Knowledge Economy Indicators

Workpackages 4 and 5

Time Series Modeling with Busy

Deliverables 4.3 and 5.7

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http://kei.publicstatistics.net/

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Chapter 1
Introduction

In the following pages the main BUSY software characteristics will be described. From this description a main conclusion for the KEI project can be derived easily: BUSY cannot be applied to the KEI data set for two main reasons:

1) A theoretical one is that Knowledge economy can be considered a novel economic phenomenon with peculiar characteristics. This is one of the main statement of the KEI project. For this reason is then impossible to use time series analysis for deriving conclusions on a KBE.
2) A practical one is that even if one would like to apply BUSY in the context of a KBE, a basic difficulty is that for many KEI indicators simply do not exist historical data (just to give an example think about internet use, ICT and so on).

The BUSY program makes available a selection of statistical techniques designed for conducting business cycle analysis on a possibly large set of time series.

Two types of statistical procedures for business cycle analysis are offered. The first is an NBER-type of analysis that is based on descriptive statistics such as cross-correlations, coherences and phases of the cross-spectra and Bry and Boschan dating procedure (see Bry and Boschan, 1971). The second is based on dynamic factor models, following the work by Forni et al. (1999, 2000). Both are aimed at building composite indices that are leading, coincident or lagging with respect to a reference series. These composite indices are the main support of the business cycle analysis.

The analysis takes place only after the series have been transformed so as to be second-moment stationary. Several stationary transformations are proposed to the users. Input data can be either in human readable or in Excel formats. The series used do not need to have the same sample. All computational results are displayed in an HTML file that can be read either in BUSY or in Excel. The composite indices produced are exportable either in Excel or in human readable file.

Chapter 2
Data transformation

The techniques implemented for business cycle analysis rely in first place on second moment analysis. It is hence assumed that the series are second-order stationary - i.e. with mean and auto-covariances that are finite and do not depend on time. Because most economic time series do not satisfy these conditions, they need to be transformed. Let $\mathbf{x}_{it}$ be a time series with sample length T, i.e. i=1,•••, N, .t=1, •••,T. Besides a log-transformation that serves at the cases where the series variance increase together with the mean, BUSY proposes three types of filtering operations according to the general expression:

$$\mathbf{z}_{it} = \nu(\mathbf{L})\mathbf{x}_{it} = \sum_{\ell=-\mathbf{m}}^{\mathbf{M}} \nu_{\ell}\mathbf{x}_{it+\ell} \quad (1)$$

where **L** is the lag operator.

1. <u>First-order difference</u>: trivially, $\nu(\mathbf{L})$ = **1 - L** so $\mathbf{z}_{it} = \mathbf{x}_{it} - \mathbf{x}_{it-1}$: only the growth of the series is considered, the most simple detrending operation.

2. <u>Hodrick-Prescott filter</u>: This filter has been designed by Hodrick and Prescott (1997) as a detrending tool. The filter is $\nu(\mathbf{L}) = \dfrac{\lambda}{(1-\mathbf{L}^2)(1-\mathbf{L}^{-2})+\lambda}$ where λ is the inverse signal to noise ratio, i.e. the ratio between of the variance of the innovations in the short-term component and of the variance of the innovations in the long-term component. For quarterly series, typical values are λ=1600, 400. Trivially, the larger λ the smoother the long-term components. See also Harvey and Jaeger (1993).

3. <u>Baxter-King filter</u> Instead of removing the long-term component of the series, it is possible to directly extract movements whose periodicity lies within a certain range. This can be done using the so-called band-pass filters. Typical ranges of periodicity for business cycle analysis are [6,32] for quarterly data, [18,96] for monthly data, corresponding in both cases to fluctuations with periodicity in the range 1.5 to 8 years.

Baxter and King (1999) proposed a popular band-pass filter that preserves movements within any given range of periodicity [a,b]. For a given filter length K, i.e. M=m=K, the Baxter-King filter has weights given by:

$$\nu_k = \frac{\sin kb - \sin ka}{k\pi} - \frac{1}{2K+1}\sum_{k=-K}^{k=K} \frac{\sin kb - \sin ka}{k\pi}$$

Users can set the range of periodicity [a,b] together with the filter length, *K*.

When either Hodirck-Prescott or Baxter-King are used, the series can be extended with forecasts according the user wish. Finally, BUSY also proposes the possibility of removing a linear deterministic trend as a fourth option.

Once the series are made second-order stationary, the analysis can start. The first possibility is to use an NBER-type of approach.

Chapter 3
NBER-Type of Analysis

The NBER-approach relies on descriptive statistics and on the detection of turning points. It is based on a large amount of empirical experience, as it has been developed since the 1940's. It has proved to be well suited to the analysis of the US business cycle. Although it is a heuristic approach, it is now a reference for macroeconomists (see for example Zarnowitz, 1992).

The main point is to analyse the behaviour of a dataset with respect to that reference series and to aggregate series with similar behaviour into composite indices. The main steps of the analysis are description, classification by category and aggregation of series that belong to the same category into a composite index. The analysis supposes that one series has been set as the reference series; typical examples of reference series are GDP or Industrial Production Indices.

3.1 Descriptive statistics

The descriptive statistics used are essentially bivariate. Let $\mathbf{z_{1t}}$ be the reference series. Three different statistics are produced:

3.1.1 Cross correlations with reference series:

$$\rho_{1i}(\mathbf{k}) = \frac{\mathbf{Cov(z_{1t}, z_{it-k})}}{\sqrt{\mathbf{Var(z_{1t})Var(z_{it})}}} \qquad (2)$$

for i=1,…,N. The BUSY output file displays the contemporaneous cross-correlation and the maximum cross-correlation together with its lag. Notice if that maximum is found for k positive, then this indicates a leading behaviour of series *i* with respect to series 1. All cross-correlations are visible in graphics.

3.1.2 Coherence with reference series

The cross-spectrum between series 1 and *j* is given by

$$\mathbf{f_{1j}(\omega_\ell) = \frac{1}{2\pi}\sum_{k=-\infty}^{\infty}\gamma_{1j}(k)e^{-i\omega_\ell k}}$$

(3)

where ω_ℓ is a frequency within [-π,π] and γ(k) denotes the cross-covariances of lag k. As the cross-covariances are not symmetric, the cross-spectrum takes in general complex values. The squared coherence is defined as the ratio of squared modulus to the product of the spectrum of the reference series and of the j-th series according to:

$$\mathbf{Coh(w)^2 = \frac{\left|f_{1j}(\omega)\right|^2}{f_{11}(\omega)f_{jj}(\omega)}}$$

When estimating spectra and cross-spectra, a smoothing is performed according to:

$$\mathbf{\bar{f}_{1j}(\omega_\ell) = \sum_{m=-M}^{M}W(m)f_{1j}(\omega_{\ell+m})} \tag{4}$$

The term $\mathbf{W(m)}$ is known as the spectral window. A lag window could have been introduced directly in (3). However, as the cross-spectrum (3) is estimated directly via Fast Fourier Transform, the use of lag window instead of spectral one is not relevant. Two optional spectral windows are proposed: Bartlett or Parzen. For further details, see for example Fuller (1992, p.384), Priestley (1981, pp.432-444).

BUSY produces the squared coherence between every series and the reference one as averages over range of frequencies or periodicities. For business cycle analysis, economists are usually interested in the periodicity range 1.5 to 8 years, so high coherences within these periods shows that the series contains an information about the cyclical behaviour of the GDP.

3.1.3 Mean delay

Since the cross-spectrum has in general complex values, it can be written in polar coordinates as:

$$\mathbf{f_{1j}(\omega) = \left|f_{1j}(\omega)\right|e^{-iPh(\omega)}}$$

where **Ph(ω)**, the argument of the cross-spectrum, is the phase of the j-th series on the first one. The mean delay is defined as the ratio **Ph(ω)/ω**: it measures the lags in the movements of a series with respect to another one. Consider for example the relationship $\mathbf{y_t = x_{t-2}}$. It can be seen that the cross-spectrum between $\mathbf{y_t}$ and $\mathbf{x_t}$ is

$$\mathbf{f_{xy}(\omega) = f_{xx}(\omega)e^{-2i\omega}}.$$

Since $\mathbf{f_{xx}(\omega)}$ is real, it is seen that **Ph(ω)=2ω** and that the mean delay is **Ph(ω)/ω=2**, that is $\mathbf{y_t}$ lags $\mathbf{x_t}$ by two periods, or in other words $\mathbf{x_t}$ is leading by two periods. Positive (negative) values imply that the j-th series is leading (lagging) with respect to the reference one. BUSY reports the mean delays in average over ranges of periodicity.

For further references, see for example Harvey (1981) and Brockwell and Davis (1991).

3.1.4 Turning points

The turning point detection procedure is based on the one built by Bry and Boschan (1971), with some updates and adaptation to the case of quarterly series. The procedure can be described as follows.

1. The original Bry and Boschan procedure starts with a detrending moving average. As our series have already been detrended either via first-order difference, Hodrick-Prescott or Baxter-King filtering, that first stage is skipped as irrelevant.
2. On the transformed series, a Spencer moving average is applied in order to obtain the so-called Spencer curve. The Spencer moving average is defined as:

$$\mathbf{\nu(L) = \frac{1}{320}\begin{bmatrix} 74 + 67(L + L^{-1}) + 46(L^2 + L^{-2}) + 21(L^3 + L^{-3}) + 3(L^4 + L^{-4}) - 5(L^5 + L^{-5}) - \\ -6(L^6 + L^{-6}) - 3(L^7 + L^{-7}) \end{bmatrix}}$$

At both ends of the series, following the original procedure, the data are extended assuming that the growth rate of the first (last) 4 observations is constant in the previous (next) seven periods.

3. The stationary series is corrected for outliers. Outliers are identified as the points that lie outside the range $\mathbf{[\bar{z}_{it} - \alpha\sigma(z_{it}), \bar{z}_{it} + \alpha\sigma(z_{it})]}$, where $\mathbf{\bar{z}_{it}}$ denotes the sample mean of the I-th series and $\mathbf{\sigma(z_{it})}$the sample standard deviation. Outlying points are replaced by their equivalent on the Spencer curve. Passing the Spencer moving average on the outlier-corrected series yields an outlier-corrected Spencer curve.

4. For monthly data, a 2x12 centred Moving Average (MA) is applied on the outlier-corrected data in order to obtain the "first cycle" curve. For quarterly series, 2x4MA are used instead. The use of 2x12 or 2x4 MA instead of 4-term or 12-term is recommended as both are symmetric and hence do not cause any phase shift in output.

5. A first set of potential turning points are searched for in the MA12 or 2x12MA filtered series, and it is used to look for the corresponding turning points on the Spencer curve. The turning points are looked for in the interval [t-nterm,t+nterm] where the default is nterm=5.

6. A minimum phase length of 1.25*MQ periods, MQ denoting data periodicity - i.e. 4 or 12 for quarterly or for monthly series, from a peak (trough) to a peak (trough) is imposed. The succession peak-trough is checked and imposed if necessary.

7. The Months for Cyclical Dominance (MCD), i.e. the minimum month-delay for which the average of absolute deviations of growth in Spencer cycle is larger than that in the irregular component is computed. Then, the outlier-corrected series is passed through a moving average of length MCD. A new set of turning points is looked for on the basis of the complementary turning points that have been found on the Spencer curve. Again the succession of turns and minimum distance of 1.25*MQ from peak to peak or from trough to trough are imposed.

8. These last set of turning points are cleaned by removing the turns found in the first six or last six observations, and by imposing a minimum phase length - i.e. distance peak (trough) to trough (peak) of 5 observations.

The OECD phase average trend procedure is not implemented because the data should already have been detrended.

The turning points found in the reference series are produced together with the leads and lags of those found in the other series. Several descriptive statistics such as average lag, median lag and about the phases and length of the cycle found are given. The transformed series together with the turns can be seen in graphics.

3.1.5 Classification and composing

On the basis of the results of the previous operations, a classification of the series as leading, coincident and lagging can be operated. That classification should be performed manually by the users. In order to help users, we briefly develop below some guidelines:

1. Check that the series has some coherence with the reference one at the business cycle frequencies. Low coherence indicates a very idiosyncratic behaviour. Such series would not be much useful in explaining the common movements in the dataset, they can be let unclassified. A possible threshold is 0.4.

2. Similarly, check the maximum cross-correlation value: series that have maximum cross-correlation with the reference series lower than a threshold should be excluded from the analysis. Again, a reasonable threshold can be 0.4.

3. For those series with large enough coherence and cross-correlations with the reference series, check the mean delay in the business cycle frequency range and the lag where the maximum cross-correlation is found. Several cases can occur, the most straightforward being:

   i. The maximum cross-correlation is the contemporaneous one and the mean delay is less than one in absolute value: this can be seen as strong evidence for a behaviour that is coincident with the reference series.

   ii. The maximum cross-correlation occurs between lags 1 and 3 and the mean delay is between 1 and 3: there is some evidence for a behaviour that leads that of the reference series.

   iii. The maximum cross-correlation occurs between lags -1 and -3 and the mean delay is between -1 and -3, there is some evidence for a behaviour that lags that of the reference series.

It is good to supplement these prior opinions with the check of the turning point occurrences with respect to the reference series ones. This will also be of help for all those cases where the evidence will not be that crystal-clear.

The series belonging to the same group can then be aggregated in order to produce a composite index that is a candidate for describing the common cycle movements in the data set. A good procedure is to re-run the descriptive statistics on them including the detection of turns, so as to check that the candidate index behave as expected.

See also Altissimo et al. (1998).

Chapter 4
Dynamic factor models

Factor models, or index models, are an alternative to the heuristic NBER approach. These models consider that a common force drive the dynamics of all variables. This common force, also known as common factor, is typically of low dimension and is not directly observed because every macroeconomic variable embodies some idiosyncratic noise or short-term movements. Factor models clean from every variable from these idiosyncratic movements and estimate the common component in every series. The operation of classification and of aggregation take then place on the variables cleaned of idiosyncratic movements, i.e. on the series common component.

Because it is computationally more suited to the case of large data set, BUSY implements non-parametric versions of the factor models (for parametric version, see for example Sargent and Sims, 1977, Stock and Watson, 1993). Non parametric factor model can be either static (see Stock and Watson, 2002) or dynamic (see Forni et al., 1999, 2000). The advantage of considering the dynamic version is that the classification of the series with respect to the reference one is a by-product of the decomposition procedure. It is this approach that BUSY proposes. An important feature of dynamic factor models is that they provide a statistical framework for a business cycle analysis in large-scale data sets where all the different steps of the analysis are nested into a unified theoretical setting.

As described by Forni et al. (1999, 2000), the generalised dynamic factor model assumes that N second-order stationary variables denoted $\mathbf{z}_{it}$, i=1, …,N observed at time t share q orthogonal common factors $\mathbf{y}_{1t},\ldots, \mathbf{y}_{qt}$. Let $\mathbf{Z}_t$ and $\mathbf{Y}_t$ denote the N × 1 vector of observations and the q × 1 vector of unobservable common factors, respectively. Writing $\mathbf{C}_q(\mathbf{L})\mathbf{Y}_t$ the linear projection of $\mathbf{Z}_t$ on the space generated by $\{\mathbf{y}_{1t},\ldots, \mathbf{y}_{qt}\}$, the vector of observations verifies:

$$\mathbf{Z}_t = \mathbf{C}_q(\mathbf{L})\mathbf{Y}_t + \boldsymbol{\zeta}_t = \boldsymbol{\chi}_t^q + \boldsymbol{\zeta}_t \qquad (4.1)$$

where $\boldsymbol{\zeta}_t$ is a N × 1 vector of possibly cross-correlated idiosyncratic components and the N × 1 vector $\boldsymbol{\chi}_t^q$ contains the common part of the series. Orthogonality between common factors and idiosyncratic parts implies the spectral density matrix (sdm) relationship

$$\Sigma(\omega) = \Sigma_{\chi}^{q}(\omega) + \Sigma_{\zeta}(\omega)$$

(4.2)

where $\omega \in [-\pi, \pi]$ is a frequency and $\Sigma(\omega)$, $\Sigma_{\chi}^{q}(\omega)$, $\Sigma_{\zeta}(\omega)$ are the sdm of the series, of the common and of the idiosyncratic parts respectively.The vector of common parts, χ_t^q, can be estimated using the dynamic principal components developed in Forni et al. (1999,2000) as summarised below.

Let us denote $\mathbf{p_j}(\omega) = \{p_{j1}(\omega)\dots, p_{jN}(\omega)\}$ the j-th eigenvector of the N × N sdm matrix $\Sigma(\omega)$ associated with the j-th eigenvalue $\lambda_j(\omega)$, the eigenvalues being classified in descending order. The N vectors $\mathbf{p_j}(\omega)$, j=1,…,N, represent an orthonormal system of eigen vectors for $\mathbf{I_N}$. It can be checked that the projection of $\mathbf{Z_t}$ on the first *q* eigenvectors verifies

$$\chi_t^{q*} = \mathbf{K}^q(\mathbf{L})\mathbf{Z_t}$$

(4.3)

where the N × N matrix of filters is such that

$$\mathbf{K}^q(\mathbf{L}) = \mathbf{p_1}(\mathbf{L}^{-1})'\mathbf{p_1}(\mathbf{L}\ ) + \dots + \mathbf{p_q}(\mathbf{L}^{-1})'\mathbf{p_q}(\mathbf{L}\ )$$

Under certain assumptions, Forni et al. (2000) showed that χ_t^{q*} is a consistent estimator of χ_t^q. The N × N matrix of polynomials $\mathbf{K}^q(\mathbf{L})$ is computed first in the frequency domain as:

$$\mathbf{K}^q(\omega) = \mathbf{p_1}(\omega)'\mathbf{p_1}(\omega\ ) + \dots + \mathbf{p_q}(\omega)'\mathbf{p_q}(\omega\ )$$

For instance the ij-th entry in the matrix $\mathbf{K}^q(\omega)$ is

$$K_{ij}^q(\omega) = p_{1i}(\omega)'\, p_{1j}(\omega\ ) + \dots + p_{qi}(\omega)' p_{qj}(\omega\ )$$

By computing the matrix $\mathbf{K}^q(\omega)$ at the 2M+1 frequencies:

$$\omega_1 = 0, \omega_2 = \frac{2\pi}{2M+1}, \dots, \omega_{2M+1} = 2M\frac{2\pi}{2M+1},$$

the weights of the polynomial $K_{ij}^{q}(L)=\sum_{k=-M}^{M}K_{ijk}^{q}L^{k}$ that loads the j-th variable for the estimation of i-th common component can be recovered by inverse Fourier Transform as in:

$$K_{ijk}^{q}=\frac{1}{2M+1}\sum_{k=0}^{2M+1}K_{ij}^{q}(\omega_{k})e^{ik\omega_{k}},$$

(4.4)

Using this methodology BUSY estimates the common component and the idiosyncratic part in every series, the decomposition being such that:

$$\mathbf{Z}_{t}=\chi_{t}^{q^{*}}+\zeta_{t}^{*}$$

The common components obtained can be saved in output.

The classification of all series according to the behaviour the common parts with respect to that of the reference series is performed by computing the mean delays in the first row of the common components spectral density matrix $\Sigma_{\chi}^{q}(\omega)$. For example, if the mean delay is between -1 and 1, meaning between one period lead and one period lag, then the series is classified as coincident. Conversely, if the mean delay is higher than 1 (-1), than the series can be classified as leading (lagging) by more than one period. The building of composite indexes is based on the common parts of all series, similarly to NBER-type of approach. Also, because the common component of every series is cleaned of idiosyncratic short-term noise, the dating of turns can be improved when performed directly on the common components.

There are several parameters in this procedure. First of all, the choice of *q,* the number of factors, is not a trivial issue. Forni et al. (1999, 2000) propose to check the behaviour of the first *q* eigenvalues when the cross-section dimension expands to infinity, and a graphical tool for that it offered in BUSY. This graphic is a good support because, if there are q common factors in the data set, then only the first q eigenvalues should have a divergent behaviour.

A common practice is to select *q* so that a large enough proportion of the series variance is explained. Typical thresholds are between 50% and 70%. BUSY allows to set that proportion and, of course, to directly set the number of factors.

Another important parameter is M, the number of frequencies where the spectral density matrix is evaluated. Forni et al (1999, 2000) propose round(√T/4), T being the number of observations. It is the default value implemented.

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