**Exercises for**
**"Mathematics for Economists"**
**Trier, Winter Semester 2019**

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## Exercises for Chapter 1:

### Simple Algebra

Simplify the following terms as much as possible:

1. $5(x-3) - 2x(x+y-1)$
2. $\dfrac{x^{a+b}}{x^{2a}}$
3. $\dfrac{y^6 + 2 \cdot y^5 + 6 \cdot y^4}{y^4}$
4. $\dfrac{y^2 + 10 \cdot y + 50}{(y+5)^2 + 25}$
5. $(3 \cdot y + 2 \cdot x + c)^k \cdot \left(\dfrac{3 \cdot y}{4} + \dfrac{x}{2} + \dfrac{c}{4}\right)^{-k}$
6. $\dfrac{K^2}{L \cdot (L + K^2/L + 2 \cdot K)} + \dfrac{L \cdot (L + 2 \cdot K)}{(L+K)^2}$

Find all values for the variable $x$ that solve the equation (consider all other elements as constants):

7. $x^2 = 16x$
8. $\dfrac{3x+3}{6} + b = 2a$
9. $4x^2 + 12x - 18 = 0$
10. $P(x^2 + \dfrac{R}{P}x) + C = 0$

### Equations

1. Solve the following system of equations with *two* different algebraic methods:

$$\begin{aligned} 8x + 25y &= 100 \\ x + 10y &= -50 \end{aligned}$$

2. Assume that $x_1$ and $x_2$ are solutions to the equation

$$a \cdot x^2 + b \cdot x + c = 0 \ .$$

Find the most compact form for $x_1 + x_2$. Hint: use the quadratic formula.

3. Assume again that $x_1$ and $x_2$ are solutions to the equation

$$a \cdot x^2 + b \cdot x + c = 0 \ .$$

Find the most compact form for $x_1 \cdot x_2$. Hint: Use the quadratic formula or the rule that

$$a \cdot x^2 + b \cdot x + c = 0 \qquad \Leftrightarrow \qquad a\,(x - x_1)\,(x - x_2) = 0 \ .$$

**Exercises for Chapter 2:**

**Summation**

Find closed form terms (without summation signs) for the following sums and simplify them as far as possible.

1. $$\sum_{x=1}^{n+1} x^2$$
2. $$\sum_{x=1}^{10} x+1$$
3. $$\sum_{L=1}^{10} (L+1)$$
4. $$\sum_{a=1}^{n} 2^a$$
5. $$\sum_{b=10}^{n+11} b^3$$

Rewrite the following expressions with summation sign(s):

6. $x_1(x_1-1)+2x_2(x_2-1)+3x_3(x_3-1)$
7. $x_1(x_{11}-1)+x_1(x_{12}-1)+2x_2(x_{21}-1)+2x_2(x_{22}-1)+3x_3(x_{31}-1)+3x_3(x_{32}-1)$

**Sets**

Given the sets $S_1 = \{2,4,6\}$, $S_2 = \{7,2,6\}$, $S_3 = \{4,2,6\}$, and $S_4 = \{2,4\}$, which of the following statements are true?

(a) $S_1 = S_3$
(b) $S_1 = \mathbb{R}$
(c) $8 \in S_2$
(d) $3 \notin S_2$
(e) $4 \notin S_3$
(f) $S_4 \subset \mathbb{R}$
(g) $S_1 \supset S_4$
(h) $\varnothing \subset S_2$
(i) $S_3 \supset \{1,2\}$

Referring to the four sets given, find

(j) $S_1 \cup S_2$
(k) $S_4 \cap S_2 \cap S_1$
(l) $S_3 \cup S_1 \cup S_4$

**Exercises for Chapter 3:**

**Two Functions With Two Unknowns**

Assume that supply and demand in a market follow the following equations (with quantities $Q$ and price $P$):

$$\begin{aligned} \text{demand} &: \quad Q = 10 - P \\ \text{supply} &: \quad Q = \frac{P^2}{5} \end{aligned}$$

Calculate the price $P$ and the quantity $Q$ that solve these two equations. This solution represents the market equilibrium. Sketch the two functions in one diagram and indicate the equilibrium.

**Graphical Representation of Functions**

Sketch the following functions:

$$\begin{aligned} f(x) &= 20 - 2x^{0.5} \\ g(x) &= -0.25x^2 + x - 10 \end{aligned}$$

**Inversion of Functions**

Sketch the function

$$y = f(x) = \frac{2}{x-1}$$

Find the inverse of this function and denote it by $g(y)$.

**Computing With Functions**

**1.** Find the range and domain of the following two functions:

$$f(u) = e^u \qquad \text{and} \qquad g(x) = \frac{x}{x-50}$$

Find the combinations $f \circ g$ and $g \circ f$.

**2.** Find the range and domain of the following two functions:

$$f(x) = \sqrt{x} \qquad \text{and} \qquad g(x) = 2x^2 + 2x + 2$$

## Exercises for Chapter 4

### Differentiation

Calculate the first and the second order derivatives of the following functions:

1. $f(x) = 20x^4 + 10x$
2. $f(x) = e^{4x} + \ln(6x)$
3. $f(x) = e^{4x} \cdot \ln(6x) + \ln(a)$
4. $f(x) = e^{\ln(4x)}$
5. $f(x) = r(k - x^2)^r$
6. $f(x) = ax^4 + x^{-1}e^{2x}$

**Exercises for Chapter 5:**

**Extreme Values**

Consider the following three functions:

$$\begin{aligned} f(x) &= (x-6)^2 - 10 \\ g(x) &= \frac{1}{3}x^3 + 0.7x^2 - x \\ h(x) &= 2x^3 - \frac{3}{4}x^4 \end{aligned}$$

For each of these functions,

**a)** calculate the first, second and third order derivatives (if possible),

**b)** find out if there are points at which the dependent variable is equal to zero,

**c)** find all extreme and inflection points (both local and global), and

**d)** sketch the graph.

**Exercises for Chapter 6:**

**Partial Derivatives**

Find every first- and second-order partial derivative (including cross second-order partial derivatives) of the following functions:

$$\begin{aligned} f(x,y) &= 2x^2 + e^{4y} \\ g(x,y) &= 2y\ln(x) \\ h(x,y) &= 2xy^4 - e^{xy} \end{aligned}$$

**Production Functions As Examples for Functions With Many Variables**

**1.** The production function is given by:

$$Q(K,L) = \left(K - \frac{L}{4}\right)^{\frac{2}{3}} \cdot L^{\frac{1}{3}}$$

where $Q$ is the quantity produced while $K$ and $L$ denote the amount of labour and capital that are used in the production of $Q$. What is the degree of homogeneity?

**2.** Consider the production function:

$$Q(K,L) = a \cdot K^b L^c$$

where $Q$ is the quantity produced while $K$ and $L$ denote the amount of labour and capital that are used in the production of $Q$. Furthermore, $a$, $b$, and $c$ are parameters with $a, b, c > 0$.

**a)** Find the degree of homogeneity.

**b)** For which positive values of the parameters $(a, b, c)$ does the production function exhibit positive first-order partial derivatives but negative second-order partial derivatives?

**Exercises for Chapter 7:**

Consider the function

$$f(x, y) = x^3 + 2xy - 5x - y^2$$

Calculate the first and second order partial derivatives (if possible). Also find and classify the stationary points.

**Exercises for Chapter 8:**

**Constrained Optimization**

Solve the following optimization problems with the Lagrange multiplier method. When you find more than one stationary point, compute the corresponding function values and check which of them is a maximum.

1. $\max f(x, y) = xy$ subject to $n(x, y, z) = x + 3y = 18$
2. $\max f(x, y) = x(y + 12)$ subject to $n(x, y, z) = x + y = 8$
3. $\max f(x, y, z) = x + y + z^2$ subject to $n(x, y, z) = x^2 + y^2 + z^2 = 1$

**Revenue Maximization With Given Budget (Cost)**

Suppose that you are running a factory, producing some sort of widget that requires steel as raw material. Your costs are human labour, which is € 20 per hour of labour ($h$), and the steel itself, which runs for € 160 per ton of steel ($s$). Suppose your revenue, $R$, is given by the following equation:

$$R(h, s) = 200h^{2/3}s^{1/3}$$

If your budget is € 19,200, what is the maximum possible revenue? Use the Lagrange approach (hint: have a look at method 4 of Section 1.5).

**Utility Maximization With Two Goods**

A household lives on water ($W$) and bread ($B$). The household's utility is given by the following function:

$$U(W, B) = 100WB + W + 2B$$

**(a)** Using the Lagrange multiplier method, calculate the optimal consumption plan if the household has a budget of 1000 € and water costs 2 € per unit while bread costs 4 € per unit (assume that all money not spend on water or bread is lost).

**(b)** What would happen to the consumption plan if all prices doubled?

**(c)** What would be the impact of an increase of the price of water to 8 € whereas the price of bread remains at 4 €?

**(d)** Assume that another household has the same budget, but its utility function is

$$U(W, B) = 2W + 3B$$

What would be the optimal consumption? Assume that the original price structure is still in place.

**Exercises for Chapter 9:**

**Basic Operations**

**1.** The following matrices are given:

$$\mathbf{A} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 4 & 4 \\ 5 & 5 & 5 \end{bmatrix} \qquad \mathbf{B} = \begin{bmatrix} 4 & 4 & 4 \\ 1 & 2 & 2 \\ 5 & 3 & 2 \end{bmatrix} \qquad \mathbf{C} = [2]$$

Calculate:

**(a)** $\mathbf{A} + \mathbf{B}$

**(b)** $\mathbf{A} - \mathbf{B}$

**(c)** $\mathbf{A}' + \mathbf{B}'$

**(d)** $\mathbf{CA}$

**(e)** $\mathbf{BC}$

**2.** The following two matrices (vectors) are given:

$$\mathbf{A} = \begin{bmatrix} 2 \\ 4 \\ 5 \end{bmatrix} \qquad \mathbf{B} = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$$

Calculate $\mathbf{C} = \mathbf{AB}$.

**3.** The following two matrices are given:

$$\mathbf{A} = \begin{bmatrix} 4 & 3 & 2 \\ 5 & 6 & 7 \end{bmatrix} \qquad \mathbf{B} = \begin{bmatrix} 1 & 7 \\ 5 & 8 \\ 6 & 9 \end{bmatrix}$$

Calculate $\mathbf{C} = \mathbf{AB}$.

**Rank of a Matrix**

Show that the matrix

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & 2 \\ -1 & 2 & 0 \\ 1 & -2 & 0 \end{bmatrix}$$

is a singular matrix.

**Definiteness of a Matrix**

Examine whether the matrix

$$\mathbf{A} = \begin{bmatrix} -1 & 3 \\ -1 & -2 \end{bmatrix}$$

is negative definite.