Monetäre Märkte und Zinsbildung

Geldpolitik im neukeynesianischen Modell

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- $\lambda = (1 - \beta\theta)(1 - \theta)/\theta$ (θ denotes the proportion of firms which cannot adjust their prices in any given period, β denotes the policy maker's discount factor.

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- Maximization problem:

$$\max -\frac{1}{2} E_t \left\{ \sum_{i=0}^{\infty} \delta^i \left[\alpha x_{t+i}^2 + \pi_{t+i}^2 \right] \right\} \tag{7}$$

Strict inflation targeting - Assumptions

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$$\begin{aligned}\mathcal{L} = & - \frac{1}{2} E_t \left\{ \sum_{i=0}^{\infty} \beta^i \left[\pi_{t+i}^2 \right] \right\} + \\ & + \sum_{i=0}^{\infty} \nu_{t+i} \left[-\varphi \left(i_{t+i} - \pi_{t+i+1}^e \right) + x_{t+i+1}^e + g_{t+i} - x_{t+i} \right] \\ & + \sum_{i=0}^{\infty} \delta_{t+i} \left[\lambda x_{t+i} + \beta \pi_{t+i+1}^e + u_{t+i} - \pi_{t+i} \right]\end{aligned} \tag{9}$$

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$\Longrightarrow$ Maximization is done with respect to x_{t+i}, π_{t+i}, i_t, ν_{t+i} and δ_{t+i}.

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  - With respect to δ_t:
$$\pi_t = \lambda x_t + \beta \pi^e_{t+1} + u_t \tag{14}$$

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\begin{aligned}
x_t &= -\varphi\left(i_t - \pi^e_{t+1}\right) + x^e_{t+1} + g_t \Longleftrightarrow \qquad (21)\\
i_t &= \pi^e_{t+1} - \frac{1}{\varphi}\left(x_t - x^e_{t+1} - g_t\right) = \\
&= 0 - \frac{1}{\varphi}\left[\left(-\frac{1}{\lambda}u_t\right) - \left(-\frac{1}{\lambda}\rho u_t\right) - g_t\right] = \\
&= \frac{1}{\varphi}\left[\left(\frac{1}{\lambda}u_t\right) - \left(\frac{1}{\lambda}\rho u_t\right) + g_t\right] = \\
&= \frac{1-\rho}{\lambda\varphi}u_t + \frac{1}{\varphi}g_t.
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$$\max -\frac{1}{2} E_t \left\{ \sum_{i=0}^{\infty} \delta^i \left[\alpha x_{t+i}^2 + \pi_{t+i}^2 \right] \right\} \tag{22}$$

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$$\pi_t = \lambda x_t + \beta \pi^e_{t+1} + u_t \tag{28}$$

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- Plugging (25) into (24) and rearranging the resulting expression yields:

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- Plugging this result into equation (28) and collecting terms gives:

$$\pi_t = \frac{\alpha\beta}{\alpha+\lambda^2}\pi^e_{t+1} + \frac{\alpha}{\alpha+\lambda^2}u_t. \tag{32}$$

Flexible inflation targeting: First-order conditions

- Solving this equation forward (and assuming that $\lim_{i \to \infty} \left(\frac{\alpha\beta}{\alpha+\lambda^2} \right)^i \pi^e_{t+i+1} = 0$) we obtain:

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$$\pi_t = \frac{\alpha}{\lambda^2+\alpha(1-\beta\rho)}u_t = \alpha q u_t \qquad (33)$$

with $q = \frac{1}{\lambda^2+\alpha(1-\beta\rho)}$.

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- Then:

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and

$$x^e_{t+1} = -\lambda q u^e_{t+1} = -\lambda q \rho u_t. \tag{36}$$

Flexible inflation targeting: First-order conditions

- From the IS curve we have:

$$
\begin{aligned}
i_t^{opt} &= \pi_{t+1}^e + \frac{1}{\varphi}\left(-x_t + x_{t+1}^e + g_t\right) = \\
&= \left(1 + \frac{\lambda(1-\rho)}{\alpha\varphi\rho}\right)\alpha q\rho u_t + \frac{1}{\varphi}g_t = \\
&= \gamma_\pi \pi_{t+1}^e + \frac{1}{\varphi}g_t
\end{aligned} \tag{37}
$$

with $\gamma_\pi = 1 + \frac{\lambda(1-\rho)}{\alpha\varphi\rho}$.