of real wages as given by supply and demand curves for labour.

Put differently, we may conclude that in Keynes' view there are two alternative theories of employment: a labour market theory which he attributed to the classical school and an aggregates goods market theory which he propagated himself. Both can be seen as having the self-same analytical foundations in a way which was elaborated above. It is exactly because of such a common analytical background that the two diverging employment theoretical schools can then be asked to be very specific about the assumptions upon which they reject the results of their opponents. The Keynesian answer was that the labour market is obviously not working when there is involuntary unemployment in the strict sense. Therefore this market can not be postulated to determine employment. Keynes' proposal is that aggregate supply and demand must be brought in in order to do the unfinished job of the 1abour market.

The classical answer, in so far as it defends a real wage analysis, must necessarily reject the aggregate supply and analysis of the determination of employment. But if they can be brought to admit that these aggregate concepts have at least formal validity then they must deal with them in such a way as to warrant that their existence does not interfere with the real wage determination of employment. The classical answer to this problem must be that there is a perpetual identity of aggregate supply and demand so that their equality gives no determinate solution for the level of employment. In the rejection of this implicit postulate Keynes obviously sees his differentia specifica and the analytical prerequisite of his aggregate supply and demand theory of economic activity.

From the historical point of view it is to be noted that after the publication of the General Theory Keynes was quite concerned that the essential point of his critique of the classics might get lost - not so much by his opponents but by the suposedly "Keynesian" disciples. Thus, in response to a draft of Harrod's (1937) "Mr. Keynes and Traditional Theory", Keynes (JMK,XiV,p.85) complained:

> "You don't mention effective demand ... To me, the ost extraordinary thing regarded historically, is the complete disappearance of the theory of the demand and supply for output as a whole,... One of the most important transitions for me, after my Treatise on Money had been published, was suddenly realizing this."

This was no isolated complaint. In quite a similar vein Keynes (JMK,XXiX,p.215) criticized Lerner's (1936) exposition of his theory:

> "... you scarcely touch on...the breaking away from the assumption in some

shape of Say's Law. This could be described as the re-discovery of there
being a problem of the equilibrium of the supply and demand of output as a
whole, in short, of effective demand."

It is in this context that we must read Keynes' (1936;p.xxv) remark in the preface to the
German edition of the General Theory.

"Alfred Marshall['s]... theory of output and consumption as a whole, as
distinct from his theory of the production and distribution of a given output,
was never separately exposed. Whether he himself felt the need of such a
theory, I am not sure."

This last quote requires close reading because it refers us back to Clower's (1975;p.4)
seemingly similar view that Marshall "never felt the need to model the workings of the
economic system as a whole". But Keynes' remark is crucially different in relating not to
'economic systems a whole' but to 'output as a whole'.

In this context Keynes' unfortunate terminology must again be deplored. In connection
with effective demand Keynes oscillates between two equally misleading terms, namely
'aggregate supply price' and 'output as a whole'. As already remarked, the term "value"
in the sense of a price-quantity scalar would have been more appropriate.

With this distinction in mind, Keynes' speculation concerning Marshall's theory as given
in the last quote can be interpreted to mean: Marshall's theoretical perspective is one of
showing a determinate solution for a vector of prices and a vector of quantities when
ressources are given. This solution is seen as resulting from a corresponding system of
demand and supply functions. But in addition to the problem of co-ordinating these
vectors there is the problem of co-ordinating also certain price-quantity scalars, in
particular the scalar value of the sum of goods produced on the one hand and the scalar
value of goods demanded on the other. The former problem involves the general
equilibrium perspective, the latter problem relates to Keynes' 'effective demand' concept.

It would have been extremely strange if Keynes had overlooked Marshall's general
equilibrium perspective (see above,p.17f). We concluded above already that this
perspective was accepted by Keynes himself and that it was used for his internal,
'positive', critique of the classics. The additional perspective of aggregate scalar value
was then distinguished and used for an exposition at Keynes' 'negative' critique, of his
denial of the validity of Say's Law.

6. The significance of the reconstructed classical model

a) The classical model and the "Cambridge" paradigm of aggregate analysis

The present reconstruction of the classical model as received by Keynes might be helpful in taking a stand in the long-lasting debate concerning the paradigm of sectoral disaggregation in macroeconomic model building. There have been lengthy investigations purporting to convey that Keynes' macroeconomic model must have been of a single-sector structure [Froyen (1976)].

Harry Johnson (1978;p.149) claimed that even the entire Cambridge School for a long time refrained from extending its thinking to two-sector disaggregation:

> "The Cambridge mind, however, never stretched to the two-sector model, until they began working on the line of proving that capitalism could not work."

It is not quite clear from this remark which Cambridge authors Johnson had in mind as being critical of the working of capitalism - presumably it was Joan Robinson and Piero Sraffa. It certainly cannot have been Pigou (1933). He never questioned the working of capitalism as such. What Pigou (1933) did criticize was the application of disaggregated real wage analysis for a theory of aggregate employment (see sect. 2a) above). It is quite clear that with his distinction of wage-goods and non-wage-goods the 'Cambridge mind' did stretch to a macroeconomic model extending to two sectors of production, at least.

Johnson's characterization of the Cambridge paradigm seems to result from a discontentment with the list of classical means of increasing employment which was given by Keynes (1936;p.7) and which appeared above as being indeed 'comprehensive' in the reconstructed classical model, as it was claimed to be by Keynes in that context. One aspect entirely absent in that list was the question of sectoral intensities in the technology of using capital and labour. But Johnson (1978;p.149) combined his characterization of the analysis of the Cambridge school with an alternative employment theoretic statement which must appear as the basis of his criticism:

> "In a two-sector model there are two...industries using labour and capital in different proportions. A shift of production towards the labour-intensive industry increases the demand for labour,... " .

What Johnson had in mind in this context seems to have been a particular two-sector model in which capital is not only "malleable" - a condition which gives sectoral production functions with variable input combinations of capital and labour so that "well behaved" production functions in the neoclassical sense may be generated. For Johnson's thought experiment to work capital must in addition be also "shiftable" from one sector to the other. Otherwise the increased production will not raise the profit

maximal real wage but it will lower it due to decreasing returns.

Thus, reconstructing the reasoning behind H.G. Johnson's above quoted statement about the limitations of the "Cambridge mind" reveals that this derogatory statement itself rests upon a two-sector model which is rather limited in its analytical applicability.

We hope to have demonstrated in sufficient detail above that it was none other but A.C. Pigou (1933) as exponent of the orthodox Cambridge tradition who insisted on the complexities of sectoral real wages and aggregate employment. Since we also discussed in detail how this model was taken up by Keynes in, both, his "positive" and his "negative" critique of the classics, we believe to be now in the position to insist: The Cambridge tradition was well aware of the complexities involved in aggregate analysis. The basic model contained, both, in Keynes' analysis and in that of Pigou, was one of sectoral disaggregation. It is one of capital being non-shiftable between sectors in the short run but sufficiently "malleable" to warrant the postulation of differentiable sectoral production functions.

b) The classical reconstruction and the Keynesian microfoundation

The question of disaggregation, just discussed, must be discerned from the question of the microfoundation of sectoral and macroeconomic variables. In the more recent Keynes-Classics debate the latter topic has also been discussed at considerable length. The literature concerning Keynes' effective demand analysis as reviewed briefly above (sect.5bg)) touched on these problems. They are part of the larger complex of "reductionism" of Keynesian aggregate analysis (see Coddington (1982) for an elaboration of this concept).

The comparative ease with which Keynes ' effective demand analysis could be related to the Pigovian model (see the discussion in the context of fig.2 above) and the reconstruction of the Pigovian model from the marginal analytic microfoundations of the Marshallian School might suggest that the outcome of our analysis is devised to suggest that this was the microfoundation which Keynes himself intended to be seen behind his aggregate analysis. But we were eager to preserve Keynes' own characterization in this context of there existing a 'correspondence' between some of his own and classical analysis. This does not necessarily mean that Keynes wanted to adopt and to propagate the classical microfoundations for his own analysis.

In fact, soon after the publication of the General Theory, Keynes disassociated himself emphatically from "the old associations of my mind" (see JMK,XXiX,p.246). Since this seems to be a proclamation of considerable importance for the characterization of Keynes' post-General Theory program, we will inspect it in some detail. The immediate

context of this statement reads as follows:

> "I have got bogged in an attempt to bring my own terms into rather closer conformity with the algebra of others than the case really permits....It amounts to very little, contributes nothing to the understanding of the argument and is simply encouraging the reader to waste his time in a rather futile sort of way. I am conscious that this, like a good deal else in the book, is largely the product of the old associations of my mind, the result of always trying to see the new theory in its relation to the old and to discover more affinities than really exist."

From this passage the impression must arise that Keynes considered as a grave mistake his old emphasis of a correspondence between his own analysis and "the algebra of others", and hence in particular that of Pigou (1933).

But what, exactly, triggered off such a sweeping reconsideration? The wider context of Keynes' 'second thought' was a correspondence with H.Townshend, a former student and extremely dilligent reader of the General Theory who spotted a number of definitorial inconsistencies in Keynes' treatment of user cost in chs.4 and 20,GT. (See JMK,XXiX,p.239-247).

The specific point of Townshend may be illustrated with reference to our Keynes-Pigou correspondence of equs.(34) and (35). According to those equations we have

$$(37) \qquad D_w = Z_w \mathrel{\hat{=}} \frac{F(x)}{F'(x)} + \frac{G(y)}{G'(y)}$$

If user cost (depreciation etc.) are zero, then there is no problem with such a correspondence. But if there are user cost functions U1(x), U2(y), as Keynes insisted that there are, we have a difference between the total value of turnover on the one hand and the total value of income (YW) on the other:

$$(38) \qquad \frac{p_1}{w}\left\{F(x) + U_1(x)\right\} + \frac{p_2}{w}\left\{G(y) + U_2(y)\right\} \neq Y_w$$

But then, if sectoral effective demand must meet the sectoral turnover as planned by profit maximizing firms, its sum cannot be equal to national income since specific

sectoral turnover contains non-income generating payments from one firm to an other firm. How should one, according to Keynes, take account of this problem?

In a reply to Townshend, Keynes made some tentative reformlulations of his original algebraic formulations but soon confessed his frustration with his attempt to relate his own concepts to "the algebra of others". To take full account of Townshend's critical question concerning user cost really requires a rather involved modelling of production of commodities not only by means of labour and other factors but also by other commodities produced within the same period of production. It is therefore quite understandable that a number of post-Keynesian authors propagated Sraffa's (1960) 'neo-Ricardian' approach as the one most appropriate for a discussion of the Keynesian economy. This solution to the user cost problem would then imply a dramatic break with the Marshallian marginal analytic microfoundations which were here taken as the common analytical ground between Keynes and the Classics.

But there is a further alternative in order to take account of the difference between turnover and income. It is the explicit modelling of depreciation of capital in an otherwise conventional model of production. This approach was implicit in Joan Robinson's (1937) 'Keynesian extension,'. Although this approach was later given up by her, we believe that a detailed inspection of the microfoundations of Joan Robinson's (1937) model might give some valueable information concerning a possible microfoundation of important aspects of Keynes' analysis which is not covered by the classical corespondence which is our present subject matter.

Taking the passage just discussed as a basis for an approach to the microfoundations of the General Theory, we conclude that it suggests two possible perspectives. If the aim is to understand Keynes' critical view of the classics, then we have to notice that he was "always trying to see" his new theory in relation to the old classical one. It was this perspective which was developed in the above. If the intention is, however, to give a positive statement of the Keynesian scheme, abstracting from Keynes' controversy with the classics, then the present perspective is not necessary and it may be even wasteful according to Keynes' own judgement.

In view of the continued interest in not only Keynesian but also classical modes of analysis we believe that it is nevertheless of continued interest to find a common analytical ground connecting the two co-existing schools.

c) The classical reconstruction and the irrelevance of functional aggregation

Having stressed the disaggregation and the microfoundations implied in the reconstructed classical model, it remains to comment on the fact that it nevertheless is a

model of aggregate economic activity as indicated by total employment.

It is well known that aggregation in economics poses formidable problems. In particular, except for trivial cases of sectoral identity, it is impossible to take neoclassical production functions which depict production and marginal productivities on different sectoral levels and to aggregate them so that the self-same type of functional relationship is then generated on the resolution level of the economy as a whole.[14] Since this Gordian knot cannot be unravelled, the modern economists do away with it by "heroic assumptions", i.e. by postulating the existence of what cannot be deduced.

It is now important to remember that this is not the way in which Keynes and Pigou resolve the aggregation problem. Nor do they claim to be able to solve the insoluble. They rather circumvent this problem by making irrelevant the question of aggregating sectoral production functions.

In arriving at a theory of aggregate employment Pigou through his j(x)-function, avoids the conventional problem of aggregating sectoral production functions by postulating an entirely new functional relationship. It was seen above that this function incorporates not only technological conditions but also information about preferences. Thus it is of entirely different conceptual status than the macroeconomic production function which seeks to deny preferences as an argument in the forrulation of macroeconomic production.

Pigou's j(x)-function which was the centerpiece of our classical reconstruction is also dimensionally interesting in taking both, dependent and independent variables as being of the same dimenisionality, namely as aggregate and sectoral labour services, respectively. This aspect of Pigou's construction survived in Keynes theory as well since he preserved Pigou's j(x)-function in the guise of the employment multiplier k'. But Keynes (1936;p.41) carried the dimensionality aspect just mentioned even further:

> "In dealing with the theory of employment I propose, therefore to make use of only two fundamental units of quantity, namely, quantities of money-value and quantities of employment."

This proposal amounted to conducting the entire analysis in terms of magnitudes of the dimension of labour since the money-values of aggregate supply (Z) and demand (D) may be easily transformed into labour-units by dividing with the wage-unit (W) giving, in dimensional notation:

$$\frac{\left(\dfrac{[\text{money price}]}{[\text{units of goods}]}\right) \bullet \left(\dfrac{[\text{units of goods}]}{[\text{time period of producition}]}\right)}{\left(\dfrac{[\text{money (wage)}]}{[\text{units of labour}]}\right)} = \left(\dfrac{[\text{units of labour}]}{[\text{time period of production}]}\right)$$

which stands for
$$\frac{Z \text{ resp. } D}{W} = Z_W \text{ resp. } D_W$$

If now ZW is set in relation to total employment N as is the case in Keynes' effective demand analysis, then we have again a relationship between a magnitude in the dimension of "labour" and total employment which is of the same dimension. In this sense Keynes' effective demand analysis may be seen as a further development of the approach taken by Pigou (1933) in the formulation of a j(x)-function.

Modern macroeconomists rarely contemplate any of the constructions just discussed. Neither the Pigovian j(x)-function nor the Keynesian Zr(Nr)-functions play any role in present-day teaching or research. There is a seeming correspondence to Keynes' effective demand analysis in so far as a considerable number of newer textbooks [e.g. Branson and Litvack (1976), Dornbusch and Fischer (1978)] use aggregate supply functions. They seem to be a straightforward adaptation of Keynes' effective demand analysis and of his concept of "output as a whole". But contrary to Keynes and Pigou these textbooks resort to single macroeconomic production functions and corresponding marginal productivity functions. In constructing on this basis a macroeconomic supply function as a relationship between a macroeconomic price index and a macroeconomic quantity index the modern authors in addition resort to the assumption of exogenously given rigid money wages. There seems to be little awareness that Keynes' proposal of doing national accounting in terms of wage units enables the economist to abstract from changes in the money wage unit and thus permit it to vary parallel with the discussion of aggregate supply.

We conclude that in several respects the Keynes-Pigou approach to the transition from sectoral to macroeconomic analytical concepts seems to be superior to modern textbook presentations of comparable concepts.

7. Summary and conclusion

In the present paper we attempted to reconstruct the authentic model of classical

analysis as perceived by Keynes in the Keynes-Classics controversy. We set out by first discussing in some detail the desirability of such a project: The question of the analytical substance of the Keynes-Classics controversy has been discussed for decades and still is of considerable significance for modern economic theorizing. Yet it is mostly agreed that existing reconstructions of the "Classics" in this contex are unsatisfactory from the point of view of authenticity.

The difficulties to give a convincing rendering of the differentia specifica of Keynes' analysis are partly understandable in view of Keynes' professed program to change the paradigm of economic analysis.[15] But such a program is, to a certain degree pre-scientific because the superiority of a new paradigm can not be proved but must be experienced, as Kuhn (1962) showed in the case of the development of the natural sciences.[16]

If that is indeed the case, than even the intention of supplying rational arguments abstracting from the actual execution of research must appear as misdirected for actually effecting a change in paradigm while the justification of such a change in view of future research must lack the power of proof.

Faced with this dilemma, Keynes reacted in part with polemics, but in part also with the demonstration that his new paradigm incorporated essential elements of the old. Thus, in the latter context Keynes could easily appear as just intending to establish himself as the 'better classical economist'.

In the present paper we put particular emphasis on this aspect of Keynes' original argumentation, disclaiming however that our reconstruction of his internal criticism of traditional classical analysis was able to represent the entire scope of his theoretical intentions. In particular, it is well known by now [see Corry(1978)] that Keynes' (1936) discussion of Pigou (1933) did not involve explicit interest theoretic considerations. Thus, due to the present intention of a recreation of the Keynes-Pigou debate as it manifested itself in the General Theory, the present analytical perspective was very limited and did not cover the interest theoretic aspects of the Keynes-Pigou debate. This latter aspect of the debate was brought into clearer relief only after the publication of the General Theory [see Ambrosi (1986)].

Keynes' criticism against Pigou amounted foremost to the demonstration that from the logic of the classical theory Pigou had to argue in the context of a general equilibrium model unless he disassociated himself from this tradition. No such disassociation is to be found in Pigou, however.

Keynes' further approach then involved making Pigou's model of an economy consistent with the classical labour supply theory. But in such a model it is not possible simply to advocate a drop in real wages in order to increase employment. The classical means of increasing employment are more complicated than this as was demonstrated by Keynes in chapter two of the General Theory. As far as we know, our interpretation is the first instance in the voluminous Keynes-Classics literature that a microfoundation was supplied for this aspect of Keynes' criticism of the classics.

The analytical framework for this discussion was a two-sector one. In developing this framework no fanciful re-interpretation of the past analysis was required since this analytical stage was set clearly by Pigou (1933) already. It is therefore not correct when Johnson (1978) claims that the 'Cambridge mind' was unable to stretch to the two-sector model of an economy unless it busied itself with the critique of capitalism.

A related aspect discussed in the above was the demonstration that Keynes was aware of an important correspondence between the Pigovian disaggregated model on the one hand and his own analysis of aggregate effective demand on the other. In order to visualize this point clearly we resorted to a detailed graphical representation of this correspondence showing that neoclassical sectoral production functions and Keynesian aggregate analysis could very well co-exist with each other. But that this possible co-existence was clearly seen by Keynes himself does not show "how little Keynes himself was concerned with intellectual consistency" as Coddington (1980) claimed. It rather shows Keynes' transition to a new type of analytical method which circuvented the well-known problems of aggregation of neoclassical sectoral productive relationships without having to negate the established theory of production on a disaggregated level of analysis.

The essential step taken by Keynes in order to resolve the aggregation problems inherited frem neoclassical analysis was his own analysis in terms of wage units.The significance of the transition to this technical device still seems to be quite unclear among economists so that Coddington (1980) rejected Chakrabarti's (1979) two-sectoral formulation of certain aspects of the General Theory in terms of wage units on the grounds that it left this type of analysis as still "mysterious" in his mind. What lies behind this concept is not mystery or extravagance but dimensionally consistent aggregation as we tried to demonstrate in the above.

In the present interpretation the transition from Pigou's analysis in terms of well behaved neoclassical production functions to the Keynesian analysis in terms of sectoral value supply functions involved a transition from an economic representation in terms of non-linear productive relationships to one in terms of linear relationships. If the implied assumption of constant elasticities of production seems to be acceptable then this procedure could show a way how to combine Keynes' acceptance of the marginal analytic first classical postulate with a seemingly contradictory fixed coefficient input-output scheme. In how far such a combination would open up the way for a new and fruitful disaggregated analysis in macroeconomic context - that is a question which must be treated in appropriate thoroughness under a different heading.

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Abstract:

The paper gives a two-sector model of Pigou's (1933) Theory of Unemployment. It is shown that in his criticism of the Classics, Keynes extended this model in several ways. First he formluated conventional labour supply considerations in the context of this model, interpreting it as one of general equilibrium. The Marshallian aspect of this view is investigated in detail. One strand of Keynes' critical discussion of the classical theory of employment is based on this extension - or rather explication - of the Pigovian model.

In his theory of effective demand Keynes pursues a different argumentation but he nevertheless does not reject the Pigouvian model as such. He rather uses a further extension of it in order to show that it is overdetermined in the case of unvoluntary employment. The main aim of this particular strand of Keynes' critique is to show that Pigou (1933), in discussing changes in the real demand function for labour, discusses changes in the sectoral structure of labour demand. What Keynes wants to establish as being of central importance for employment is, however, the level of demand.

In arguing this case, Pigou's original two sector model may be easily aggregated to give the Keynesian presentation of an economy as a whole. This is shown graphically. The last section of the paper then discusses the wider significance of this reconstruction, giving particular attention to problems of dimensionality and of aggregation.

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[1] See [i] and [iv] of the quote from Pigou in the preceeding section

[2] Pigou (1933;p.88)"As throughout this book, the whole problem is treated as a short period one, in the sense that slow-working reactions from changes in fixed capital equipment are left out of account;"

[3] In particular, labour´s partial elasticity of production ai is well defined and lies in the range 0<ai <1. See also equ. (15) below.

[4] See [iii] in the preceeding section. This assumption appeares in Keynes (1936;p.5) as the "first classical postulate".

[5] This is just a slight reformulation of Jevon´s labour supply analysis. Cf. Blaug (1936;p.314). See also Keynes (Collected Works - JMK henceforth - vol.XiV,p.25): "The theory of Pigou and of Marshall and of all modern economists is that different quantities of labour are on offer according to the amount of wage goods offered in return for a unit of labour."

[6] In the General Theory, ch. 2, Keynes refers to this assumption as the "second classical postulate".

[7] This follows from [v] above when the "trading surplus" mentioned there is assumed to be positive.

[8] See e.g. Marshall (1920;p.99)

[9] Equ. (11) is related to (9) by dividing the latter with V, the utility of non-workers, and then expanding the left and right hand sides with Q1k resp. Q2. The final step then involves the application of the definitions given in (12).

[10] See Keynes' letter to R.G.Hawtrey about this matter in JMK, XiV,p.53: Pigou (1933) "is assuming a supply schedule of labour, though an L-shaped one, which means that he is assuming that the wage stipulated by all wage earners is the same (though I do not think it would much affect his argument if he were to make the straight-line portion of his supply schedule into a curve)."

[11] Cf. Keynes in JMK,XiV,p.40f.:"I assume that ... Pigou [is] not importing into the argument any change in technical conditions which are assumed as given."

[12] Actually Pigou (1933;p.114) does admit a specific type of technological change which he characterizes as "normal type of improvement". His definition of this type of technological change permits Bi-changes but not ai-changes, since, in our symbolism it requires $\dfrac{F_x(x,T)}{\left[F_x(x,T) + F_{xT}(x,T)\right]}$ =const. to hold where T=shift parameter for technology, i.e. either B1 or a1 in the present formalism.

[13] The correspondence just noted invites a number of further comments. It should be noted that from re-writing (36) $N_1 = \alpha_1 c(N_1 + N_2)$ it follows after minor rearrangements that Keynes "employment multiplier" is $k' = \dfrac{\alpha_1 c}{(1 - \alpha_1 c)}$ in the present case. But Keynes insisted (GT,p.273) that Pigou's j' is the same as $1 + \dfrac{1}{k'}$ in his own symbolism. Replacing k' by the value just given in the preceding sentence will then generate indeed the value of j' as given above in (36). This then could serve as a further hint for the authenticity of this correspondence. Actually, equ.(36) was expressly formulated by Keynes (JMK,vol.XiV,p.373): "Thus we can sum up the factor of consumption by saying that if aggregate current output is the product of employing N men, effective demand from consumption will employ N1 men, where N1= j(N), the function j depending on the propensity to consume."

[14] The most detailed investigation of this problem seems to be the one by Streißler (1959).See also Green(1964)

[15] Pigou (1936;p.115) actually acknowledged this intention, although sardonically, in commenting on Keynes' program:"Einstein actually did for Physics what Mr. Keynes believed himself to have done for Economics."

[16] Keynes (JMK,XIII,p.470) was well aware of a comparable problem in the context of economics in stating: "In economics you cannot convict your opponent of error; you can only convince him of it." (Keynes' emphasis,GMA).