



Workpackage 7
State-of-the-Art Report on
Simulation and Indicators

Deliverable 7.1

List of contributors:

Michela Nardo, Michaela Saisana, Andrea Saltelli, Stefano Tarantola, Joint Research Centre, Ispra

Main responsibility:

Michaela Saisana, Joint Research Centre, Ispra

CIS8-CT-2004-502529 KEI

The project is supported by European Commission by funding from the Sixth Framework Programme for Research.

http://europa.eu.int/comm/research/index_en.cfm

http://europa.eu.int/comm/research/fp6/ssp/kei_en.htm

http://www.cordis.lu/citizens/kick_off3.htm

<http://kei.publicstatistics.net/>



Preface

This report is the first deliverable of the work-package 7 (WP7, simulation studies) of the project KEI (Knowledge Economy Indicators: Development of Innovative and Reliable Indicator Systems <http://kei.publicstatistics.net>). KEI is part of the *Policy Orientated Research* section of the specific programme *Integrating and Strengthening the European Research Area* in the context of the Sixth Framework Programme of the European Commission.

In WP7 a set of simulations will be carried out to test the accuracy and the reliability of indicators of the knowledge-based economy in a practical environment under different realistic assumptions and data quality standards. WP7 will also address the robustness of the composite indicators to various policy scenarios, data quality and weighting / scaling approaches. The robustness assessment will be done with regards to various indicator outcomes, such as scores and rankings for both leaders and laggards, middle-of-the-road performers, and on *status* versus progress performances.

The construction of (composite) indicators involves stages where subjective judgement has to be made: the selection of indicators, the quality of the data, the treatment of missing values, the choice of aggregation model, the weights of the indicators, etc. These subjective choices can be used to manipulate the results. It is, thus, important to identify the sources of subjective choice and of imprecise assessment and use uncertainty and sensitivity analysis to gain useful insights during the process of composite indicators building, for an appraisal of the reliability of countries' ranking.

This report is a state-of-the-art of tools of uncertainty and sensitivity analysis that can be used to assess robustness of indicator-based inferences, increase transparency and make policy inference more defensible. We will use the Technology Achievement Index (TAI), a composite indicator developed by the United Nations (2001) (Human Development Report), to elucidate the various steps in the uncertainty and sensitivity analysis (a detailed description of the composite indicator is given in the Appendix).

The opinions expressed in the present report are those of the authors. The material contained in it will also feed in the ongoing joint OECD-JRC review of suggested practices for composite indicators building, see: <http://farmweb.jrc.cec.eu.int/CI/>.

Contents

List of figures	VII
List of tables	IX
1 Why robustness analysis	1
2 Uncertainty and sensitivity analysis	3
2.1 Set up of the analysis	5
2.1.1 Output variables of interest	5
2.1.2 General framework for the analysis	6
2.1.3 Inclusion – exclusion of individual indicators	6
2.1.4 Data quality	6
2.1.5 Normalisation	7
2.1.6 Weighting schemes	7
2.1.7 Aggregation systems	9
2.1.8 Uncertainty analysis	10
2.1.9 Sensitivity analysis using variance-based techniques	12
2.2 Results	16
2.2.1 First analysis	16
2.2.2 Second analysis	21
3 Conclusions	23
A Appendix	27

List of Figures

2.1	Scatterplots of weights	8
2.2	Uncertainty analysis results	16
2.3	Sensitivity analysis results based on the first order indices	17
2.4	Sensitivity analysis results based on the total effect indices	18
2.5	Ranking of the Netherlands	19
2.6	Result of UA for the output variable	20
2.7	Average shift in countries' ranking	20
2.8	Uncertainty analysis results	21

List of Tables

2.1	Sobol' sensitivity measures I	19
2.2	Sobol' sensitivity measures II	22
A.1	List of indicators of the Technology Achievement Index	28
A.2	Raw data for the indicators of the Technology Achievement Index	30

Chapter 1

Why robustness analysis

Indicators and composite indicators are increasingly recognized as a useful tool for policy making and public communication in conveying information on countries' performance in fields such as environment, economy, society, or technological development. They are useful in ranking countries in benchmarking exercises. However, they can send misleading or non-robust policy messages if they are poorly constructed or misinterpreted.

The construction of (composite) indicators involves making choices. This introduces issues of uncertainty such as selection of data, imprecision of the data, data imputation methods, data normalisation, weighting schemes, weights' values and aggregation methods.

As no model (construction path of the indicator or composite indicator) is a priori better than another (as each model serves different interests), a plurality of methods should be initially considered, provided that internal coherence is assured. The (composite) indicator is no longer a magic number corresponding to crisp data treatment, weighting set or aggregation method, but reflects uncertainty and ambiguity in a more transparent and defensible fashion.

All these sources of subjective judgement will affect the message brought by the (composite) indicator in a way that deserves analysis and corroboration. For example, changes in weights will almost in all cases lead to changes in rankings of countries. It is seldom that top performers become worse due to changes in weights but a change in ranking from e.g. ranking 2 to ranking 4 is not uncommon even in well-constructed composite indicators.

The robustness analysis tries to answer different questions, such as:

- (a) Does the use of one construction strategy versus another in building the composite indicator provide actually a partial picture of the countries' performance? In other words, how do the results of the composite indicator compare to a deterministic approach in building the composite indicator?
- (b) How much do the uncertainties affect the results of a composite indicator with respect to a deterministic approach used in building the composite indicator?
- (c) Which countries have large uncertainty bounds in their ranking and which are the uncertain factors that affect their rankings?

Uncertainty and sensitivity analysis can be used iteratively and contribute to the well-structuring of the composite indicator, to provide information on the robustness of country rankings and to identify ways to reduce uncertainty in country rankings, for better monitoring and policy-actions.

Chapter 2

Uncertainty and sensitivity analysis

A combination of uncertainty and sensitivity analysis can help to gauge the robustness of indicators and composite indicators, to increase their transparency and to help framing a debate around it. Uncertainty analysis (UA) focuses on how uncertainty in the input factors propagates through the structure of the (composite) indicator and affects the (composite) indicator values. Sensitivity analysis (SA) studies how much each individual source of uncertainty contributes to the output variance. In the field of building composite indicators, UA is more often adopted than SA (Jamison and Sandbu, 2001; Freudenberg, 2003) and the two types of analysis are almost always treated separately. A synergistic use of UA and SA is proven to be more powerful (Saisana *et al.*, 2005 ; Tarantola *et al.*, 2002).

In this section we describe the general procedures to assess uncertainty in composite indicators building. In particular, we shall try to tackle all possible sources of uncertainty, which arise from:

- i. selection of component indicators,
- ii. data quality,
- iii. data editing,
- iv. data normalisation,
- v. weighting scheme,
- vi. weights' values,
- vii. composite indicator formula

We will exemplify it with the example of the Technology Achievement Index (TAI), a composite indicator developed by the United Nations. We build an error propagation analysis, as complete as possible given the example, to the effect of showing the machinery at work on a rather complicate setting, on which we want to test different index architectures. In practical applications it might happen that the aggregation formula and the weighting scheme are dictated by the purpose of the index and/or by an agreement

among the parties involved in the index construction and use, thus making the UA/SA simpler.

With reference to the uncertainty sources (i to vii above), the approach taken to propagate uncertainties could include in theory all of the steps below:

- i. inclusion – exclusion of indicators,
- ii. modelling of data error, e.g. based of available information on variance estimation.
- iii. alternative editing schemes, e.g. multiple imputation.
- iv. using alternative data normalisation schemes, to remove the incomparability between data.
- v. using several weighting schemes, i.e. two participatory methods (budget allocation BAL and analytic hierarchy process AHP), and one based on data endogenous weighting (benefit of the doubt BOD)
- vi. using several aggregation systems, i.e. linear (LIN), geometric (GME) and a non-compensatory multi-criteria analysis (MCA).
- vii. weights' values, sampled from distributions when appropriate to the weighting scheme.

First TAI analysis. In a first analysis, in order to use the geometric mean aggregation approach GME, we shall omit (iii), i.e. we shall discard all countries with incomplete information. This is because with imputation, we might generate zeros that might be untreatable by GME. In a second analysis described later we shall relax this assumption. Also modelling of the data error, point (ii) above, will not be included as in the case of TAI no standard error estimate is available for the indicators. In a general case, based on estimate of the standard error associated to each individual indicator, we could sample an error for each one assuming e.g. a Gaussian error distribution.

Furthermore, not all combinations of choices under (i) to (vii) above are feasible with our TAI index. In particular

- A. When using LIN for aggregation and BAL or AHP for weighting, the option *use of raw data* for normalisation is a forbidden combination.
- B. When using LIN for aggregation and BOD for weighting, the options *use of raw data and standardisation* for normalisation are forbidden combinations.
- C. When using GME for aggregation, then BOD for weighting is a forbidden combination¹. Furthermore when using BAL and AHP, the option *standardisation* for normalisation is a forbidden combination.

¹the BoD approach we have applied here is based on a linear optimization approach (see section 2.1.6) , yet BoD using geometric optimization could be found in Charnes *et al.* (1983) and Banker and Maindiratta (1986).

- D. When using MCA for aggregation, then BOD and AHP for weighting are forbidden combinations.

A few technicalities are also worth mentioning.

- E. As all weights for both AHP and BAL are given by the experts, we sample the expert rather than the weight to preserve coherence among weights, e.g. to avoid generating combinations of weights that no expert would have advocated for.
- F. When using BOD, the exclusion of an indicator leads to a total re-run of the optimisation algorithm. When using BAL or AHP a simple rescaling of the weights to unit is sufficient.

Second TAI analysis. This differs from the first analysis in that we assume that stakeholders have converged to using LIN aggregation. In this case we can allow for alternative editing schemes, point (iii) above and consider all countries as in the original TAI. This analysis aims at answering mainly two questions:

- (a) Does the use of one strategy versus another in indicator building (i to vii above) provide a biased picture of the countries' performance? How does this compare to the original TAI?
- (b) To what extent do the uncertain input factors (used to generate the alternatives i to vii above) affect the countries' rankings with respect to the original TAI?

2.1 Set up of the analysis

2.1.1 Output variables of interest

Let

$$CI_c = f_{rs}(I_{1,c}, I_{2,c}, \dots, I_{Q,c}, w_{s,1}, w_{s,2}, \dots, w_{s,Q}) \quad (2.1)$$

be the index value for country c , $c = 1, \dots, M$, according the weighting model f_{rs} , $r = 1, 2, 3$, $s = 1, 2, 3$ where the index r refers to the aggregation system (LIN, GME, MCA) and the index s refers to the weighting scheme (BAL, AHP, BOD). The composite indicator is based on Q indicators $I_{1,c}, I_{2,c}, \dots, I_{Q,c}$ for that country and scheme-dependent weights $w_{s,1}, w_{s,2}, \dots, w_{s,Q}$ for the indicators.

The ranking assigned by the composite indicator to a given country, i.e. $Rank(CI_c)$ will be an output of interest for the uncertainty – sensitivity analysis.

Additionally, the average shift in countries' ranking will be explored. This latter statistics captures in a single number the relative shift in the position of the entire system of

countries. It can be quantified as the average of the absolute differences in countries' ranking with respect to a reference ranking over the M countries:

$$\bar{R}_S = \frac{1}{M} \sum_{c=1}^M |Rank_{ref}(CI_c) - Rank(CI_c)| \quad (2.2)$$

The reference ranking for the TAI analysis is the original rank given to the country by the deterministic version of the composite indicator.

The investigation of $Rank(CI_c)$ and \bar{R}_S will be the scope of the uncertainty and sensitivity analysis (in both analyses), targeting the questions raised in the introduction on the quality of the composite indicator. We always work on $Rank(CI_c)$ rather than on the raw values of CI_c as the multi criteria approach MCA only produces rankings for countries.

2.1.2 General framework for the analysis

As described in the following sections, we shall frame the analysis as a single Monte Carlo experiment, e.g. by plugging all uncertainty sources simultaneously, as to capture all possible synergistic effects among uncertain input factors. This will involve the use of triggers, e.g. the use of uncertain input factors used to decide e.g. which aggregation system and weighting scheme to adopt. To stay with the example, a discrete uncertain factor which can take integer values between 1 and 3 will be used to decide upon the aggregation system and another also varying in the same range for the weighting scheme. Other trigger factors will be generated to select which indicators to omit, the editing scheme (for the second TAI analysis only), the normalisation scheme and so on, till a full set of input variables is available to compute for the given run the statistics $Rank(CI_c)$, \bar{R}_S described above.

2.1.3 Inclusion – exclusion of individual indicators

No more than one indicator at a time is excluded for simplicity. A single random variable is used to decide if any indicator will be omitted and which one. Note that an indicator can also be practically neglected as a result of the weight assignment procedure. Imagine a very low weight is assigned by an expert to an indicator q . Every time we select that expert in a run of the Monte Carlo simulation, the relative indicator q will be almost neglected for that run.

2.1.4 Data quality

This is not considered here as discussed above.

2.1.5 Normalisation

Several methods are available to normalise indicators. The methods that are most frequently met in the literature are based on the re-scaled values (equation 2.3) or on the standardised values (equation 2.4) or on the raw indicator values (equation 2.5).

$$I_{q,c} = \frac{x_{q,c} - \min(x_q)}{\text{range}(x_q)} \quad (2.3)$$

$$I_{q,c} = \frac{x_{q,c} - \text{mean}(x_q)}{\text{std}(x_q)} \quad (2.4)$$

$$I_{q,c} = x_{q,c} \quad (2.5)$$

where $I_{q,c}$ is the normalised and $x_{q,c}$ is the raw value of the indicator x_q for country c .

2.1.6 Weighting schemes

The difficulty in assessing properly the relative importance of the indicators is one of the most debated problems in building composite indicators (Cox *et al.*, 1992). In our analysis we employ two participatory approaches, budget allocation and analytic hierarchy process, to allow for an expression of the relative importance of the indicators from the societal viewpoint. Two pilot surveys have been carried out across 20 informed interviewees at the authors' institute.

In the *budget allocation* (BAL) the interviewees were invited to distribute a budget of points over the eight indicators, paying more for those indicators whose importance they wanted to emphasize (Moldan and Billharz, 1997). In the *Analytic Hierarchy Process* (AHP) the strength of preference per pairs of indicators was expressed on a semantic scale of 1 (equality) to 9 (i.e. an indicator can be voted to be 9 times more important than the one to which it is being compared). The relative weights of the eight indicators were then calculated using an eigenvector technique, which allows to resolve inconsistencies, e.g a better than b better than c better than a loops (Saaty, 1980).

Figure 2.1.6 presents the eight scatterplots of the weights for each indicator. Each point in a scatterplot represents the weight given to the indicator by one interviewee when requested in a BAL or an AHP approach. The deviation of the weights from the 45° line of perfect agreement between the two weighting schemes is an interesting feature of this analysis, revealing the human tendency to reply differently to different formulations of the same question. Both weighting approaches have advantages and limitations. The weights provided by BAL are less spread than AHP for each indicator and the variance of the weights across the eight indicators is smaller for BAL than for AHP. However, AHP is based on pair-wise comparisons, where perception is high enough to make a distinction between indicators. In BAL all the indicators are compared at a glance, and

this might lead to circular thinking across indicators, creating difficulties in assigning weights, particularly when the number of indicators is high.

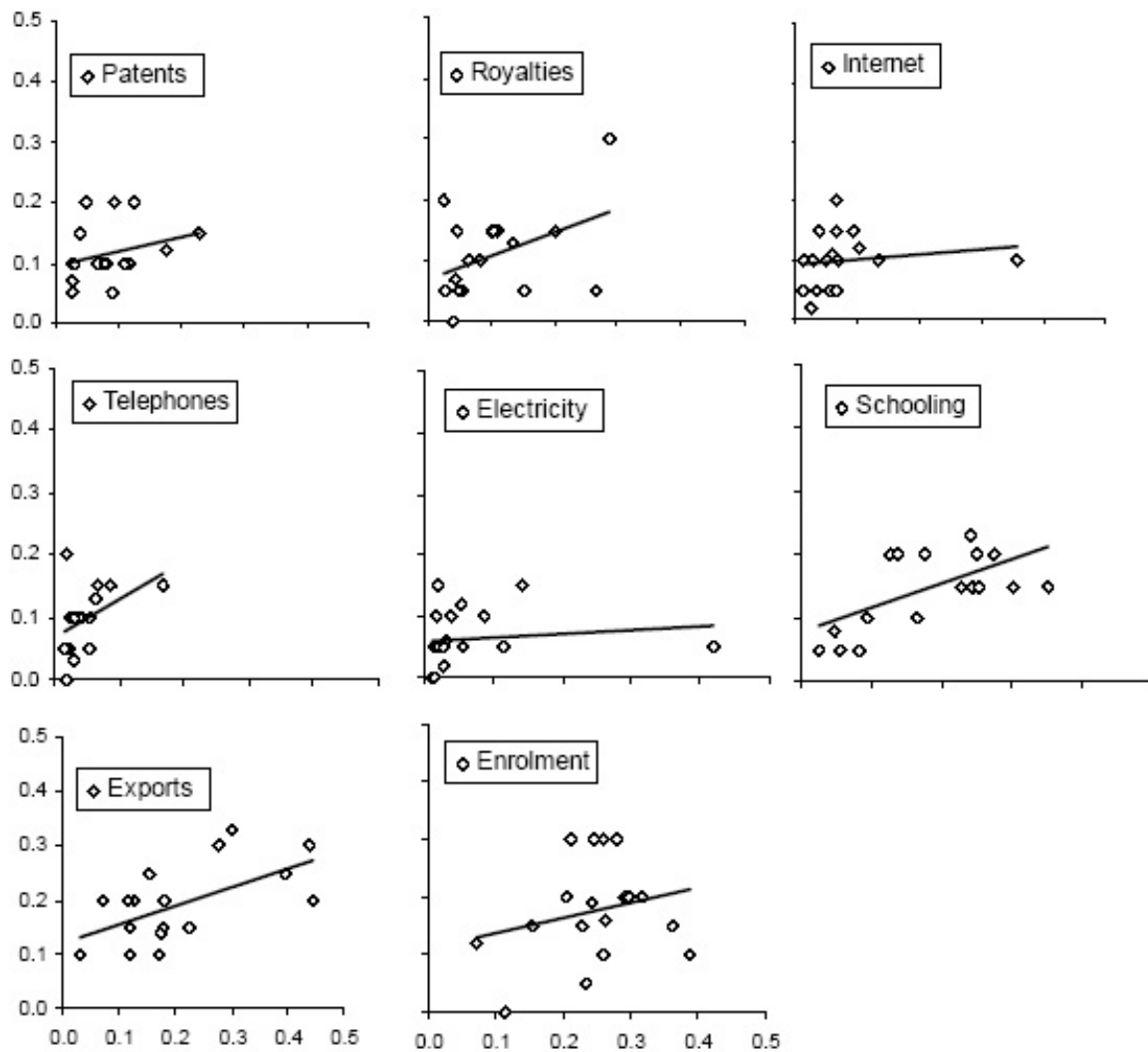


Figure 2.1: Scatterplots of weights (range between 0.0 and 0.5) for the eight indicators of TAI. The weights have been derived from pilot surveys of 20 informed interviewees using budget allocation (vertical axes) and analytic hierarchy process (horizontal axes). Best fit linear regression lines are indicated.

The *benefit-of-the-doubt* procedure (BOD) (Melyn and Moesen, 1991; Cherchye *et al.*, 2003) is a particular case of data envelopment analysis. This weighting method can be seen as a tool for identifying benchmarks without imposing strong normative judgments. This weighting method involves linear optimisation techniques and allows countries to emphasise and prioritise those aspects for which they perform relatively well. The weights, therefore, are country-dependent and sensitive to the benchmarks. In general, even using the best combination of weights for a given country, other countries may show better performance. The optimization process could lead to many zero weights if no restrictions on the weights were imposed. In such cases, many countries would be considered as

benchmarks. Bounding restrictions on weights are hence necessary for this method to be of practical use. For the eight indicators in our case we have used absolute restrictions on weights with the lower bound set to 5% and the upper bound to 30%.

2.1.7 Aggregation systems

The literature of composite indicators offers several examples of aggregation techniques. Additive techniques are the most frequent ones, whilst less widespread are multiplicative (or geometric) aggregations or non-linear aggregations (e.g. multi-criteria analysis). In our analysis we have considered all three types of aggregation.

The *linear aggregation* (equation 2.6) has been employed as it is the most widely used. Several authors, however, note that an additive aggregation function for a given set of indicators exists if and only if these indicators are mutually preferentially independent (Keeney and Raiffa, 1976). Preferential independence is a very strong condition since it implies that the trade-off ratio between two indicators is independent of the values of the remaining indicators (Ting, 1971). Furthermore, an undesirable feature in additive aggregations might be the full compensability that they imply: poor performance in some indicators can be compensated by sufficiently high values in other indicators. The use of a *geometric aggregation* (equation 2.7) would partially solve the problem. The use of geometric aggregation can also be justified on the grounds of the different countries' incentives in a benchmarking exercise. Countries with low values in the indicators would prefer a linear rather than a geometric aggregation, so as to achieve a higher ranking. On the other hand, a country would need to increase those sectors/activities with the lowest value in order to have the highest chance to improve its position in the ranking if the aggregation is geometric rather than linear (Zimmermann and Zysno, 1983).

$$CI_c = \sum_{q=1}^Q w_q I_{q,c} \quad (2.6)$$

$$CI_c = \prod_{q=1}^Q (w_q I_{q,c})^{1/Q} \quad (2.7)$$

where $\sum_q w_q = 1$, $0 \leq w_q \leq 1$, $c = 1, \dots, M$.

The *multi-criteria analysis* (MCA) tries to resolve the conflict arising in country comparisons as some indicators are in favour of one country while other indicators are in favour of another. This conflict can be treated in the light of a non-compensatory logic and taking into account the absence of preference independence within a discrete multi-criteria approach (Munda, 1995). The approach employs a mathematical formulation (Condorcet ranking procedure) to rank in a complete pre-order (i.e. without any incomparability relation) all the countries from the best to the worst one after a pair-wise comparison of countries across the whole set of the available indicators (Munda and Nardo, 2003). We offer here a 'hand waving' description of the method. Imagine we have just three countries, A, B and C, and we want to compare them with one another. We build to this

effect an ‘outsourcing matrix’ whose entries e_{ij} tell us how much country ‘i’ does better than country ‘j’. The entry e_{ij} is in fact the sum of all weights of all indicators for which country ‘i’ does better than country ‘j’. Likewise, e_{ji} will be the sum of all weights for which the reverse is true. If the two countries do equally well on one variable, its weight is split between e_{ij} and e_{ji} . As a result $e_{ij} + e_{ji} = 1$, provided that weights are scaled to unity sum. We now write down all permutations of county order (ABC,ACB,BAC,BCA,CAB,CBA) and compute for each of them the ordered sum of the scores, e.g. for ABC we compute $Y = e_{AB} + e_{AC} + e_{BC}$. We do this for all permutations and we consider the ranking with the highest total score Y. Note that this ordering is only based on the weights, and on the sign of the difference between countries values for a given indicator, the magnitude of the difference being ignored. Hence, to exemplify, a country that does marginally better on many indicators comes out better than a country that does much better on a few ones.

We summarise next the combinations of normalisation methods, weighting schemes and aggregation systems that we have used. Equation 2.3 is used in conjunction with all weighting schemes (BAL, AHP and BOD) for all aggregation systems (LIN, GME, MCA). Equation 2.4 is used in conjunction with weighting schemes (BAL, AHP) for aggregation systems (LIN, MCA). Finally, Equation 2.5 is used in conjunction with weighting schemes (BAL, AHP) for aggregation systems (GME, MCA).

2.1.8 Uncertainty analysis

All points of the (i) to (vii) chain of composite indicator building can introduce uncertainty in the output variables $Rank(CI_c)$ and \bar{R}_S . Thus we shall translate all these uncertainties into a set of scalar input factors, to be sampled from their distributions. As a result, all outputs $Rank(CI_c)$ and \bar{R}_S are non-linear functions of the uncertain input factors, and the estimation of the probability distribution functions (pdf) of $Rank(CI_c)$ and \bar{R}_S is the purpose of the uncertainty analysis. The UA procedure is essentially based on simulations that are carried out using equation 2.1 that constitutes our *model*. As the model is in fact a computer programme that implements steps (i) to (vii) above, the uncertainty analysis operates on a *computational model*. Various methods are available for evaluating output uncertainty.

In the following, the Monte Carlo approach is presented, which is based on performing multiple evaluations of the model with k randomly selected model input factors. The procedure involves three steps:

Step 1. Assign a pdf to each input factor X_i , $i = 1, 2 \dots k$. The first input factor, X_1 is used for the selection of the editing scheme (for the second TAI analysis only):

X_1	Editing
1	Use bivariate correlation to impute missing data
2	Assign zero to missing datum

The second input factor X_2 is the trigger to select the normalisation method.

X_2	Normalisation method
1	Rescaling (Equation 2.3)
2	Standardisation (Equation 2.4)
3	None (Equation 2.5)

Both X_1 and X_2 are discrete random variables. In practice, they are generated drawing a random number ζ uniformly distributed in $[0,1]$ and applying the so called Russian roulette algorithm, e.g. for X_1 we select 1 if $\zeta \in [0, 0.5)$ and 2 if $\zeta \in [0.5, 1]$. The uncertain factor X_3 is generated to select which indicator –if any, should be omitted. The procedure is

ζ	X_3 , excluded indicator
$[0, \frac{1}{Q+1})$	None ($X_3 = 0$) all indicators included)
$[\frac{1}{Q+1}, \frac{2}{Q+1})$	$X_3 = 1$
\dots	\dots
$[\frac{Q}{Q+1}, 1]$	$X_3 = Q$

i.e. with probability $\frac{1}{Q+1}$ no indicator will be excluded, while with probability $[1 - \frac{1}{Q+1}]$ one of the Q indicators will be excluded with equal probability. Clearly we could have made the probability of $X_3 = 0$ larger or smaller than $\frac{1}{Q+1}$ and still sample the values $X_3 = 1, 2, \dots, Q$ with equal probability. We anticipate here that a scatter-plot based sensitivity analysis will allow us to track which indicator – when excluded – affects the output the most. Also recall that whenever an indicator is excluded, the weights of the other factors are rescaled to 1 to make the composite index comparable if either BAL or AHP is selected. When BOD is selected the exclusion of an indicator leads to a re-execution of the optimisation algorithm.

Trigger X_4 is used to select the aggregation system

X_4	Aggregation system
1	LIN (Equation 2.6)
2	GME (Equation 2.7)
3	MCA

X_5 is the trigger to select the weighting scheme:

The last uncertain factor X_6 is used to select the expert. In our experiment we had 20 experts, and once an expert is selected at runtime via the trigger X_6 , the weights assigned by that expert (either for the BAL or AHP schemes) are assigned to the data. Clearly, the selection of the expert has no bearing when BOD is selected ($X_5 = 3$). All the same this uncertain factor will be generated at each individual Monte Carlo simulation. This

X_5	Weighting Scheme
1	BAL
2	AHP
3	BOD

is because the row dimension of the Monte Carlo sample (called constructive dimension) should be fixed in a Monte Carlo experiment, i.e. even if some of the sampled factors will not be active at a particular run, they will be all the same generated by the random sample generation algorithm.

The constructive dimension of this Monte Carlo experiment, e.g. the number of random numbers to be generated for each trial, is hence $k = 6$.

That described here is only a possible design of the uncertainty/sensitivity analysis. The analysis could be designed in alternative ways, to investigate particular characteristics of the model. During the KEI project, several simulations shall be executed.

Step 2. Having generated the input factors distributions in step 1, we can now generate randomly N combinations of independent input factors \mathbf{X}^l , $l = 1, 2, \dots, N$ (a set $\mathbf{X}^l = X_1^l, X_2^l, \dots, X_k^l$ of input factors is called a sample). For each trial sample \mathbf{X}^l the computational model can be evaluated, generating values for the scalar output variable Y^l , where Y^l is either $Rank(CI_c)$, the value of the ranking assigned by the composite indicator to each country, or \bar{R}_S , the averaged shift in countries' ranking.

Step 3. We can now close the loop over l , and analyse the resulting output vector \mathbf{Y}^l , with $l = 1, \dots, N$.

The sequence of \mathbf{Y}^l allows the estimation of the empirical probability distribution function (pdf) of the output. The distribution reflects the uncertainty of the output due to the uncertainty in the input. Its characteristics such as the variance and higher order moments, can be estimated with an arbitrary level of precision that is depends on the number of simulations N .

2.1.9 Sensitivity analysis using variance-based techniques

A necessary step when designing a sensitivity analysis is to identify the output variables of interest. Ideally these should be relevant to the issue tackled by the model, as opposed to just relevant to the model *per se* (Saltelli *et al.*, 2000b, 2004).

In the following, we shall apply sensitivity analysis to output variables $Rank(CI_c)$, and \bar{R}_S , for their bearing on the quality assessment of our composite indicator.

It has been noted earlier in this work that composite indicators can be considered as models. When –as in the present analysis– several layers of uncertainty are simultaneously activated, composite indicators turn out to be non linear, possibly non additive models. As argued by practitioners (Saltelli *et al.*, 2000a, Environmental Protection Agency -EPA, 2004), robust, *model-free* techniques for sensitivity analysis should be used for non linear

models. Variance-based techniques for sensitivity analysis are model free and display additional properties convenient for the present analysis:

- they allow an exploration of the whole range of variation of the input factors, instead of just sampling factors over a limited number of values, as done e.g. in fractional factorial design (Box *et al.*, 1995);
- they are quantitative, and can distinguish main effects (first order) from interaction effects (higher order).
- they are easy to interpret and to explain
- they allow for a sensitivity analysis whereby uncertain input factors are treated in groups instead of individually
- they can be justified in terms of rigorous settings for sensitivity analysis, as we shall discuss later in this section.

How do we compute a variance based sensitivity measure for a given input factor X_i ? We start from the fractional contribution to the model output variance (i.e. the variance of Y where Y is either $Rank(CI_c)$, and \bar{R}_G) due to the uncertainty in X_i . This is expressed as:

$$V_i = V_{X_i}(E_{\mathbf{X}_{-i}}(Y | X_i)) \quad (2.8)$$

One way of reading Equation 2.8 is the following. Imagine we fix the factor X_i , e.g. to a specific value x_i^* in its range, and we compute the mean of the output Y averaging over all factors but factor X_i : $E_{\mathbf{X}_{-i}}(Y | X_i = x_i^*)$. Imagine then to take the variance of the resulting function of x_i^* over all possible x_i^* values. The result is given by Equation 2.8, where the dependence from x_i^* has been dropped, since we have averaged over it. V_i is a number between 0 (when X_i does not give a contribution to Y at the first order), and $V(Y)$, the unconditional variance of Y , when all factors other than X_i are non influential at any order. The meaning of *order* will be explained in a moment. Note that it is always true that:

$$V_{X_i}(E_{\mathbf{X}_{-i}}(Y | X_i)) + E_{X_i}(V_{\mathbf{X}_{-i}}(Y | X_i)) = V(Y) \quad (2.9)$$

where the first term in equation 2.9 is called a main effect, and the second one is the residual. An important factor should have a small residual, e.g. a small value of $E_{X_i}(V_{\mathbf{X}_{-i}}(Y | X_i))$. This is intuitive as the residual measures the expected reduced variance that one would achieve if one could fix X_i . Let us write the reduced variance as $V_{\mathbf{X}_{-i}}(Y | X_i = x_i^*)$, a variance conditional on x_i^* . Then the residual $E_{X_i}(V_{\mathbf{X}_{-i}}(Y | X_i))$ is the expected value of such conditional variance, averaged over all possible values of x_i^* and this should be small if X_i is influential. A first order sensitivity index is obtained by dividing the first order term by the unconditional variance:

$$S_i = \frac{V_{X_i}(E_{\mathbf{X}_{-i}}(Y | X_i))}{V(Y)} = \frac{V_i}{V(Y)} \quad (2.10)$$

One can compute conditional variances corresponding to more than one factor, e.g. for two factors X_i and X_j one can compute $V_{X_i X_j}(E_{\mathbf{X}_{-ij}}(Y | X_i, X_j))$, and from this a second order variance contribution can be written as:

$$V_{ij} = V_{X_i X_j}(E_{\mathbf{X}_{-ij}}(Y | X_i, X_j)) - V_{X_i}(E_{\mathbf{X}_{-i}}(Y | X_i)) - V_{X_j}(E_{\mathbf{X}_{-j}}(Y | X_j)) \quad (2.11)$$

where clearly V_{ij} is only different from zero if $V_{X_i X_j}(E_{\mathbf{X}_{-ij}}(Y | X_i, X_j))$ is larger than the sum of the first order terms relative to factors X_i and X_j .

When all k factors are mutually independent, the sensitivity indices can be computed using the following decomposition formula for the total output variance $V(Y)$

$$V(Y) = \sum_i V_i + \sum_i \sum_{j>i} V_{ij} + \sum_i \sum_{j>i} \sum_{\substack{l>j \\ j>i}} V_{ijl} + \dots + V_{12\dots k} \quad (2.12)$$

Terms above the first order in equation 2.12 are known as interactions. A model without interactions among its input factors is said to be additive. In this case, $\sum_{i=1}^k V_i = V(Y)$,

$\sum_{i=1}^k S_i = 1$ and the first order conditional variances of equation 2.8 are all what we need to know to decompose the model output variance. For a non-additive model, higher order sensitivity indices, responsible for interaction effects among sets of input factors, have to be computed. However, higher order sensitivity indices are usually not estimated, as in a model with k factors the total number of indices (including the S_i 's) that should be estimated is as high as $2^k - 1$. For this reason, a more compact sensitivity measure is used. This is the total effect sensitivity index, which concentrates in one single term all the interactions involving a given factor X_i . To exemplify, for a model of $k=3$ independent factors, the three total sensitivity indices would be:

$$S_{T1} = \frac{V(Y) - V_{X_2 X_3}(E_{X_1}(Y | X_2, X_3))}{V(Y)} = S_1 + S_{12} + S_{13} + S_{123} \quad (2.13)$$

And analogously:

$$\begin{aligned} S_{T2} &= S_2 + S_{12} + S_{23} + S_{123} \\ S_{T3} &= S_3 + S_{13} + S_{23} + S_{123} \end{aligned} \quad (2.14)$$

The conditional variance $V_{X_2 X_3}(E_{X_1}(Y | X_2, X_3))$ in equation 2.13 can be written in general terms as $V_{\mathbf{X}_{-i}}(E_{X_i}(Y | \mathbf{X}_{-i}))$ (Homma and Saltelli, 1996). It expresses the total contribution to the variance of Y due to non- X_i i.e. to the $k-1$ remaining factors, so that $V(Y) - V_{\mathbf{X}_{-i}}(E_{X_i}(Y | \mathbf{X}_{-i}))$ includes all terms, i.e. a first order as well as interactions in equation 2.12, that involve factor X_i . In general $\sum_{i=1}^k S_{T_i} \geq 1$. Given the algebraic relation 2.9, the total effect sensitivity index can also be written as:

$$S_{T_i} = \frac{V(Y) - V_{\mathbf{X}_{-i}}(E_{X_i}(Y | \mathbf{X}_{-i}))}{V(Y)} = \frac{E_{\mathbf{X}_{-i}}(V_{X_i}(Y | \mathbf{X}_{-i}))}{V(Y)} \quad (2.15)$$

For a given factor X_i a significant difference between S_{T_i} and S_i flags an important role of interactions for that factor in Y . Highlighting interactions among input factors helps us improving our understanding of the model structure. Estimators for both (S_i, S_{T_i}) are provided by a variety of methods reviewed in Chan *et al.* (2000). Here the method of Sobol' (1993), in its improved version by Saltelli (2002), is used. The method of Sobol' is based on LP τ sequences, which are quasi random sequences, to produce sample points that best scan the entire space of possible combinations between the input factors (Sobol', 1976). Quasi-random sequences are used in place of random points to guarantee convergence of estimates in the classical sense. Moreover, Sobol' sequences usually result in better convergence when employed in numerical integration (see Bratley and Fox (1988) for a good summary description). The pair (S_i, S_{T_i}) gives a fairly good description of the model sensitivities at a reasonable cost, which for the improved Sobol' method is of $2n(k+1)$ model evaluations, where n represents the sample size required to approximate the multidimensional integrations implicit in the E and V operators above to a plain sum. n can vary in the hundred-to-thousand range.

When the uncertain input factors X_i are dependent, the output variance cannot be decomposed as in equation 2.12. The S_i, S_{T_i} indices, as defined by equation 2.8 and equation 2.15 are still valid sensitivity measures for X_i , though their interpretation changes as, e.g. S_i carries over also the effects of other factors that can be positively or negatively correlated to X_i (see Saltelli and Tarantola, 2002), while S_{T_i} can no longer be decomposed meaningfully into main effect and interaction effects. The usefulness of S_i, S_{T_i} , also for the case of non-independent input factors, is also linked to their interpretation in terms of *settings* for sensitivity analysis. We offer here a description of two settings linked to S_i, S_{T_i} . A justification is in Saltelli *et al.* (2004).

Factors' Prioritisation (FP) Setting. One must bet on a factor that, once *discovered* in its true value and fixed, would reduce the most $V(Y)$. Of course one does not know where the true values are for the factors. The best choice one can make is the factor with the highest S_i , whether the model is additive or not and whether the factors are independent or not.

Factors' Fixing (FF) Setting: Can one fix a factor [or a subset of input factors] at any given value over their range of uncertainty without reducing significantly the variance of the output? One can only fix those (sets of) factors whose S_{T_i} is zero.

The extended variance-based methods, including the improved version of Sobol', for both dependent and independent input factors, are implemented in the freely distributed software SIMLAB (Saltelli *et al.*, 2004).

2.2 Results

2.2.1 First analysis

The first analysis was run without imputation, i.e. by censoring all countries with missing data. As a result, only 34 countries could in theory be analysed. We further dropped countries from ranking 24 (original TAI), Hong Kong, as this is the first country with missing data, and it was preferred to analyse the set of countries whose ranking was not altered the omission of missing records. The uncertainty analysis for the remaining 23 countries is given in Figure 2.2.1 for the rankings, with countries ordered by their original TAI position, going from Finland, ranking = 1, to Slovenia, ranking = 23. The width of the 5th – 95th percentile bounds, and the fact that the ordering by the median values (black mark) often is at odd with the ordering of the original TAI (grey mark), shows that the acknowledgement of all uncertainty sources, including 3 alternative aggregation systems, results in considerable differences between the new and the original TAI, although one still sees the difference between the group of leaders and that of laggards. If the uncertainty plugged into the system were a true reflection of the status of knowledge and of the (lack of) consensus among experts on how TAI should be built, we would have to conclude that TAI is not a robust measure of country technology achievement.

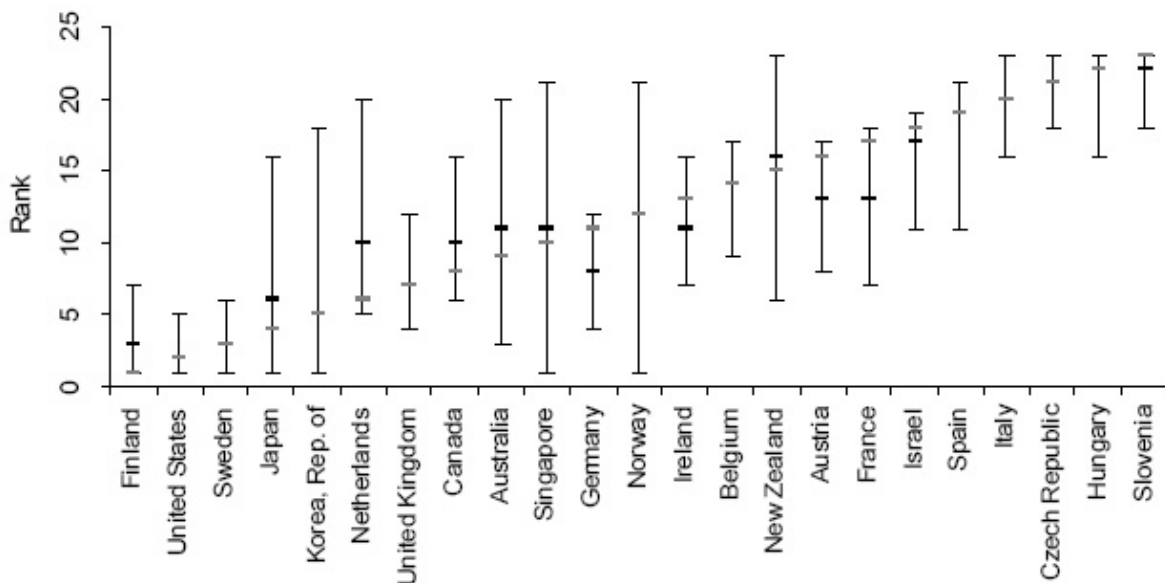


Figure 2.2: Uncertainty analysis results showing the countries' ranking according to the original TAI 2001 (light grey marks), and the median (black marks) and the corresponding 5th and 95th percentiles (bounds) of the distribution of the MC-TAI for 23 countries. Uncertain input factors: normalisation method, inclusion-exclusion of an indicator aggregation system, weighting scheme, expert selection. Countries are ordered according to the original TAI values.

Keeping up with this example, we show in Figure 2.2.1 a sensitivity analysis based on the first order indices calculated using the method of Sobol' (1993) in its improved version due

to Saltelli (2002). In fact we present the total variance for each country's ranking and how much of it can be decomposed according to the first order conditional variances. We can roughly say that aggregation system, followed by the inclusion-exclusion of indicator and expert selection are the most influential input factors. The countries with the highest total variance in rankings are the middle-performing countries in Figure 2.2.1, while the leaders and laggards in technology achievement present low total variance. The non-additive, part of the variances that is not explained by the first order sensitivity indices ranges from 35% for the Netherlands to 73% for United Kingdom, whilst for most countries it exceeds 50%. This underlines the necessity for computing higher order sensitivity indices that capture the interaction effects among the input factors.

Figure 2.2.1 shows the total effect sensitivity indices for the variances of each country's rankings. The total effect sensitivity indices concentrate in one single term all the interactions involving each input factor and they clearly add up to a number greater than one due to the existing interactions. Again interactions seem to exist among the influential factors already identified.

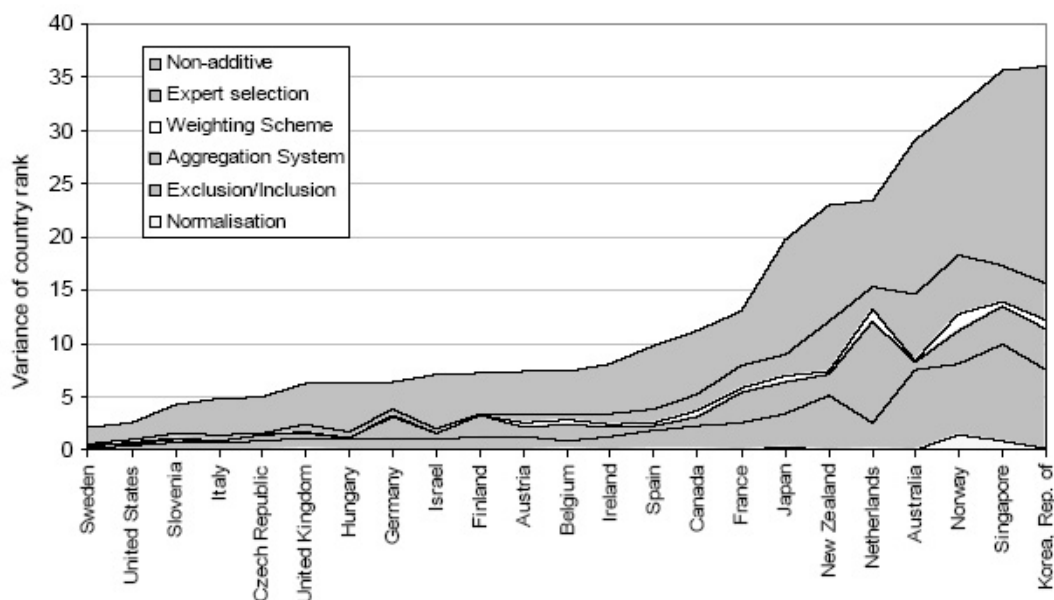


Figure 2.3: Sensitivity analysis results based on the first order indices. Decomposition of country's variance according to the first order conditional variances. Aggregation system, followed by the inclusion-exclusion of indicator and expert selection are the most influential input factors. The part of the variance that is not explained by the first order indices is noted as non-additive. Countries are ordered in ascending order of total variance.

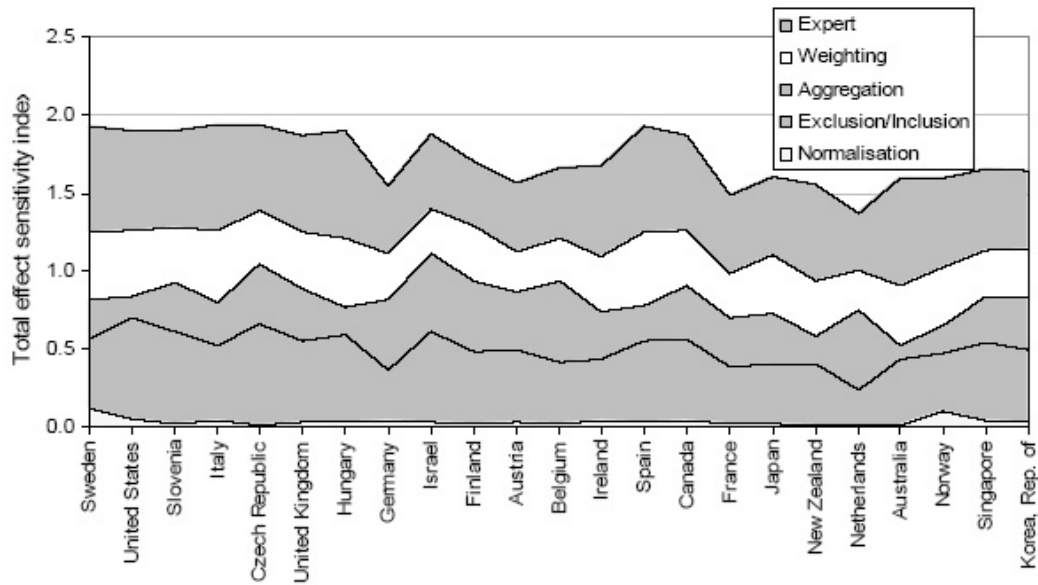


Figure 2.4: Sensitivity analysis results based on the total effect indices. Aggregation system inclusion-exclusion of indicator and expert selection present most of the interaction effects. Countries are ordered in ascending order of total variance.

If the TAI model was additive with no interactions between the input factors, the non-additive part of the variance in Figure 2.2.1 would have been zero (in other words the first order sensitivity indices would have summed to 1) and the sum of the total effect sensitivity indices in Figure 2.2.1 would have been 1. Yet, the sensitivity indices show the high degree of non linearity and additivity for the TAI model, and of the importance of the interactions. For instance, the high effect of interactions for Netherlands, which also had a large percentile bounds, can be further explored. In Figure 2.2.1 we see that this country is favoured by combination of *geometric mean system* with *BAL weighting* and unfavoured by combination of *Multi criteria system* with *AHP weighting*. This is a clear interaction effect. In depth analysis of the output data reveals that as far as inclusion – exclusion is concerned, it is the exclusion of the indicator *Royalties* leading to worse ranking for the Netherlands under any aggregation system.

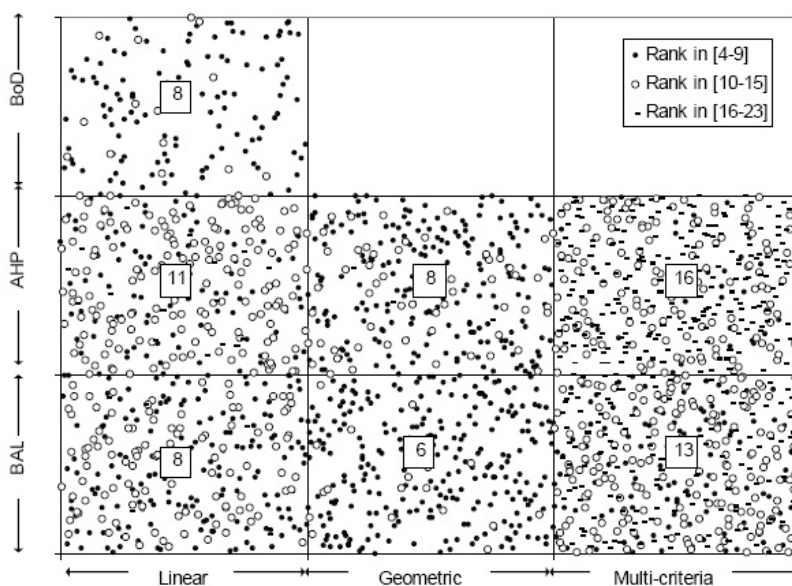


Figure 2.5: Ranking of the Netherlands for different combinations of aggregation system and weighting scheme. Average ranking per case is indicated in the box. The interaction effect between aggregation system and weighting scheme is clear.

Coming to the output variable average shift in ranking (Equation 2.2) with respect to the original TAI ranking we see in Figure 2.2.1 the histogram of the values. The mean value is almost 3 positions, with a standard deviation slightly above 1 position. The input factors affecting this variable the most are aggregation system plus inclusion – exclusion at the first order, while if the interactions are considered both weighting scheme and expert choice become important (Table 2.2.1). This effect can be seen in Figure 2.2.1 where the effect of MCA in spreading the countries rankings can be appreciated. In some cases the average shift in country’s ranking when using MCA can be as high as 9 places.

<i>Input Factors</i>	First order (S_i)	Total effect (S_{T_i})	$S_{T_i} - S_i$
Normalisation	0.000	0.008	0.008
Exclusion/Inclusion of indicator	0.148	<u>0.435</u>	<u>0.286</u>
Aggregation system	<u>0.245</u>	0.425	0.180
Weighting Scheme	0.038	<u>0.327</u>	<u>0.288</u>
Expert selection	0.068	<u>0.402</u>	<u>0.334</u>
Sum	0.499	1.597	

Table 2.1: Sobol’ sensitivity measures of first order and total effect for the output: Average shift in countries’ ranking with respect to the original TAI. Significant values are underlined.

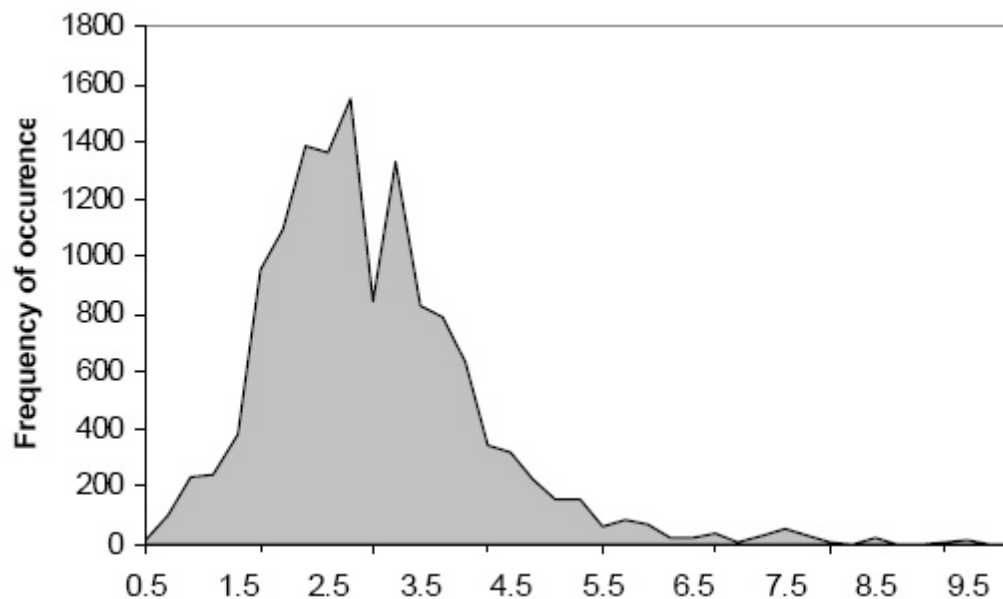


Figure 2.6: Result of UA for the output variable. Average shift in countries' ranking with respect to the original TAI. Uncertain input factors: normalisation method, inclusion-exclusion of an indicator, aggregation system, weighting scheme, expert selection.

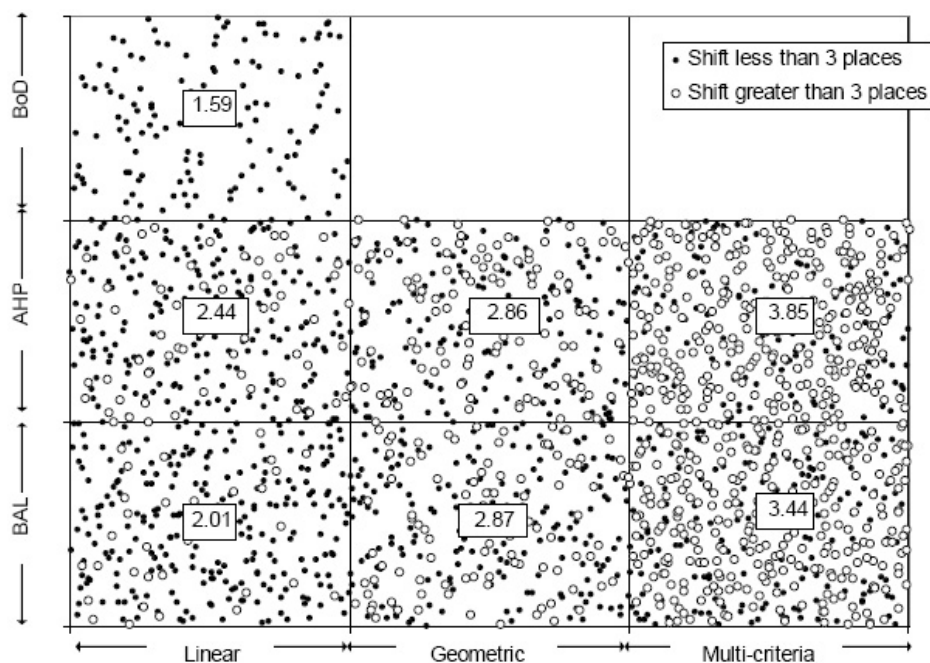


Figure 2.7: Average shift in countries' ranking with respect to the original TAI for different combinations of aggregation system and weighting scheme. Average value per case is indicated in the box.

2.2.2 Second analysis

In the second analysis we consider that TAI stakeholders have eventually converged to the linear aggregation system, as in the original TAI. This analysis is conducted using zeros for the missing values in the full data set. As a result, all 72 countries have been included. The uncertainty analysis plot (Figure 2.2.2) shows now a much more robust behaviour of the composite indicator, with fewer inversion of ranking when median-TAI and original TAI are compared. As far as the sensitivity is concerned, the consideration of uncertainty arising from imputation does not seem to make a significant contribution to the output uncertainties, which are also in this case dominated by weighting, inclusion-exclusion, expert selection. Even when, as in the case of Malaysia, imputation by bivariate approach ends into an unrealistic number of patents being imputed for this country (234 patents granted to residents per million people), the uncertainty of its ranking is insensitive to imputation. The sensitivity analysis results for the variable average shift in ranking (Equation 2.2) is shown in Table 2.2.2. Interactions are now between expert selection and weighting, and considerably less with interaction with inclusion-exclusion.

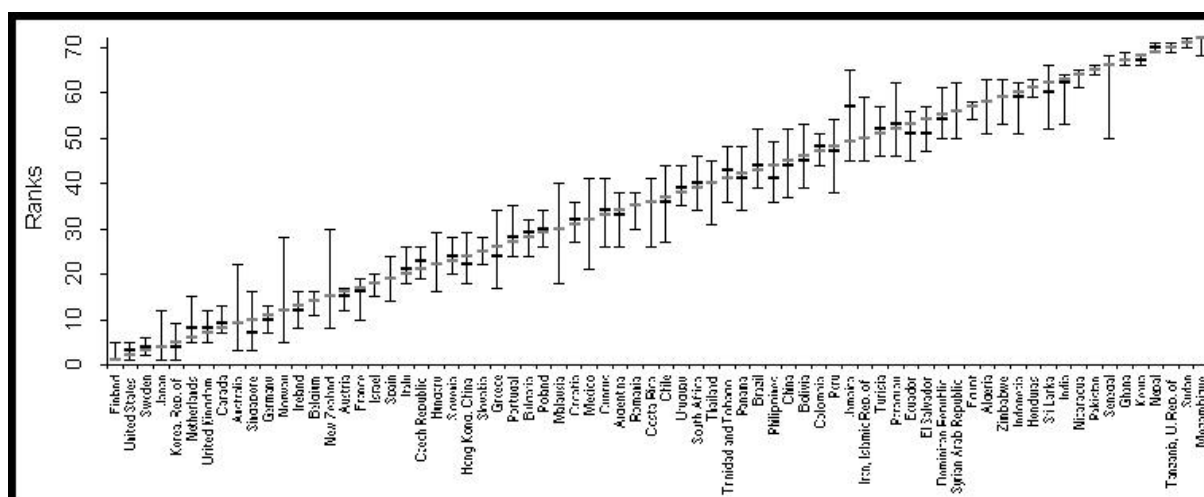


Figure 2.8: Uncertainty analysis results showing the countries' ranking according to the original TAI 2001 (light grey marks), and the median (black mark) and the corresponding 5th and 95th percentiles (bounds) of the distribution of the MC-TAI for 72 countries. Uncertain input factors: imputation, normalisation method, inclusion-exclusion of an indicator, weighting scheme, expert selection. A linear aggregation system is used. Countries are ordered according to the original TAI values.

<i>Input Factors</i>	First order (S_i)	Total effect (S_{Ti})	$S_{Ti} - S_i$
Imputation	0.001	0.005	0.004
Normalisation	0.000	0.021	0.021
Exclusion/Inclusion of indicator	0.135	0.214	0.078
Weighting Scheme	<u>0.212</u>	<u>0.623</u>	<u>0.410</u>
Expert selection	<u>0.202</u>	<u>0.592</u>	<u>0.390</u>
Sum	0.550	1.453	

Table 2.2: Sobol' sensitivity measure of first order and total effect for the output: Average shift in countries' ranking with respect to the original TAI. Significant values are underlined.

Chapter 3

Conclusions

The exercise carried out in this report gives an overview of the state-of-the-art methods available for the robustness assessment of (composite) indicators and illustrates the methodologies that will be used in the simulation studies that will be conducted during KEI on the knowledge-based economy.

The analyses carried out in this report on the TAI index answer the following questions:

(a) Does the use of one strategy versus another in indicator building (the steps i to vii described above) provide a biased picture of the countries' performance? How does this compare to the original TAI index?

The answer to this question is that much depends on the severity of the uncertainties. As shown by the two analyses, if the builders of the composite indicator disagree on the aggregation system, there is not much hope that a robust composite indicator will emerge, not even by the best provision of uncertainty and sensitivity analysis. If uncertainties exist in the context of a well established theoretical approach, e.g. the developer of the composite indicator favours a participatory approach within a linear aggregation scheme, then the analysis shows that the countries ranking is fairly robust in spite of the uncertainties.

(b) To what extent do the uncertain input factors (used to generate the alternatives i to vii above) affect the countries' rankings with respect to the original, deterministic TAI?

Both imputation and normalisation do not affect significantly countries ranking when uncertainties of higher order are present. In this exercise the most relevant uncertainties were expert selection and weighing scheme. In other words, when the weights are uncertain, it is unlikely that normalisation and editing will affect sensibly the country ranks.

In this example, the choice of the aggregation system is of paramount importance. Once the system is fixed, then it is the choice of the aggregation methods and of the experts that – together with indicator inclusion – exclusion, dominates the uncertainty in the country rankings. It is important to mention that even in the second analysis, when the aggregation system is fixed, the composite indicator model is strongly non additive, which reinforces the case for the use of quantitative, Monte Carlo based approaches to robustness analysis.

Bibliography

- Banker, R. D. and Maindiratta (1986) Piecewise Loglinear Estimates of Efficient Production Surfaces.
Management Science **32**, 126–135.
- Box, G., Hunter, W. and Hunter, J. (1995) *Statistics for experimenters*.
New York: John Wiley and Sons.
- Bratley, P. and Fox, B. L. (1988) ALGORITHM 659 Implementing Sobol’s quasirandom sequence generator.
ACM Trans. Math. Software **14**, 88–100.
- Chan, K., Tarantola, S., Saltelli, A. and Sobol, I. M. (2000) Variance based methods.
In *Sensitivity Analysis* (eds A. Saltelli, K. Chan, M. Scott), pp. 167–197. New York: John Wiley & Sons.
- Charnes, A., Cooper, W. W., Seiford, L. M. and Stutz (1983) Invariant Multiplicative Efficiency and Piecewise Cobb-Douglas Envelopments.
Operations Research Letters **2**, 101–103.
- Cherchye, L., Moesen, W. and Van Puyenbroeck, T. (2003) Legitimately Diverse, Yet Comparable: on Synthesizing Social Inclusion Performance in the EU , Econ WPA working paper.
In *the Journal of Common Market Studies*, 42, NR 4, Dec. 2004.
Forthcoming, with some minor revisions.
- Cox, D., Fitzpatrick, R., Fletcher, A., Gore, S., Spiegelhalter, D. and Jones, D. (1992) Quality-of-life assessment: can we keep it simple?
J.R. Statist.Soc. **155(3)**, 353–393.
- Environmental Protection Agency -EPA (2004) Council for Regulatory Environmental Modeling (CREM).
Draft Guidance on the Development, Evaluation, and Application of Regulatory Environmental Models, http://www.epa.gov/osp/crem/library/CREM%20Guidance%20Draft%2012_03.pdf.
- Freudenberg, M. (2003) Composite Indicators of Country Performance: a Critical Assessment.
STI working paper 2003/16, OECD, Paris.
- Homma, T. and Saltelli, A. (1996) Importance measures in global sensitivity analysis of model output.

- Reliability Engineering and System Safety* **52** (1), 1–17.
- Jamison, D. and Sandbu, M. (2001) WHO ranking of health system performance. *Science* **293**, 1595–1596.
- Keeney, R. and Raiffa, H. (1976) *Decision with multiple objectives: preferences and value trade-offs*.
Wiley, New York.
- Melyn, W. and Moesen, W. (1991) Towards a synthetic indicator of macroeconomic performance: Unequal weighting when limited information is available.
Public Economics Research paper 17, CES, KU Leuven.
- Moldan, B. and Billharz, S. (1997) *Sustainability indicators: Report of the Project on Indicators of Sustainable Development*.
Scope 58. Wiley Pub.
- Munda, G. (1995) *Multicriteria evaluation in a fuzzy environment*.
Physica-Verlag, Contributions to Economics Series, Heidelberg.
- Munda, G. and Nardo, M. (2003) *On the Construction of Composite Indicators for Ranking Countries*.
mimeo, Universitat Autònoma de Barcelona.
- Saaty, T. (1980) *The Analytic Hierarchy Process*.
N.Y.: McGraw - Hill Book Co.
- Saisana, M., Saltelli, A. and Tarantola, S. (2005) Uncertainty and Sensitivity Analysis as tools for the quality assessment of composite indicators.
Journal of the Royal Statistical Society Series A **168**, 1–17.
- Saltelli, A. (2002) Making best use of model valuations to compute sensitivity indices.
Computer Physics Communications **145**, 280–297.
- Saltelli, A., Chan, K. and Scott, M. (2000a) *Sensitivity analysis, Probability and Statistics series*.
New York: John Wiley & Sons.
- Saltelli, A. and Tarantola, S. (2002) On the relative importance of input factors in mathematical models: safety assessment for nuclear waste disposal.
Journal of American Statistical Association **97**, 702–709.
- Saltelli, A., Tarantola, S. and Campolongo, F. (2000b) Sensitivity analysis as an ingredient of modelling.
Statistical Science **15**, 377–395.
- Saltelli, A., Tarantola, S., Campolongo, F. and Ratto, M. (2004) *Sensitivity Analysis in practice, a guide to assessing scientific models*.
New York: John Wiley & Sons.
A software for sensitivity analysis is available at <http://www.jrc.cec.eu.int/uasa/prj-sa-soft.asp>.

- Sobol', I. M. (1976) Uniformly distributed sequences with an additional uniform property.
USSR Computat. Maths. Math. Phys. **16**, 236–242.
- Sobol', I. M. (1993) Sensitivity analysis for non-linear mathematical models.
Mathematical Modelling & Computational Experiment **1**, 407–414.
- Tarantola, S., Saisana, M., Saltelli, A., Schmiedel, F. and Leapman, N. (2002) Statistical techniques and participatory approaches for the composition of the European Internal Market Index 1992-2001, EUR 20547 EN.
European Commission: JRC-Italy.
- Ting, H. M. (1971) Aggregation of attributes for multiattributed utility assessment.
Technical Report 66, Operations Research Center, MIT Cambridge Mass.
- United Nations (2001) *Human Development Report*.
Oxford University Press, UK.
<http://www.undp.org>.
- Zimmermann, H. J. and Zysno, P. (1983) Decisions and evaluations by hierarchical aggregation of information.
Fuzzy Sets and Systems **10**, 243–260.

Appendix A

Appendix

TAI focuses on four dimensions of technological capacity (Table A):

- (a) Creation of technology. Two indicators are used to capture the level of innovation in a society: the number of patents granted per capita (to reflect the current level of invention activities), and the receipts of royalty and license fees from abroad per capita (to reflect the stock of successful innovations of the past that are still useful and hence have market value).
- (b) Diffusion of recent innovations. This diffusion is measured by two indicators: diffusion of the Internet (indispensable to participation), and by exports of high-and medium-technology products as a share of all exports.
- (c) Diffusion of old innovations. Two indicators are included here, telephones and electricity, which are especially important because they are needed to use newer technologies and are also pervasive inputs to a multitude of human activities. Both indicators are expressed as logarithms, as they are important at the earlier stages of technological advance but not at the most advanced stages. Expressing the measure in logarithms ensures that as the level increases, it contributes less to the technology achievement.
- (d) Human skills. A critical mass of skills is indispensable to technological dynamism. The foundations of such ability are basic education to develop cognitive skills and skills in science and mathematics. Two indicators are used to reflect the human skills needed to create and absorb innovations: mean years of schooling and gross enrolment ratio of tertiary students enrolled in science, mathematics and engineering.

Table A shows the raw data for the eight indicators for a set of 72 countries (original). However the original data set contains a large number of missing values, mainly due to missing data in Patents and Royalties. Note that for the first analysis described in Section 2.2.1, the set of the first 23 countries (from Finland to Slovenia) is used. The second analysis described in Section 2.2.2 is based on the entire set of 72 countries.

<i>Indicator</i>	<i>Unit</i>	<i>Definition</i>
Creation of technology		
PATENTS	Patents granted per 1,000,000 people	Number of patents granted to residents, so as to reflect the current level of invention activities (1998)
ROYALTIES	US \$ per 1,000 people	Receipts of royalty and license fees from abroad per capita, so as to reflect the stock of successful innovations of the past that are still useful and hence have market value (1999)
Diffusion of recent innovations		
INTERNET	Internet hosts per 1,000 people	Diffusion of the Internet, which is indispensable to participation in the network age (2000)
EXPORTS	%	Exports of high and medium technology products as a share of total goods exports (1999)
Diffusion of old innovations		
TELEPHONES	Telephone lines per 1,000 people (log)	Number of telephone lines (mainline and cellular), which represents old innovation needed to use newer technologies and is also pervasive input to a multitude of human activities (1999)
ELECTRICITY	kWh per capita (log)	Electricity consumption, which represents old innovation needed to use newer technologies and is also pervasive input to a multitude of human activities (1998)
Human skills		
SCHOOLING	years	Mean years of schooling (age 15 and above), which represents the basic education needed to develop cognitive skills (2000)
ENROLMENT	%	Gross enrolment ratio of tertiary students enrolled in science, mathematics and engineering, which reflects the human skills needed to create and absorb innovations (1995-1997)

Table A.1: List of indicators of the Technology Achievement Index

		PATENTS	ROYALTIES	INTERNET	EXPORTS	TELEPHONES(log)	ELECTRICITY(log)	SCHOOLING	ENROLMENT
1	Finland	187	125.6	200.2	50.7	3.08	4.15	10	27.4
2	United States	289	130	179.1	66.2	3.00	4.07	12	13.9
3	Sweden	271	156.6	125.8	59.7	3.10	4.14	11.4	15.3
4	Japan	994	64.6	49	80.8	3.00	3.86	9.5	10
5	Korea, Rep. of	779	9.8	4.8	66.7	2.97	3.65	10.8	23.2
6	Netherlands	189	151.2	136	50.9	3.02	3.77	9.4	9.5
7	United Kingdom	82	134	57.4	61.9	3.02	3.73	9.4	14.9
8	Canada	31	38.6	108	48.7	2.94	4.18	11.6	14.2
9	Australia	75	18.2	125.9	16.2	2.94	3.94	10.9	25.3
10	Singapore	8	25.5	72.3	74.9	2.95	3.83	7.1	24.2
11	Germany	235	36.8	41.2	64.2	2.94	3.75	10.2	14.4
12	Norway	103	20.2	193.6	19	3.12	4.39	11.9	11.2
13	Ireland	106	110.3	48.6	53.6	2.97	3.68	9.4	12.3
14	Belgium	72	73.9	58.9	47.6	2.91	3.86	9.3	13.6
15	New Zealand	103	13	146.7	15.4	2.86	3.91	11.7	13.1
16	Austria	165	14.8	84.2	50.3	2.99	3.79	8.4	13.6
17	France	205	33.6	36.4	58.9	2.97	3.80	7.9	12.6
18	Israel	74	43.6	43.2	45	2.96	3.74	9.6	11
19	Spain	42	8.6	21	53.4	2.86	3.62	7.3	15.6
20	Italy	13	9.8	30.4	51	3.00	3.65	7.2	13
21	Czech Republic	28	4.2	25	51.7	2.75	3.68	9.5	8.2
22	Hungary	26	6.2	21.6	63.5	2.73	3.46	9.1	7.7
23	Slovenia	105	4	20.3	49.5	2.84	3.71	7.1	10.6
24	Hong Kong, China (SAR)	6		33.6	33.6	3.08	3.72	9.4	9.8
25	Slovakia	24	2.7	10.2	48.7	2.68	3.59	9.3	9.5
26	Greece			16.4	17.9	2.92	3.57	8.7	17.2
27	Portugal	6	2.7	17.7	40.7	2.95	3.53	5.9	12
28	Bulgaria	23		3.7	30	2.60	3.50	9.5	10.3
29	Poland	30	0.6	11.4	36.2	2.56	3.39	9.8	6.6
30	Malaysia			2.4	67.4	2.53	3.41	6.8	3.3
31	Croatia	9		6.7	41.7	2.63	3.39	6.3	10.6
32	Mexico	1	0.4	9.2	66.3	2.28	3.18	7.2	5
33	Cyprus			16.9	23	2.87	3.54	9.2	4
34	Argentina	8	0.5	8.7	19	2.51	3.28	8.8	12
35	Romania	71	0.2	2.7	25.3	2.36	3.21	9.5	7.2
36	Costa Rica		0.3	4.1	52.6	2.38	3.16	6.1	5.7
37	Chile		6.6	6.2	6.1	2.55	3.32	7.6	13.2
38	Uruguay	2		19.6	13.3	2.56	3.25	7.6	7.3
39	South Africa		1.7	8.4	30.2	2.43	3.58	6.1	3.4

40	Thailand	1	0.3	1.6	48.9	2.09	3.13	6.5	4.6
41	Trinidad and Tobago			7.7	14.2	2.39	3.54	7.8	3.3
42	Panama			1.9	5.1	2.40	3.08	8.6	8.5
43	Brazil	2	0.8	7.2	32.9	2.38	3.25	4.9	3.4
44	Philippines		0.1	0.4	32.8	1.89	2.65	8.2	5.2
45	China	1	0.1	0.1	39	2.08	2.87	6.4	3.2
46	Bolivia	1	0.2	0.3	26	2.05	2.61	5.6	7.7
47	Colombia	1	0.2	1.9	13.7	2.37	2.94	5.3	5.2
48	Peru		0.2	0.7	2.9	2.03	2.81	7.6	7.5
49	Jamaica		2.4	0.4	1.5	2.41	3.35	5.3	1.6
50	Iran, Islamic Rep. of	1			2	2.12	3.13	5.3	6.5
51	Tunisia		1.1		19.7	1.98	2.92	5	3.8
52	Paraguay		35.3	0.5	2	2.14	2.88	6.2	2.2
53	Ecuador			0.3	3.2	2.09	2.80	6.4	6
54	El Salvador		0.2	0.3	19.2	2.14	2.75	5.2	3.6
55	Dominican Republic			1.7	5.7	2.17	2.80	4.9	5.7
56	Syrian Arab Republic				1.2	2.01	2.92	5.8	4.6
57	Egypt		0.7	0.1	8.8	1.89	2.94	5.5	2.9
58	Algeria				1	1.73	2.75	5.4	6
59	Zimbabwe			0.5	12	1.56	2.95	5.4	1.6
60	Indonesia			0.2	17.9	1.60	2.51	5	3.1
61	Honduras				8.2	1.76	2.65	4.8	3
62	Sri Lanka			0.2	5.2	1.69	2.39	6.9	1.4
63	India	1		0.1	16.6	1.45	2.58	5.1	1.7
64	Nicaragua			0.4	3.6	1.59	2.45	4.6	3.8
65	Pakistan			0.1	7.9	1.38	2.53	3.9	1.4
66	Senegal			0.2	28.5	1.43	2.05	2.6	0.5
67	Ghana				4.1	1.08	2.46	3.9	0.4
68	Kenya			0.2	7.2	1.04	2.11	4.2	0.3
69	Nepal			0.1	1.9	1.08	1.67	2.4	0.7
70	Tanzania, U. Rep. of				6.7	0.78	1.73	2.7	0.2
71	Sudan				0.4	0.95	1.67	2.1	0.7
72	Mozambique				12.2	0.70	1.73	1.1	0.2

Table A.2: Raw data for the indicators of the Technology Achievement Index. The first 23 countries are used in the first analysis (Section 2.2.1), while in the second analysis (Section 2.2.2) the entire set is used. Units are given in Table A.