Monetäre Märkte und Zinsbildung

Neukeynesianische monetäre Ökonomie (Walsh, Kapitel 8)

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- Both equations are derived from optimizing economy's agents.
- Hence, those micro-founded curves overcome shortcomings of the traditional AS-AD model.
  - They are derived from primitive tastes and technology assumptions.
  - They comprise forward looking behaviour of agents.
- This model represents state of the art macroeconomic modeling.

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  - Controls nominal interest rate

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- In our simplified model the monetary authority sets the nominal interest rate according to a simple rule or reaction function
- Alternatively, such a policy rule could be justified using a loss function of the central bank

Model overview

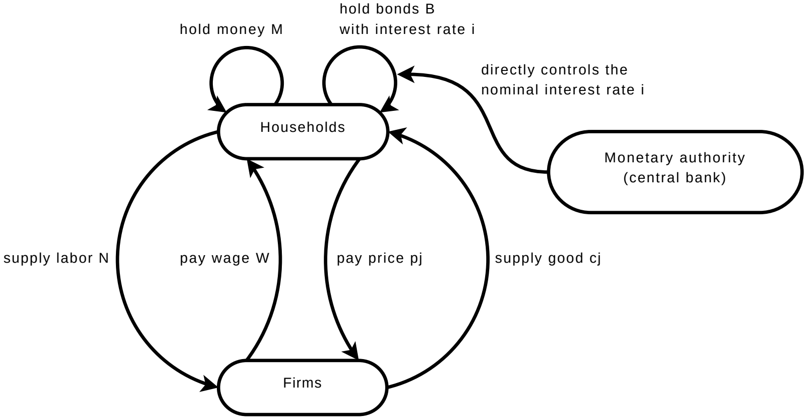

Figure: Short representation of model's agents

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  1. Agents optimize utility or profits respectively subject to a constraint.
  2. Occasionally first-order conditions are combined or rearranged.
  3. Optimality conditions are log-linearized around a steady state.

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$$\mathbb{E}_t \sum_{i=0}^{\infty} \beta^i \left[\underbrace{\frac{C_{t+i}^{1-\sigma}}{1-\sigma}}_{\text{consumption}} + \underbrace{\frac{\gamma}{1-b} \left(\frac{M_{t+i}}{P_{t+i}} \right)^{1-b}}_{\text{real money holding}} - \underbrace{\chi \frac{N_{t+i}^{1+\eta}}{1+\eta}}_{\text{working time}} \right] \quad (1)$$

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$$C_t = \left(\int_0^1 c_{jt}^{\frac{\theta-1}{\theta}} d_j \right)^{\frac{\theta}{\theta-1}} \tag{2}$$

with price elasticity of demand $\theta > 1$.

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$$\min_{c_{jt}} \int_0^1 p_{jt} c_{jt} \, dj$$

subject to

$$\left(\int_0^1 c_{jt}^{\frac{\theta-1}{\theta}} \right)^{\frac{\theta}{\theta-1}} \geq C_t, \tag{3}$$

where p_{jt} denotes the price of consumption good j.

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$$c_{jt} = \left(\frac{p_{jt}}{P_t} \right)^{-\theta} \underbrace{\left[\int_0^1 c_{jt}^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}}}_{=C_t \text{ (see (2))}}$$

$\Longrightarrow$ Interpretation?

Contents

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subject to the following budget constraint

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- It experiences disutility from working N_t.

Household's Lagrange problem II

- Again we use the Lagrange approach to solve the constrained problem:

$$\mathcal{L} = \mathbb{E}_t \sum_{i=0}^{\infty} \beta^i \left\{ \frac{C_{t+i}^{1-\sigma}}{1-\sigma} + \frac{\gamma}{1+b}\left(\frac{M_{t+i}}{P_{t+i}}\right)^{1-b} - \chi \frac{N_{t+i}^{1+\eta}}{1+\eta} \right.$$
$$+ \lambda_{t+i}\left[\left(\frac{W_{t+i}}{P_{t+i}} N_{t+i}\right) + \frac{M_{t-1+i}}{P_{t+i}} + (1+i_{t-1+i})\left(\frac{B_{t-1+i}}{P_{t+i}}\right) + \Pi_{t+i} \right.$$
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- The representative household chooses consumption, labor supply, money and bond holding optimally.

First order conditions

- Algebraically we compute the following FOC:

$$\frac{\partial \mathcal{L}}{\partial C_t} = C_t^{-\sigma} - \lambda_t \overset{!}{=} 0 \tag{I}$$

$$\frac{\partial \mathcal{L}}{\partial N_t} = -\chi N_t^{\eta} + \lambda_t \left(\frac{W_t}{P_t}\right) \overset{!}{=} 0 \tag{II}$$

$$\frac{\partial \mathcal{L}}{\partial M_t} = \gamma M_t^{-b} \frac{1}{P_t^{1-b}} + \mathbb{E}_t \left(\beta \lambda_{t+1} \frac{1}{P_{t+1}}\right) - \lambda_t \frac{1}{P_t} \overset{!}{=} 0 \tag{III}$$

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- Rearranging first-order conditions yields:

$$U'(C_t) = \beta(1 + i_t)\, E_t\left[\frac{P_t}{P_{t+1}} U'(C_{t+1})\right]. \tag{6}$$

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$$\frac{\chi N_t^{\eta}}{C_t^{-\sigma}} = \frac{W_t}{P_t} \tag{5.54}$$

- (5.54) sets the MRS between leisure and consumption equal to the real wage.

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- This is explained by a fixed capital stock in the short run.

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- Price stickyness is modeled using Calvo (1983) staggered price setting.
- Each period a fraction $1 - \omega$ of all firms adjust their prices optimally.
- The residual fraction ω are not able to adjust prices this period.
- Firms are randomly selected to belong to either fraction.
- Profits at date $t + s$ are affected by the choice in t only if the firm does not receive another opportunity to adjust prices between t and $t + s$.

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- Taking the derivative with respect to N_t yields

$$\frac{W_t}{P_t} + \varphi_t Z_t \overset{!}{=} 0$$

$$\varphi_t = \frac{W_t / P_t}{Z_t} \tag{5.55}$$

Firms' optimization problem

- Firm j sets its price in period t to maximize

$$\mathbb{E}_t \sum_{i=0}^{\infty} \omega^i \Delta_{i,t+i} \underbrace{\left[\overbrace{\left(\frac{p_{jt}}{P_{t+i}} \right) c_{jt+i}}^{\text{revenues}} - \overbrace{\varphi_{t+i} c_{jt+i}}^{\text{costs}} \right]}_{\text{profits in period } t+i},$$

where $\Delta_{i,t+i} = \beta^i \left(\frac{C_{t+i}}{C_t} \right)^{-\sigma}$ denotes a discount factor

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- When prices are flexible, all firms charge the same price, thus $p_t^* = P_t$ and $\varphi_t = 1/\mu$

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$$\left(\frac{\omega}{1-\omega}\right)\pi_t = (1-\omega\beta)\,\hat{\varphi}_t + \omega\beta\left(\frac{1}{1-\omega}\right)E_t\pi_{t+1}$$

Contents

New Keynesian Phillips curve

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- Multiplying both sides by $(1-\omega)/\omega$ yields a representation of the *New Keynesian Phillips curve* with firm's marginal cost $\hat{\varphi}_t$

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$$
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\pi_t &= \tilde{\kappa}\hat{\varphi}_t + \beta\tilde{\kappa} E_t \hat{\varphi}_{t+1} + \beta^2 E_t \pi_{t+2} \\
&\vdots \\
\pi_t &= \kappa\hat{\varphi}_t + \beta\tilde{\kappa} E_t \hat{\varphi}_{t+1} + \beta^2\tilde{\kappa} E_t \hat{\varphi}_{t+2} + \cdots + \beta^n E_t \pi_{t+n} \\
\pi_t &= \tilde{\kappa} \sum_{i=0}^{n} \beta^i E_t \hat{\varphi}_{t+i} + \beta^{n+1} E_t \pi_{t+n+1}
\end{aligned}
$$

- Letting n approach infinity yields

$$\pi_t = \tilde{\kappa} \sum_{i=0}^{\infty} \beta^i E_t \hat{\varphi}_{t+i}$$

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- Different to traditional Phillips curves inflation depends on real marginal cost rather then a measure of the output gap or the unemployment rate.
- But marginal cost can also be related to an output gap measure.

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$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t \tag{14}$$

where

$$\kappa = \gamma \tilde{\kappa} = \gamma \frac{1-\omega}{1-\beta\omega} \quad \text{and} \quad x_t \equiv \hat{y}_t - \hat{y}_t^f$$

Contents

IS curve

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$$\hat{x}_t = E_t \hat{x}_{t+1} - \left(\frac{1}{\sigma}\right)\left(\hat{\imath}_t - E_t \pi_{t+1}\right) + u_t. \quad (15)$$

$\Longrightarrow$ IS curve.