A Semi-Parametric Block Bootstrap Approach for Clustered Data

Ray Chambers & Hukum Chandra
University of Wollongong

SAE2011, Trier, Germany
12 August 2011

CSSM
Centre for Statistical & Survey Methodology

Overview of Presentation

- Background and Motivation
- Bootstrap Methods
- Empirical Evaluations
- Application to a Real Dataset
- Concluding Remarks

Background

- **Bootstrap technique:** a computer intensive and general way of measuring the accuracy of estimators **(Efron, 1979; Efron & Tibshirani, 1993)**

- Originally developed for parameter estimation given data values that are **independent and identically distributed** (*iid*)

- Random effects models for **hierarchically dependent data**, e.g. clustered data, are now widely used (e.g. in SAE)

- With such data, it is important to use bootstrap techniques that allow for the hierarchical dependence structure

- **Parametric bootstrap** based on the assumed hierarchical random effects model is widely used
  - very effective if model is **correctly specified**

Background

- If the variability assumptions of the model, e.g. the assumption that the random effects are *iid* Normal random variables, are violated, then it is hard to justify use of the parametric bootstrap **(Rasbash *et al.*, 2000; Carpenter *et al.*, 2003)**

- A semi-parametric bootstrap for multilevel modelling is described in **Carpenter *et al.* (2003)**

- We describe an alternative **semi-parametric block** bootstrap for clustered data
  - 'semi-parametric' because although the marginal model is bootstrapped parametrically, the dependence structure in the model residuals is non-parametrically block bootstrapped
  - 'block' because we are interested in a bootstrap procedure that is robust to within cluster heterogeneity

Focus on bootstrap confidence interval performance

Bootstrap Percentile Confidence Interval

- Upper and lower $\alpha/2$ values of the bootstrap distribution are used to construct a $100(1-\alpha)\%$ confidence interval

- Let $\hat{\theta}_{L,\alpha/2}$ denotes the value such that a fraction $\alpha/2$ of all bootstrap estimates are smaller, and likewise let $\hat{\theta}_{U,\alpha/2}$ be the value such that a fraction $\alpha/2$ of all bootstrap estimates are larger; then an approximate confidence interval for θ is $\left[\hat{\theta}_{L,\alpha/2}, \hat{\theta}_{U,\alpha/2}\right]$

- Width = $\hat{\theta}_{U,\alpha/2} - \hat{\theta}_{L,\alpha/2}$ and standardised width = $\left(\hat{\theta}_{U,\alpha/2} - \hat{\theta}_{L,\alpha/2}\right)/\theta$

Random Effects Model for Clustered Data

Two-level model

$$y_{ij} = \mathbf{x}_{ij}^T \boldsymbol{\beta} + u_i + e_{ij},\ \ j = 1,\ldots,n_i; i = 1,..,D$$

Group-specific random effects (level 2) $u_i \overset{IID}{\sim} N(0,\sigma_u^2)$

Individual level errors (level 1) $e_{ij} \overset{IID}{\sim} N(0,\sigma_e^2)$, $u_i \perp e_{ij} \mid x_{ij}$

Focus on bootstrap distributions for estimates $\hat{\boldsymbol{\beta}}$, $\hat{\sigma}_u^2$ and $\hat{\sigma}_e^2$

Parametric 2-Level Bootstrap (Para)

- ML/REML estimates $\hat{\beta}$, $\hat{\sigma}_u^2$ and $\hat{\sigma}_e^2$
- Simulate level 2 errors $u_i^* \overset{IID}{\sim} N(0, \hat{\sigma}_u^2),\ i = 1,..,D$
- Simulate level 1 errors $e_{ij}^* \overset{IID}{\sim} N(0, \hat{\sigma}_e^2),\ j = 1,...,n_i; i = 1,..,D$, $u_i^* \perp e_{ij}^*$
- Bootstrap Data $y_{ij}^* = x_{ij}^T \hat{\beta} + u_i^* + e_{ij}^*$
- Refit model, obtain **bootstrap parameter estimates** $\hat{\beta}^*$, $\hat{\sigma}_u^{2*}$ and $\hat{\sigma}_e^{2*}$
- Repeat B times $\Rightarrow$ B sets of bootstrap estimates
- Generate bootstrap distributions of $\hat{\beta}$, $\hat{\sigma}_u^2$ and $\hat{\sigma}_e^2$
- Bootstrap CIs 'read off' from bootstrap distributions

Semi-Parametric 2-Level Bootstrap (CGR)

(Carpenter, Goldstein and Rasbash, 2003)

- ML/REML estimates $\hat{\boldsymbol{\beta}}$, $\hat{\sigma}_u^2$ and $\hat{\sigma}_e^2$
- Level 2 residuals (EBLUPs) $\hat{u}_i\ , i = 1,.., D$
- Level 1 residuals $\hat{e}_{ij} = y_{ij} - x_{ij}^T \hat{\boldsymbol{\beta}} - \hat{u}_i, j = 1,..., n_i; i = 1,.., D$
- Centre and then rescale residuals $\hat{u}_i$ and $\hat{e}_{ij}$

Rescaling

- The empirical variance-covariance matrices of the level 1 and level 2 residuals can be different from their corresponding ML/REML estimates
- Rescale residuals to make these the same <u>before</u> bootstrapping

Rescaling (Cholesky decomposition)

- Estimate the variance-covariance matrix $\hat{\Sigma}$ of model errors $\hat{\mathbf{U}}$ (either level 2 or level 1) via ML/REML
- Cholesky decomposition $\hat{\mathbf{\Sigma}} = \mathbf{A}\mathbf{A}^T$ calculated, where $\mathbf{A}$ is a lower triangular matrix
- $\hat{\mathbf{V}}$ = empirical variance/covariance matrix of $\hat{\mathbf{U}}$
- Cholesky decomposition $\hat{\mathbf{V}} = \mathbf{B}\mathbf{B}^T$ calculated, where $\mathbf{B}$ is a lower triangular matrix
- Rescaled residuals $\hat{\mathbf{U}}^* = \hat{\mathbf{U}}\mathbf{C}$ where $\mathbf{C} = \left(\mathbf{A}\mathbf{B}^{-1}\right)^T$

- Sample **independently** with replacement from these two **new sets** of centred and rescaled residuals
- $u_i^* = srswr\left(\left\{\hat{u}_h; h = 1,\ldots.D\right\}, m = 1\right)$
- $e_{ij}^* = srswr\left\{\hat{e}_{hj}, j = 1,\ldots,n_i; h = 1,..,D\right\}$
- Bootstrap Data $\quad y_{ij}^* = x_{ij}^T\hat{\boldsymbol{\beta}} + u_i^* + e_{ij}^*$
- Refit model, obtain bootstrap parameter estimates $\hat{\boldsymbol{\beta}}^*$, $\hat{\sigma}_u^{2*}$ and $\hat{\sigma}_e^{2*}$
- Repeat this process B times
- Generate bootstrap distributions for $\hat{\boldsymbol{\beta}}$, $\hat{\sigma}_u^2$ and $\hat{\sigma}_e^2$
- Bootstrap CIs 'read off' from bootstrap distributions

Simple Semi-Parametric Block Bootstrap (SBB)

- Estimate $\hat{\boldsymbol{\beta}}$ with residuals : $r_{ij} = y_{ij} - x_{ij}^T \hat{\boldsymbol{\beta}}$, $j = 1,\dots,n_i; i = 1,..,D$
- Level 2 residuals : Group averages $\overline{r}_h = n_h^{-1} \sum_{j=1}^{n_h} r_{hj}$, $h = 1,..,D$
- Level 1 residuals : $r_{hj}^{(1)} = r_{hj} - \overline{r}_h \Rightarrow \mathbf{r}_h^{(1)}$ vector of size n_i
- Sample **independently** with replacement from these two sets of residuals
  - $\overline{r}_i^* = srswr\left(\left\{\overline{r}_h; h = 1,\dots.D\right\}, m = 1\right)$
  - $h(i) = srswr\left(\left\{1,\dots\dots, D\right\}, m = 1\right)$
  - **Block/group structure** : $\mathbf{r}_i^{(1)*} = srswr\left(\left\{\mathbf{r}_{h(i)}^{(1)}\right\}, m = n_i\right)$
- Bootstrap Data : $\mathbf{y}_i^* = \mathbf{x}_i^T \hat{\boldsymbol{\beta}} + \overline{r}_i^* \mathbf{1}_{n_i} + \mathbf{r}_i^{(1)*}$

SBB + Post Bootstrap Adjustment (SBB.Post)

- Multivariate bootstrap distribution of the variance component estimates is first transformed **('tilted')** in order to ensure that the bootstrap estimates of these components are uncorrelated

- All bootstrap distributions of model parameter estimates are then **'tethered'** to the original estimate values, using either a mean correction (for estimates, e.g. regression coefficients, defined on the entire real line) or a ratio correction (for estimates, e.g. variance components, that are strictly positive)

Tilting and Tethering (post-bootstrapping)

- $\left(\hat{\beta}_k^{**}\right)_B = \left[\hat{\beta}_k \mathbf{1}_B + \left(\hat{\beta}_k^{*}\right)_B - av_B\left(\hat{\beta}_k^{*}\right)\right]$
- $\left(\hat{\sigma}_u^{2**}\right)_B = \left(\hat{\sigma}_u^{*\text{mod}\,2}\right)_B \times \hat{\sigma}_u^2 \left\{av_B\left(\hat{\sigma}_u^{*\text{mod}\,2}\right)\right\}^{-1}$
- $\left(\hat{\sigma}_e^{2**}\right)_B = \left(\hat{\sigma}_e^{*\text{mod}\,2}\right)_B \times \hat{\sigma}_e^2 \left\{av_B\left(\hat{\sigma}_e^{*\text{mod}\,2}\right)\right\}^{-1}$

where

$$\left[\left(\hat{\sigma}_u^{*\text{mod}\,2}\right)_B \left(\hat{\sigma}_e^{*\text{mod}\,2}\right)_B\right] = \exp\left\{\mathbf{M}_B^* + \left\{\left(\mathbf{S}_B^* - \mathbf{M}_B^*\right)\left(\mathbf{C}_B^*\right)^{-1/2}\right\} \times \mathbf{D}_B^*\right\}$$

$$\mathbf{M}_B^* = \left[av_B\left(\log\hat{\sigma}_u^{*2}\right)\mathbf{1}_B \quad av_B\left(\log\hat{\sigma}_e^{*2}\right)\mathbf{1}_B\right]$$

$$\mathbf{S}_B^* = \left[\left(\log\hat{\sigma}_u^{*2}\right)_B \quad \left(\log\hat{\sigma}_e^{*2}\right)_B\right]$$

$$\mathbf{C}_B^* = \text{cov}_B\left(\mathbf{S}_B^*\right)$$

$$\mathbf{D}_B^* = \left[sd_B\left(\log\hat{\sigma}_u^{*2}\right)\mathbf{1}_B \quad sd_B\left(\log\hat{\sigma}_e^{*2}\right)\mathbf{1}_B\right]$$

Semi-Parametric Block Bootstrap with Centred and Rescaled Residuals (SBB.Prior)

- Estimate $\hat{\boldsymbol{\beta}}$ with residuals $r_{ij} = y_{ij} - x_{ij}^T\hat{\boldsymbol{\beta}}, j = 1,\ldots,n_i; i = 1,..,D$
- Level 2 residuals: Group-wise averages $\bar{r}_h = n_h^{-1}\sum_{j=1}^{n_h} r_{hj}, h = 1,..,D$
- Level 1 residuals: $r_{hj}^{(1)} = r_{hj} - \bar{r}_h \quad \Rightarrow \mathbf{r}_h^{(1)}$ vector of size n_i
- **Centre and rescale** Level 1 and Level 2 residuals: $\bar{r}_h$ and $r_{hj}^{(1)}$
- Apply SBB to these centred and rescaled residuals
- No post-bootstrap adjustment

SBB + External Calibration to Covariance Matrix of Variance Components (SBB.Prior.Adj)

- Calibrate covariance matrix of bootstrap estimates of variance components to ML/REML estimate of the covariance matrix of variance components obtained from the model - **Cholesky decomposition**

- Post-bootstrap adjustment where bootstrap distribution of variance components is tilted to recover ML/REML estimate of variance/covariance matrix of estimated variance components

Bootstrap Methods Used in Simulations

Type	Description
Para	Parametric 2-level bootstrap
CGR	Semi-Parametric 2-Level Bootstrap **(Carpenter, Goldstein and Rasbash, 2003)**
SBB	Simple semi-parametric Block bootstrap using empirical Level 1 and Level 2 residuals
SBB.Post	SBB with post-bootstrap tilting & tethering adjustments
SBB.Prior	SBB using internally rescaled empirical residuals
SBB.Prior.Adj	SBB using internally rescaled residuals with post-bootstrap adjustment to recover REML estimate of the variance of the estimated variance components

Simulation Design

- Total number of clusters: $D = 50,\ 100$
- Uniform cluster sample sizes: $n_i = 5, 20$
- $n = 250, 500, 1000, 2000$
- 1000 simulations
- 1000 Bootstrap samples per method per simulation

Model

- $y_{ij} = \beta_0 + \beta_1 x_{ij} + u_i + e_{ij}, j = 1,...,n_i; i = 1,..,D$
- $x_{ij} \sim U(0,1)$
- $u_i \overset{IID}{\sim} (0,\sigma_u^2)$, $e_{ij} \overset{IID}{\sim} (0,\sigma_e^2)$ $u_i \perp e_{ij} \mid x_{ij}$
- $\beta_0 = 1,\ \beta_1 = 2,\ \sigma_u^2 = 0.04$ and $\sigma_e^2 = 0.16$
- u_i and e_{ij} generated using **four** scenarios ...

Set A - Normal Scenario

- $u_i \sim N(0, \sigma_u^2 = 0.04)$ and $e_{ij} \sim N(0, \sigma_e^2 = 0.16)$

Set B - Chi-Square Scenario

- $u_i \sim 0.2\left[\left(\chi_1^2 - 1\right)/\sqrt{2}\right]$ and $e_{ij} \sim 0.4\left[\left(\chi_1^2 - 1\right)/\sqrt{2}\right]$

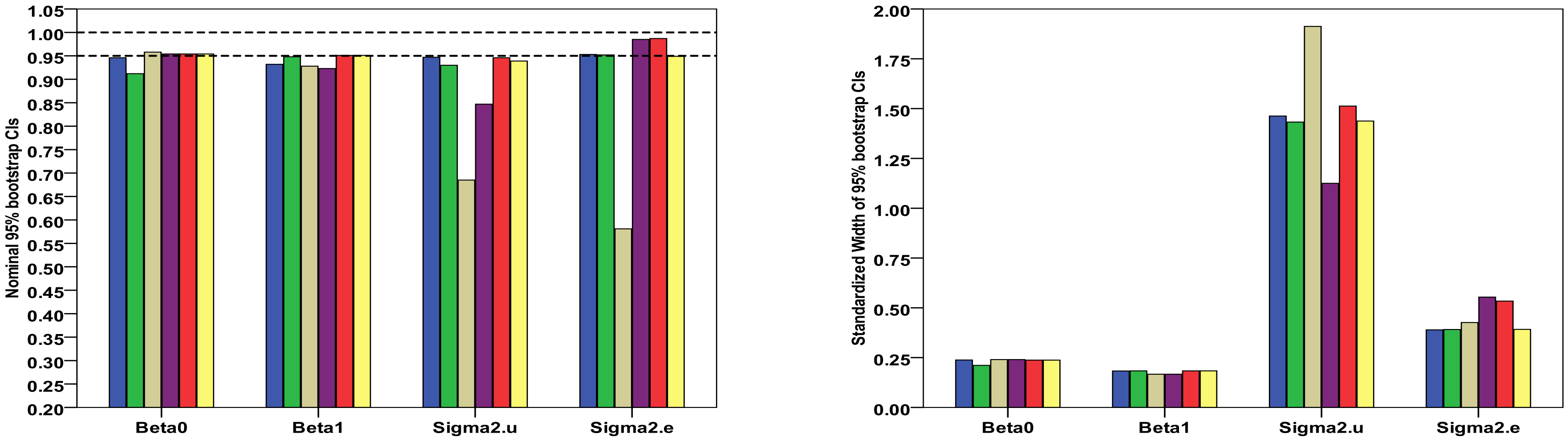
Set C - Within Cluster Auto-Correlation Scenario

- Level 2 errors normally distributed as in Set A, but Level 1 errors within each cluster independently generated as a first order auto-correlated series $e_{ij} = 0.5e_{i(j-1)} + \varepsilon_{ij}, j = 1,\ldots n_i$, with $\varepsilon_{ij} \sim N(0,1)$

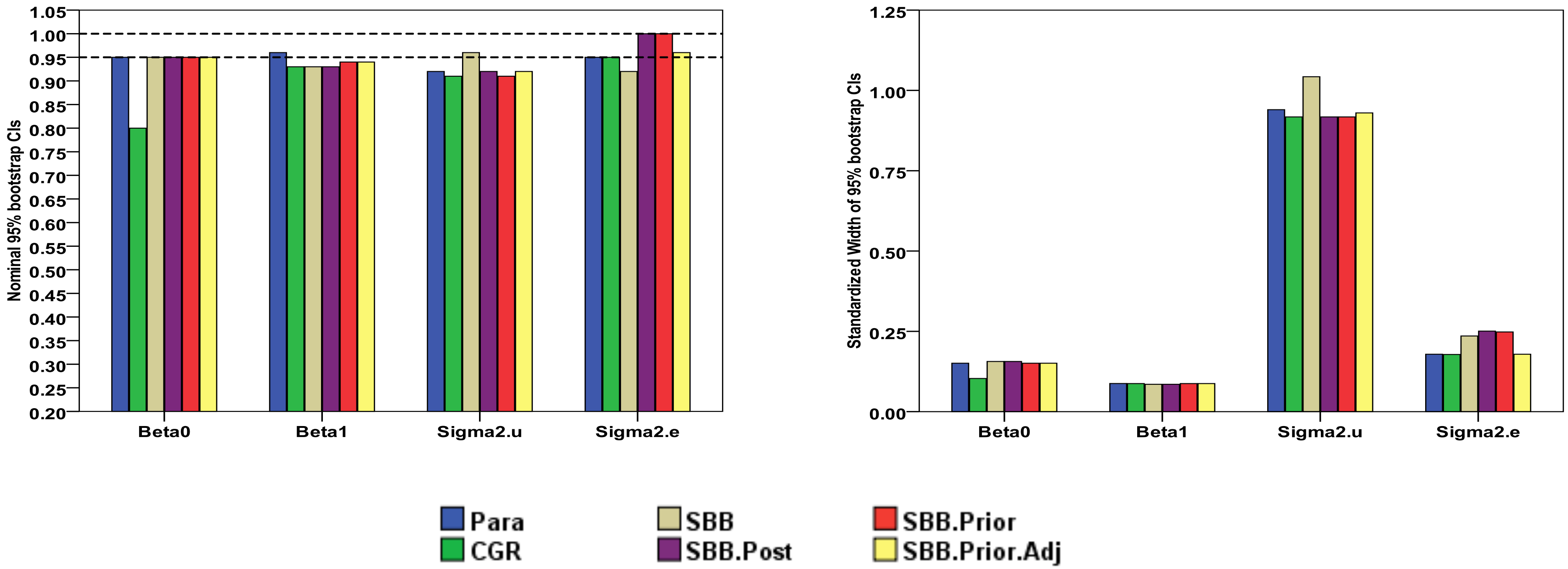
Set D - Between Cluster Auto-Correlation Scenario

- Level 2 errors normally distributed as in Set A, but Level 1 errors for entire population generated as a first order auto-correlated series $e_{ij} = 0.5e_{i(j-1)} + \varepsilon_{ij}, j = 1,\ldots n_i$, with $\varepsilon_{ij} \sim N(0,1)$, with clusters sequentially defined along the 'time' axis. This simulation approximates the type of time series problem that motivated the development of the block bootstrap

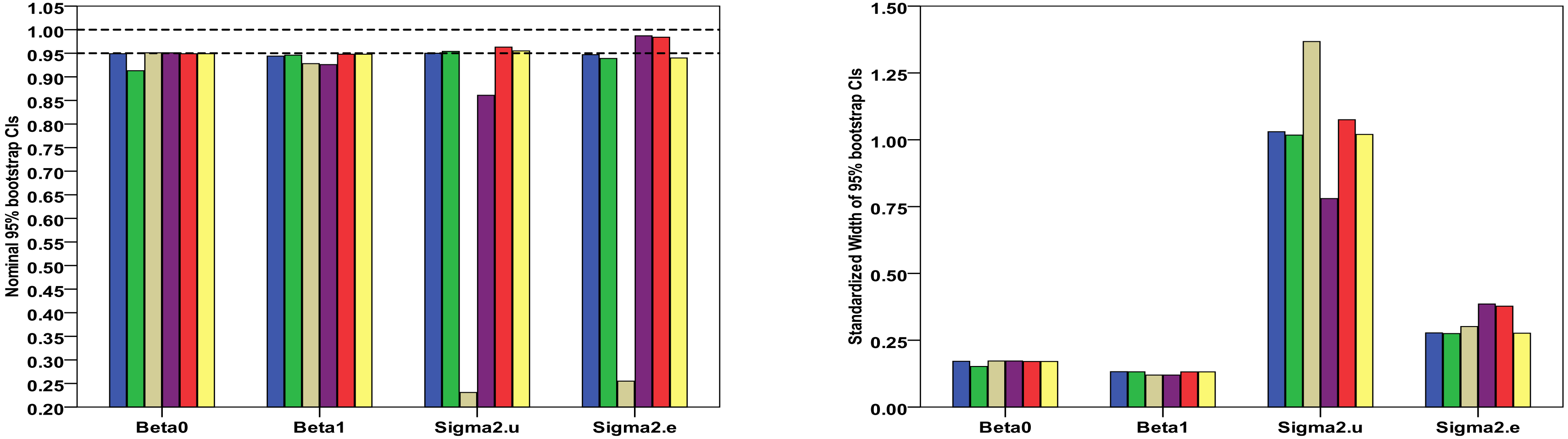
A: Normal Scenario: D=50, n_i=5 (n=250)



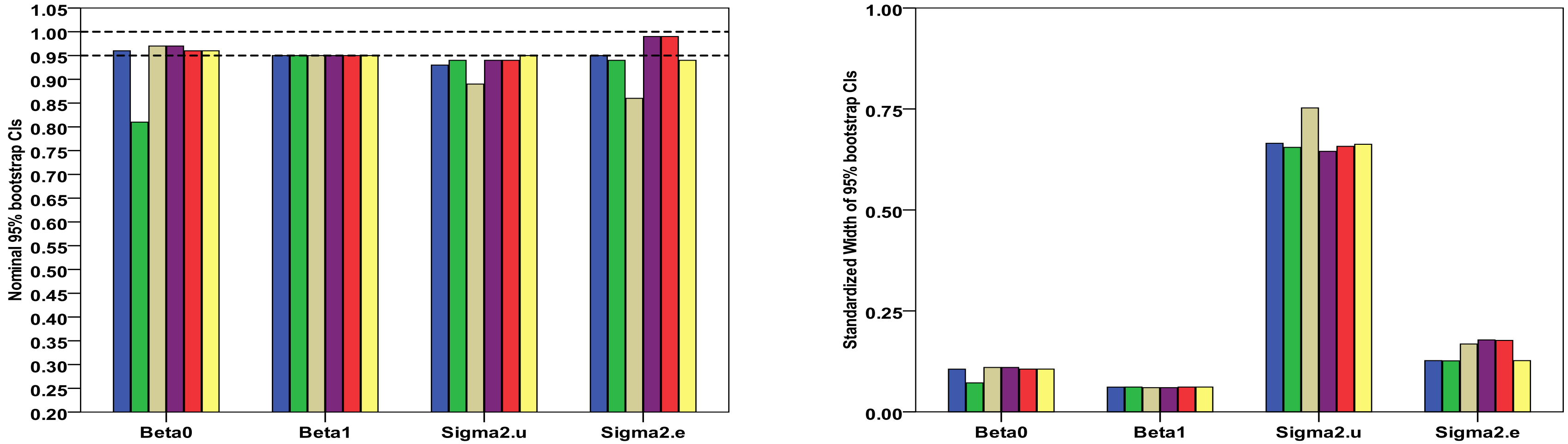
A: Normal Scenario: D=50, n_i=20 (n=1000)

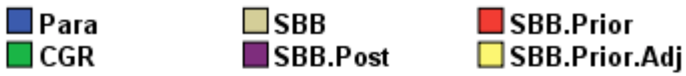


A: Normal Scenario: D=100, n_i=5 (n=500)

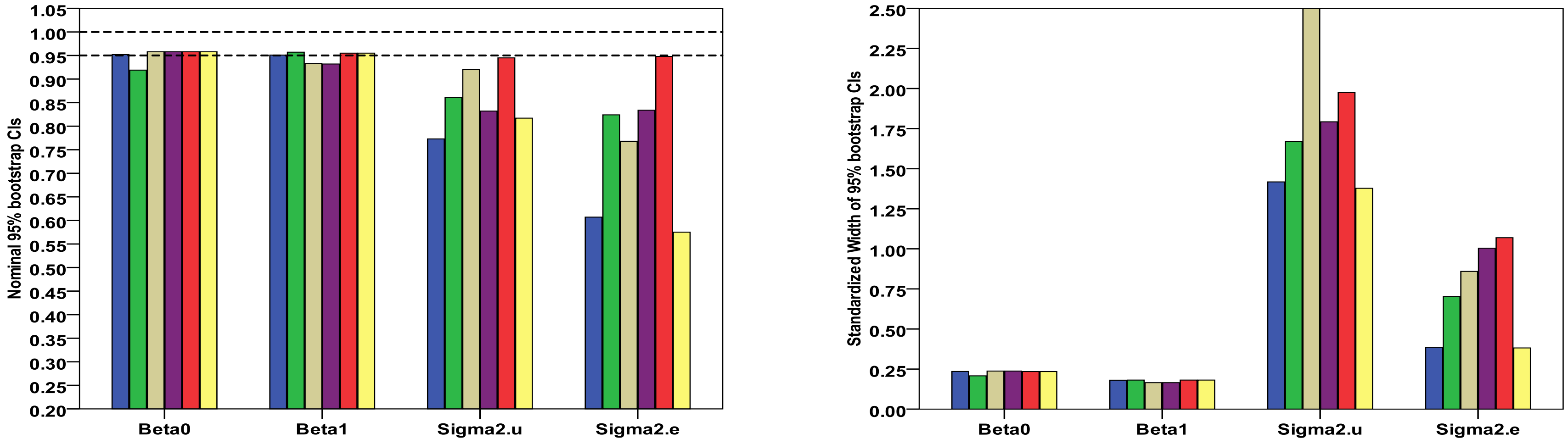


A: Normal Scenario: D=100, n_i=20 (n=2000)

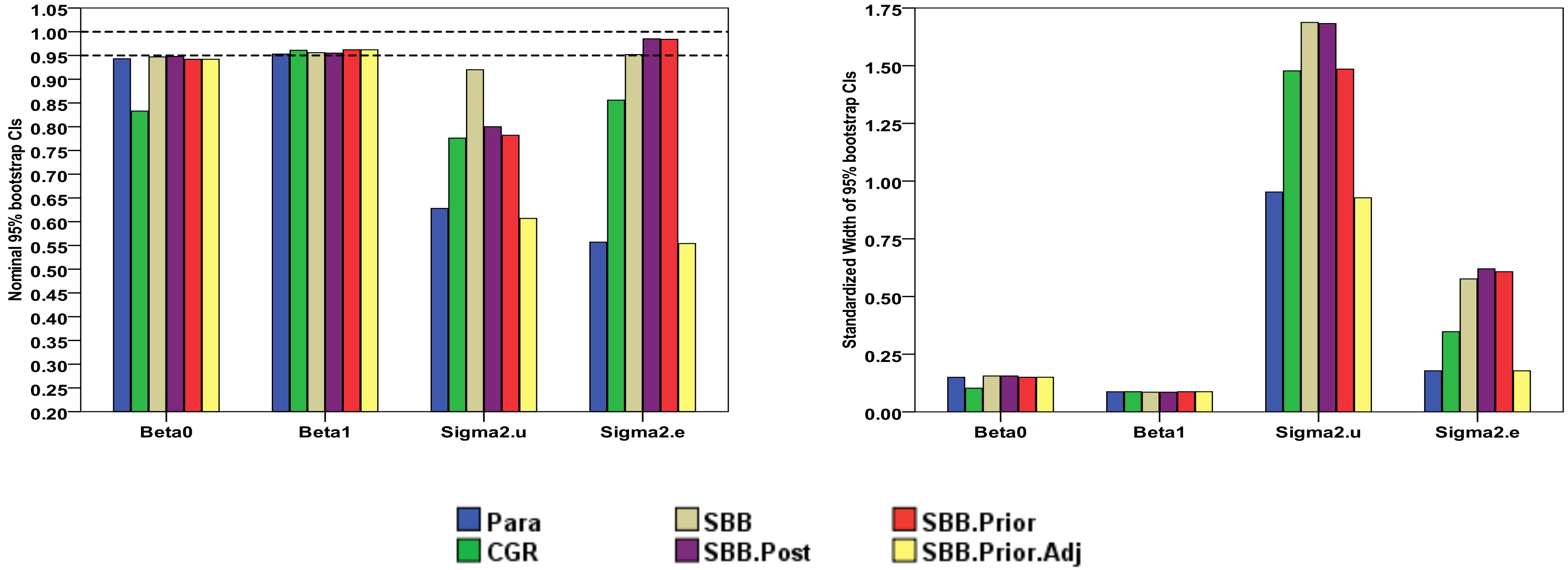




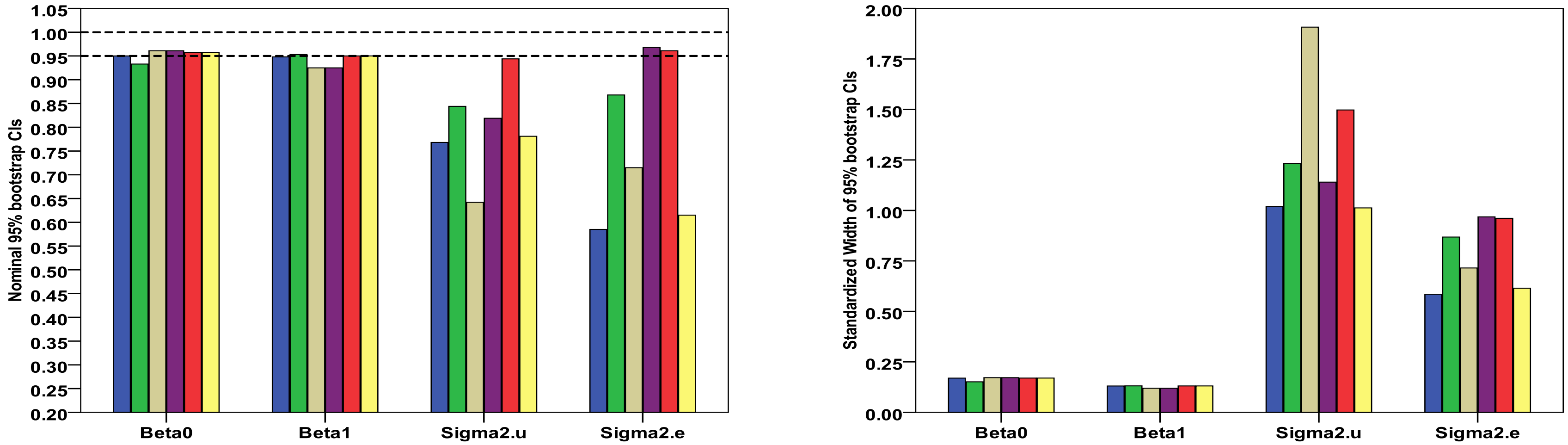
B: Chi-Square Scenario: D=50, n_i=5 (n=250)



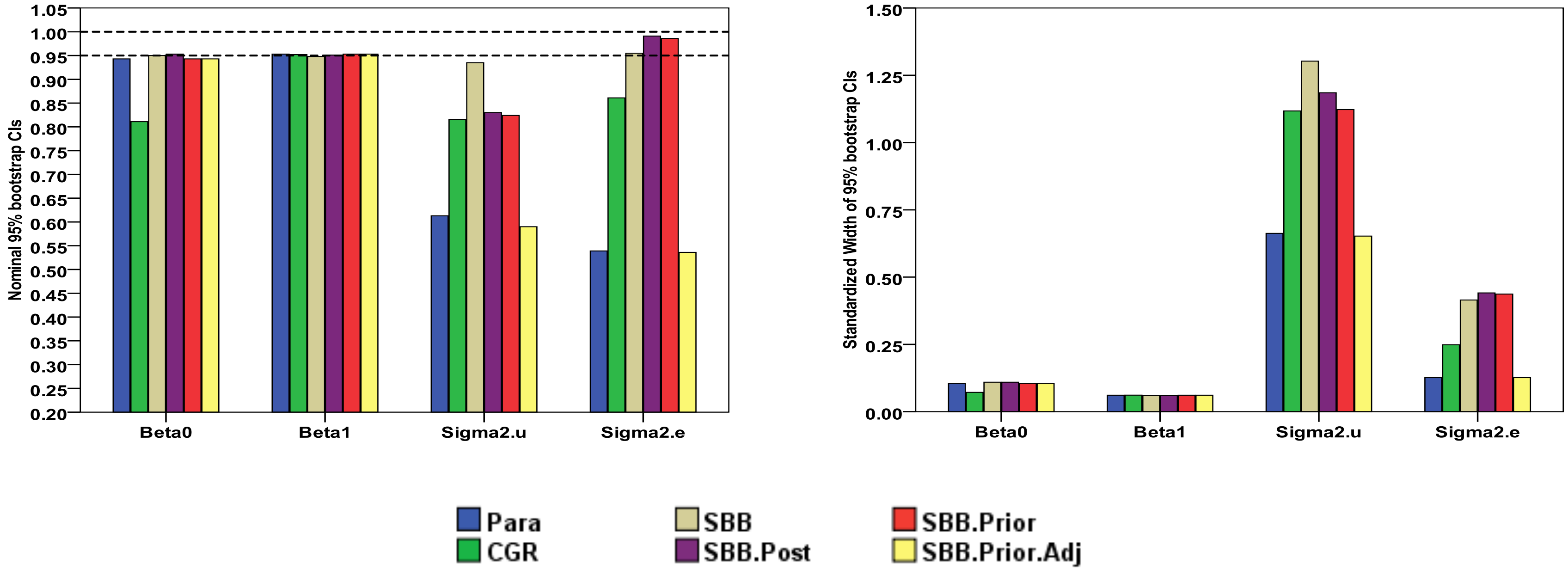
B: Chi-Square Scenario: D=50, n_i=20 (n=1000)



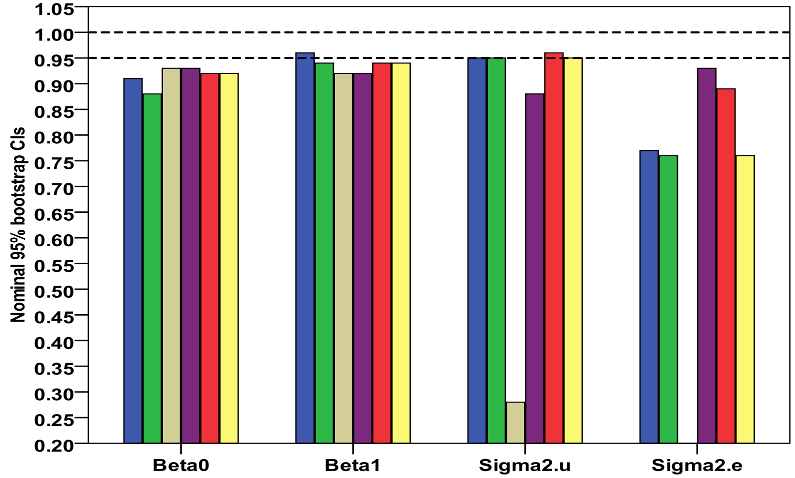
B: Chi-Square Scenario: D=100, n_i=5 (n=500)

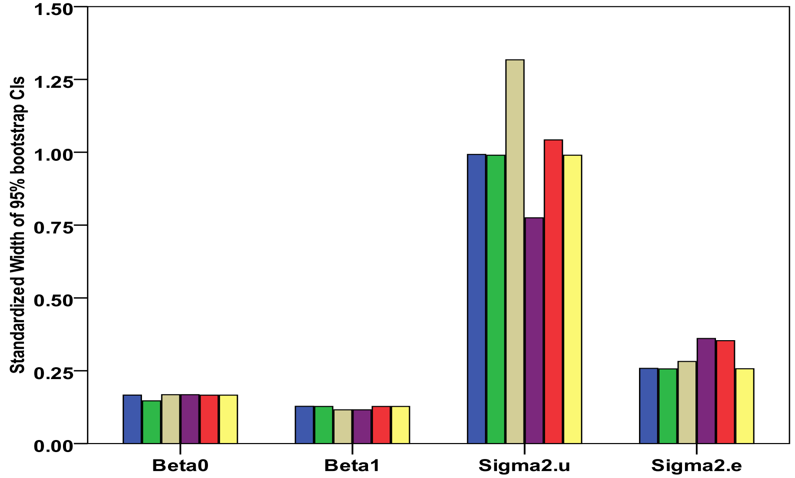


B: Chi-Square Scenario: D=100, n_i=20 (n=2000)

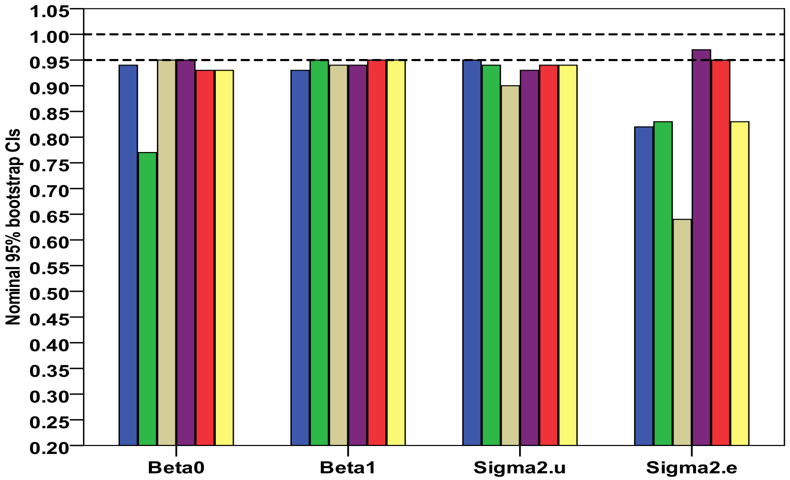


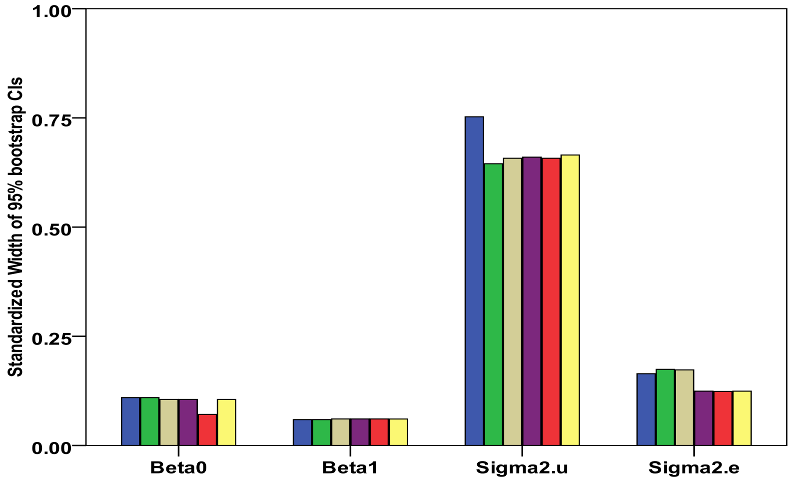
C: Within Cluster Auto-Correlation Scenario: D=100, n_i=5 (n=500)

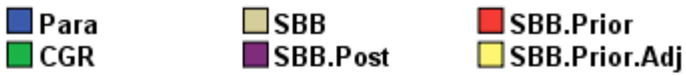




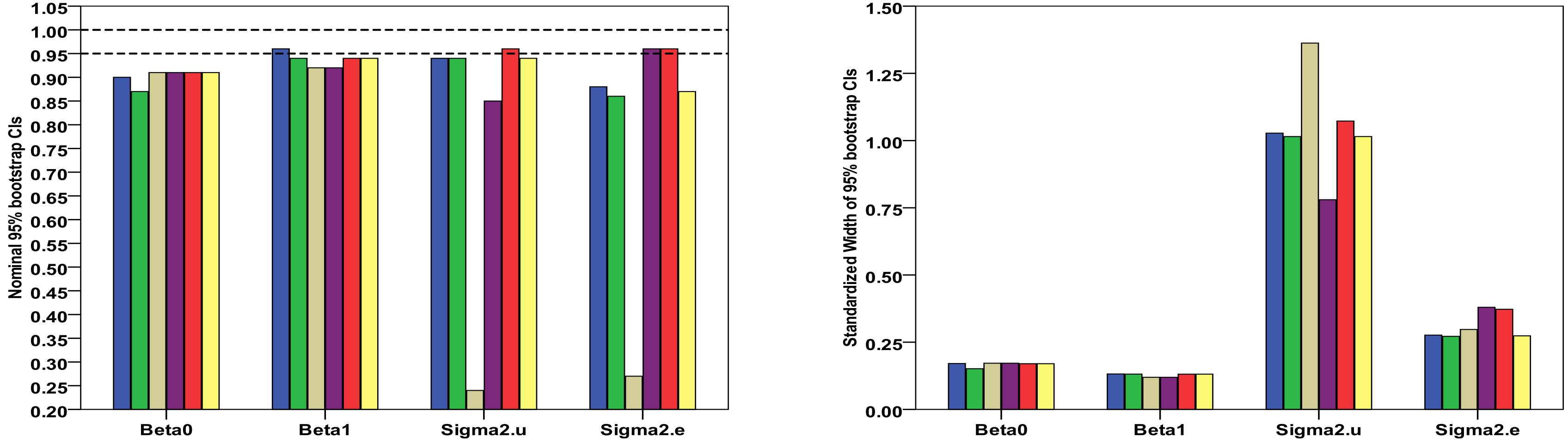
C: Within Cluster Auto-Correlation Scenario: D=100, n_i=20 (n=2000)



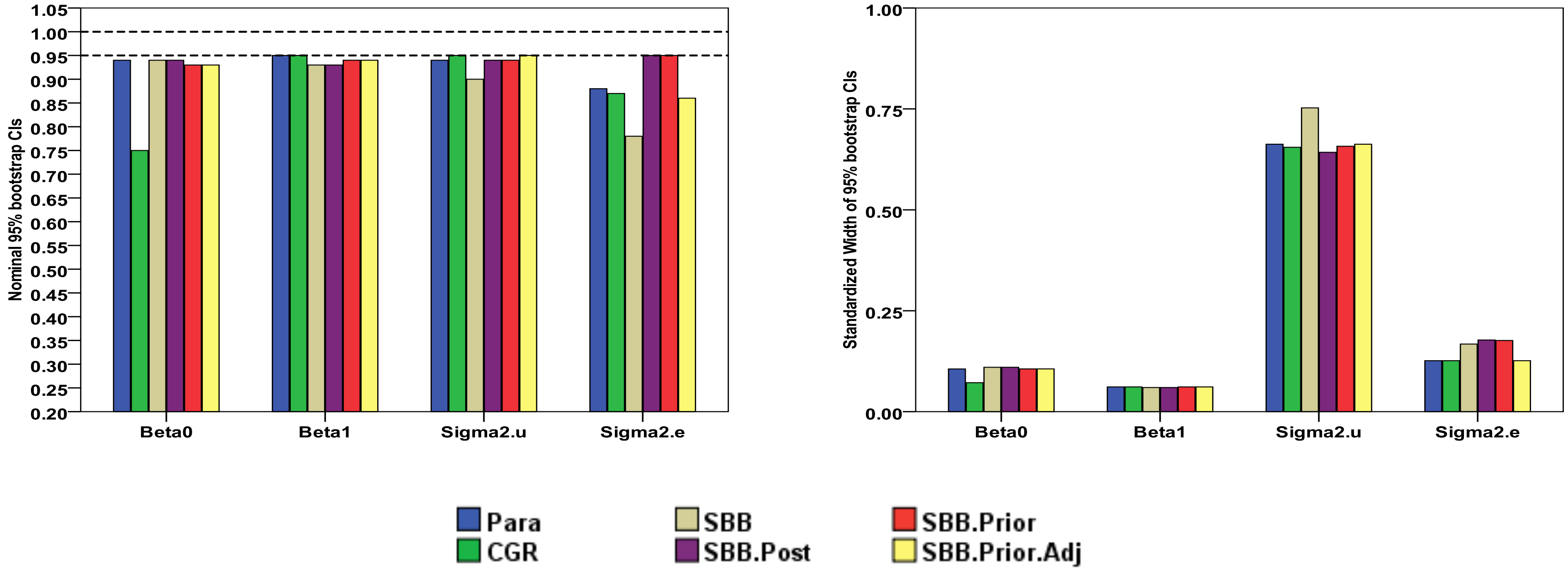




D: Between Cluster Auto-Correlation Scenario: D=100, n_i=5 (n=500)



D: Between Cluster Auto-Correlation Scenario: D=100, n_i=20 (n=1000)



Simulation Results for Spatially Correlated Data

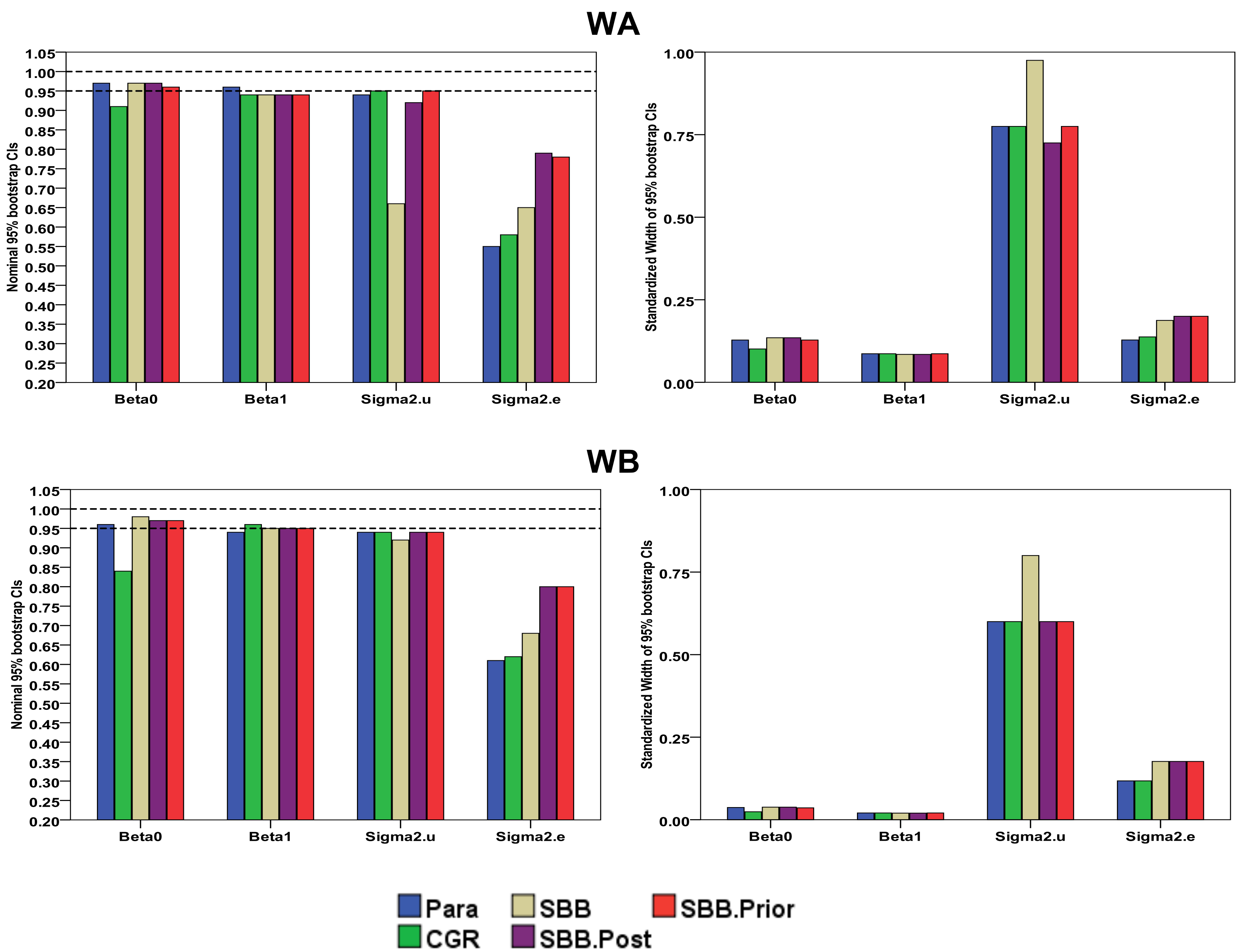
- Total number of clusters: $D = 100$
- Uniform cluster sample sizes: $n_i = 20$
- $y_{ij} = 1 + 2x_{ij} + u_i + e_{ij}, j = 1,\ldots,n_i; i = 1,..,D$
- $u_i \sim (0, \sigma_u^2)$, $\mathbf{e}_i = 0.45\mathbf{W}\mathbf{e}_i + \mathbf{z}_i$ (SAR specification)
- σ_u^2 = 0.04 (WA) & 0.005 (WB) and σ_e^2 = 0.3205 (WA) & 0.0177 (WB)

WA

0	4	3	2	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	0	4	3	2	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	4	0	4	3	2	1	0	0	0	0	0	0	0	0	0	0	0	0	0
2	3	4	0	4	3	2	1	0	0	0	0	0	0	0	0	0	0	0	0
1	2	3	4	0	4	3	2	1	0	0	0	0	0	0	0	0	0	0	0
0	1	2	3	4	0	4	3	2	1	0	0	0	0	0	0	0	0	0	0
0	0	1	2	3	4	0	4	3	2	1	0	0	0	0	0	0	0	0	0
0	0	0	1	2	3	4	0	4	3	2	1	0	0	0	0	0	0	0	0
0	0	0	0	1	2	3	4	0	4	3	2	1	0	0	0	0	0	0	0
0	0	0	0	0	1	2	3	4	0	4	3	2	1	0	0	0	0	0	0
0	0	0	0	0	0	1	2	3	4	0	4	3	2	1	0	0	0	0	0
0	0	0	0	0	0	0	1	2	3	4	0	4	3	2	1	0	0	0	0
0	0	0	0	0	0	0	0	1	2	3	4	0	4	3	2	1	0	0	0
0	0	0	0	0	0	0	0	0	1	2	3	4	0	4	3	2	1	0	0
0	0	0	0	0	0	0	0	0	0	1	2	3	4	0	4	3	2	1	0
0	0	0	0	0	0	0	0	0	0	0	1	2	3	4	0	4	3	2	1
0	0	0	0	0	0	0	0	0	0	0	0	1	2	3	4	0	4	3	2
0	0	0	0	0	0	0	0	0	0	0	0	0	1	2	3	4	0	4	3
0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	2	3	4	0	4
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	2	3	4	0

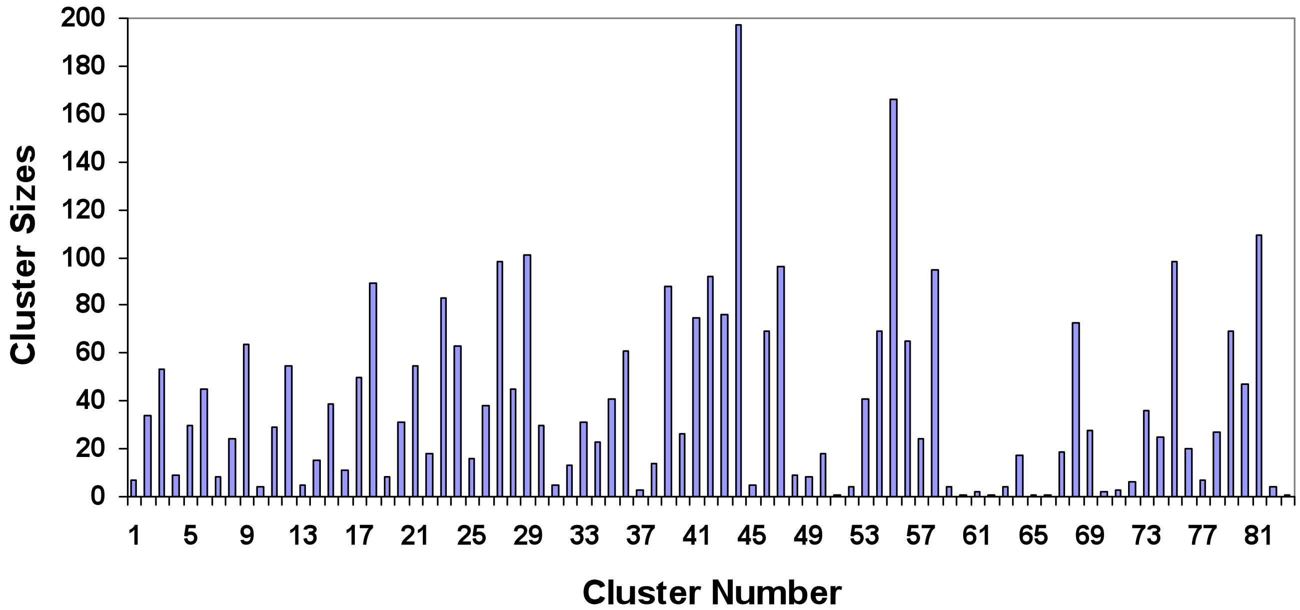
WB

0	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1
19	0	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2
18	19	0	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3
17	18	19	0	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4
16	17	18	19	0	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5
15	16	17	18	19	0	19	18	17	16	15	14	13	12	11	10	9	8	7	6
14	15	16	17	18	19	0	19	18	17	16	15	14	13	12	11	10	9	8	7
13	14	15	16	17	18	19	0	19	18	17	16	15	14	13	12	11	10	9	8
12	13	14	15	16	17	18	19	0	19	18	17	16	15	14	13	12	11	10	9
11	12	13	14	15	16	17	18	19	0	19	18	17	16	15	14	13	12	11	10
10	11	12	13	14	15	16	17	18	19	0	19	18	17	16	15	14	13	12	11
9	10	11	12	13	14	15	16	17	18	19	0	19	18	17	16	15	14	13	12
8	9	10	11	12	13	14	15	16	17	18	19	0	19	18	17	16	15	14	13
7	8	9	10	11	12	13	14	15	16	17	18	19	0	19	18	17	16	15	14
6	7	8	9	10	11	12	13	14	15	16	17	18	19	0	19	18	17	16	15
5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	0	19	18	17	16
4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	0	19	18	17
3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	0	19	18
2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	0	19
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	0


WA
Nominal 95% bootstrap CIs
Standardized Width of 95% bootstrap CIs
1.05
1.00
0.95
0.90
0.85
0.80
0.75
0.70
0.65
0.60
0.55
0.50
0.45
0.40
0.35
0.30
0.25
0.20
0.00
Beta0
Beta1
Sigma2.u
Sigma2.e
WB
Para
CGR
SBB
SBB.Post
SBB.Prior

Atlant (Australian Rain Technology) Data
(Beare *et al*. 2010)

Response Variable : Daily rainfall or log (Daily rainfall)
Explanatory Variables : 37
Cluster : 4 Day Downwind Cluster
No of Clusters : 83
Total sample size : 3177 **(Min=1, Max=197, Average=38)**

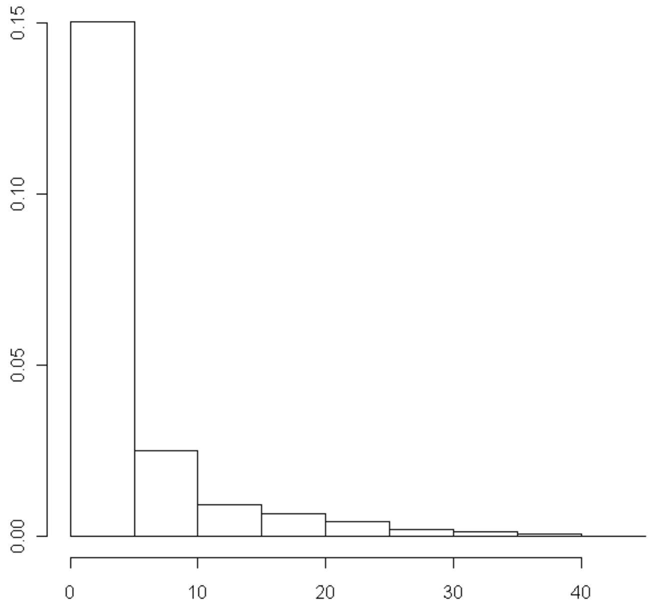


Explanatory Variables

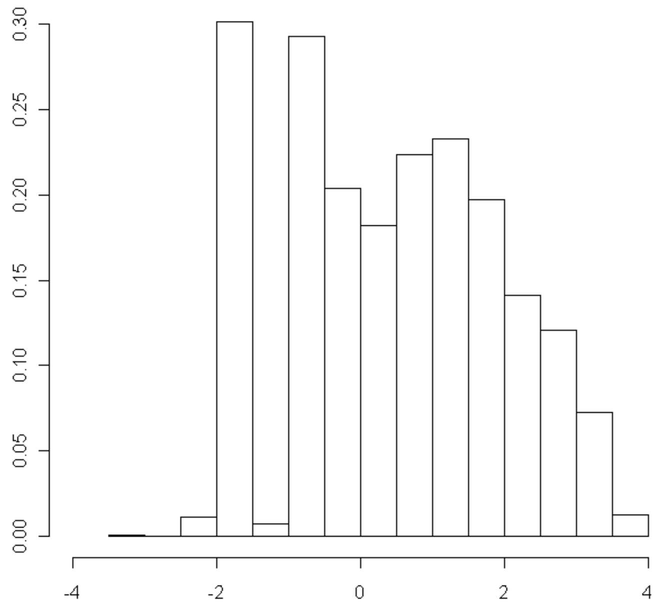
No.	Variable	No.	Variable
1	Intercept	20	Elevation (100m)
2	August/September	21	Distance from C2
3	WRE	22	C2 Theta
4	Upwind Rain 0.1mm and over	23	Lagged C2 Theta
5	Wind Speed 700	24	C2 Theta * C2 Distance
6	Lagged Wind Speed 700	25	Lag C2 Theta * C2 Distance
7	Wind Speed 850	26	Distance from C3
8	Lagged Wind Speed 850	27	C3 Theta
9	Wind Speed 925	28	Lagged C3 Theta
10	Lagged Wind Speed 925	29	C3 Theta * C3 Distance
11	SWD 700	30	Lag C3 Theta * C3 Distance
12	SLWD 700	31	**C2 On 1**
13	SWD 850	32	**C2 On 2**
14	SLWD 850	33	**C2 On 1 * C2 Distance**
15	SWD 925	34	**C2 On 2 * C2 Distance**
16	SLWD 925	35	**C3 On 1**
17	Air Temp	36	**C3 On 2**
18	Dew Point Difference	37	**C3 On 1 * C3 Distance**
19	Sea Level Pressure	38	**C3 On 2 * C3 Distance**

Response Variable

Daily Rainfall



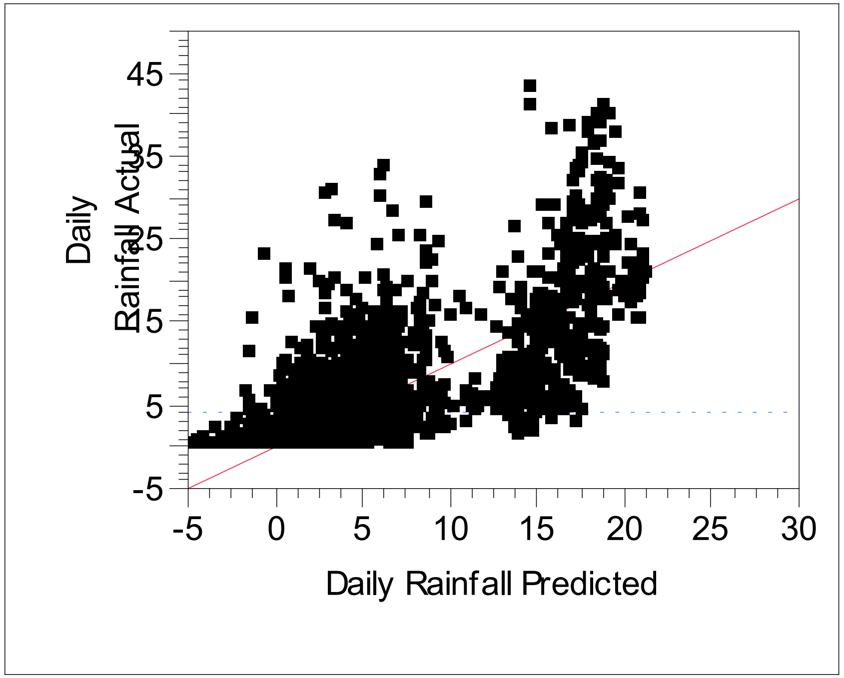
log (Daily Rainfall)



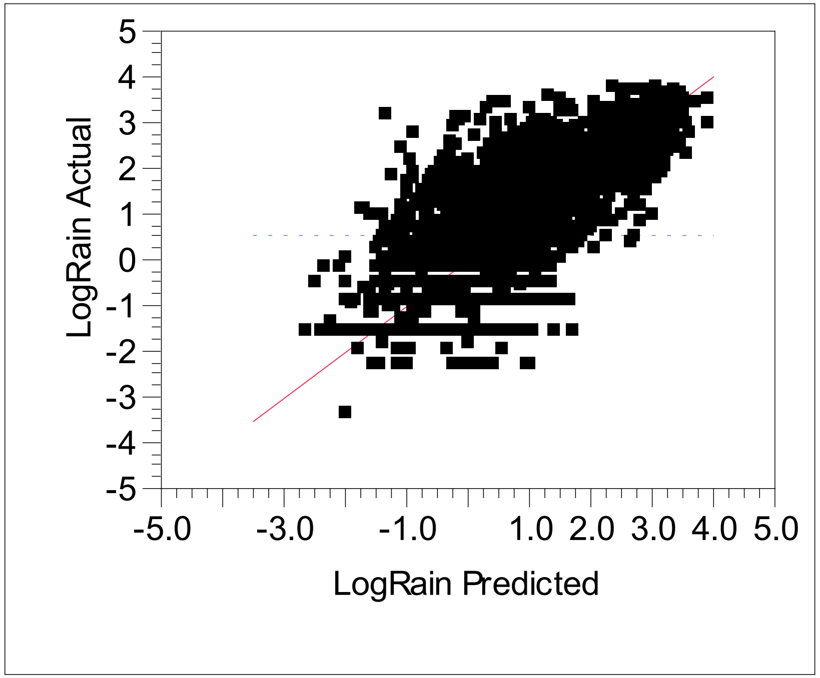
	Mean	Std Dev	Median
Rain	4.29	6.39	1.67
LogRain	0.51	1.44	0.51

Model Fit: Actual by Predicted Plot

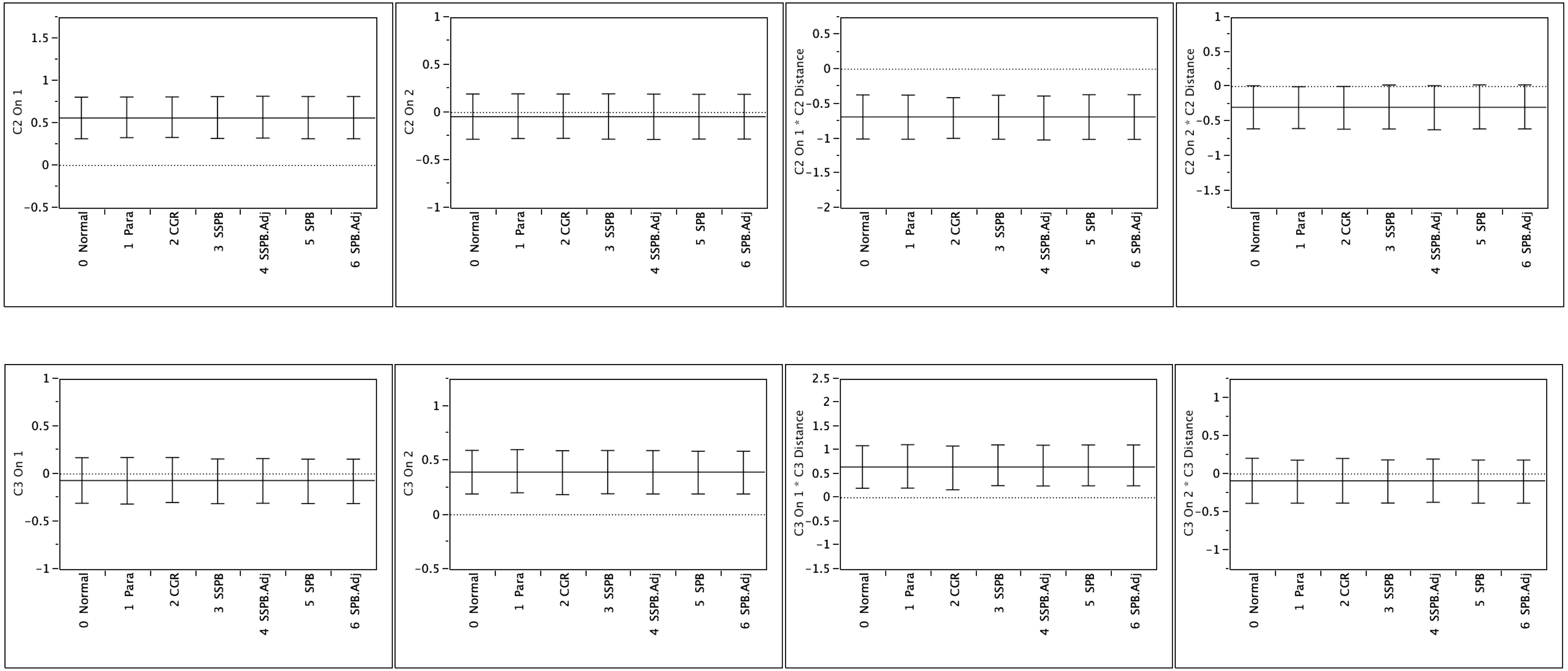
Daily Rainfall



log (Daily Rainfall)



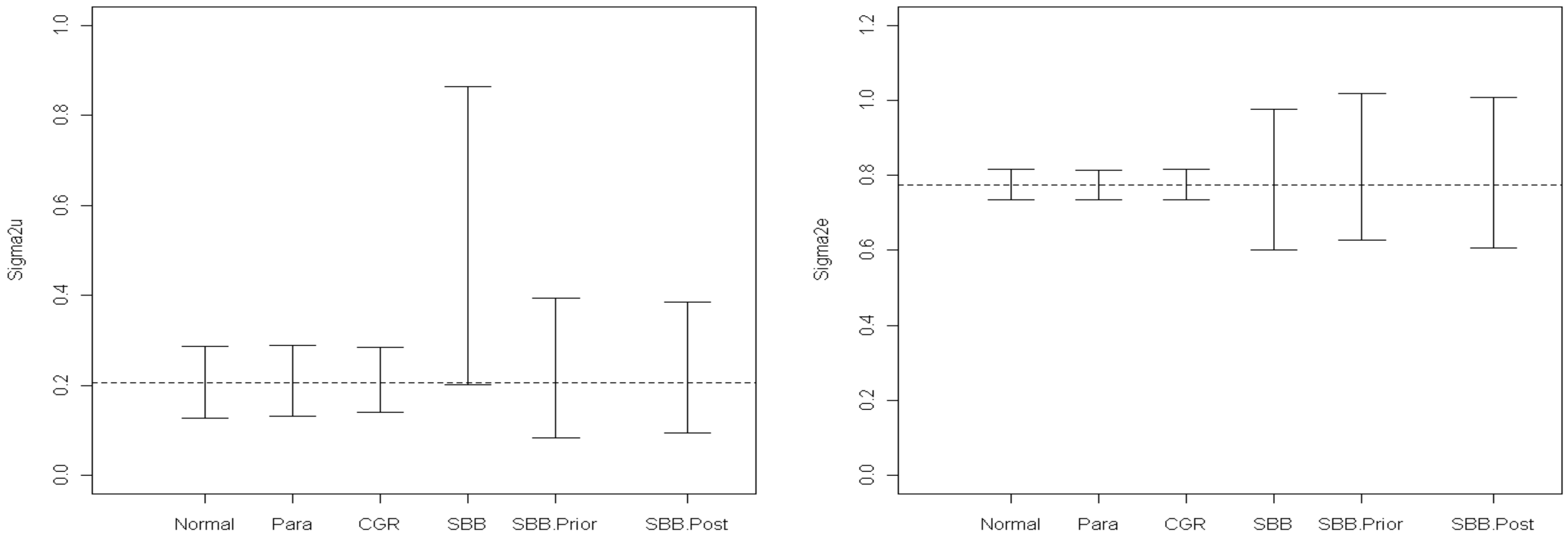
Log (Daily Rainfall): 95% CIs for 8 Effect Variables



- If the bootstrap confidence interval **fails** to include 0, then the p-value is deemed to be less than or equal to 0.05, and the effect is **significant**
- Bootstrap does not change conclusions about fixed effects parameters

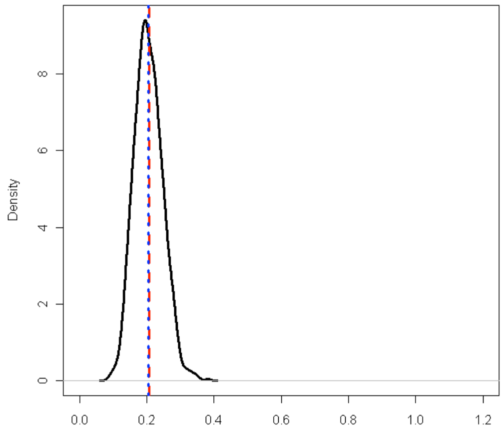
Log (Daily Rainfall): 95% CIs for Variance Components

$\hat{\sigma}_u^2 = 0.206$ $\hat{\sigma}_e^2 = 0.775$

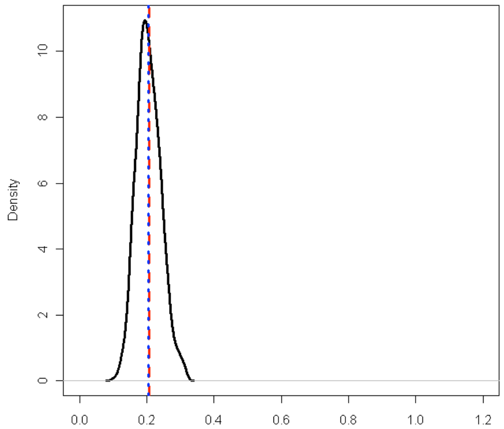


- Semiparametric block bootstrap results seem more believable ....

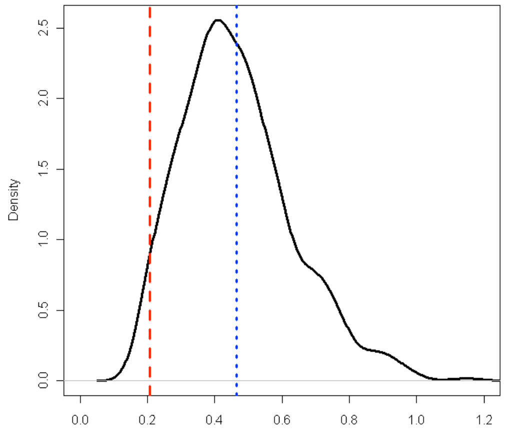
Red : $\hat{\sigma}_u^2 = 0.206$ Blue : $\bar{\hat{\sigma}}_u^{2*} = B^{-1}\sum_{b=1}^{B}\hat{\sigma}_u^{2*(b)}$



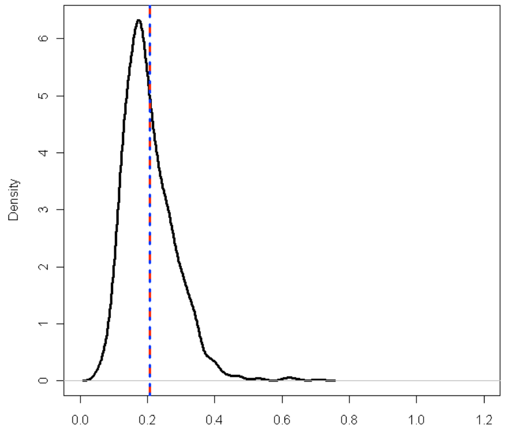
Para



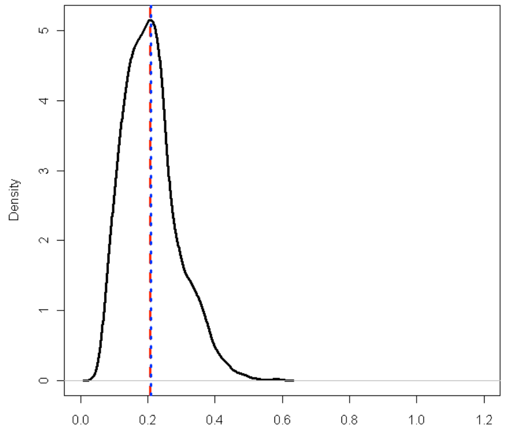
CGR



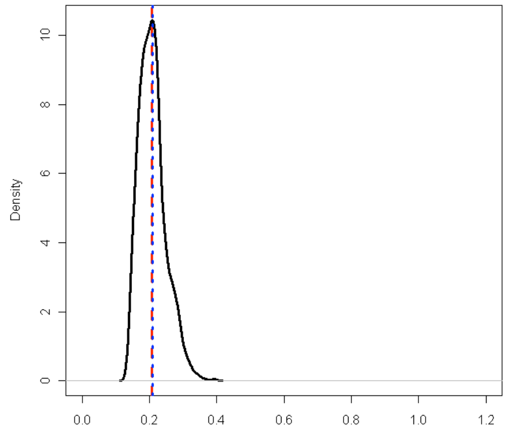
SBB



SBB.Post

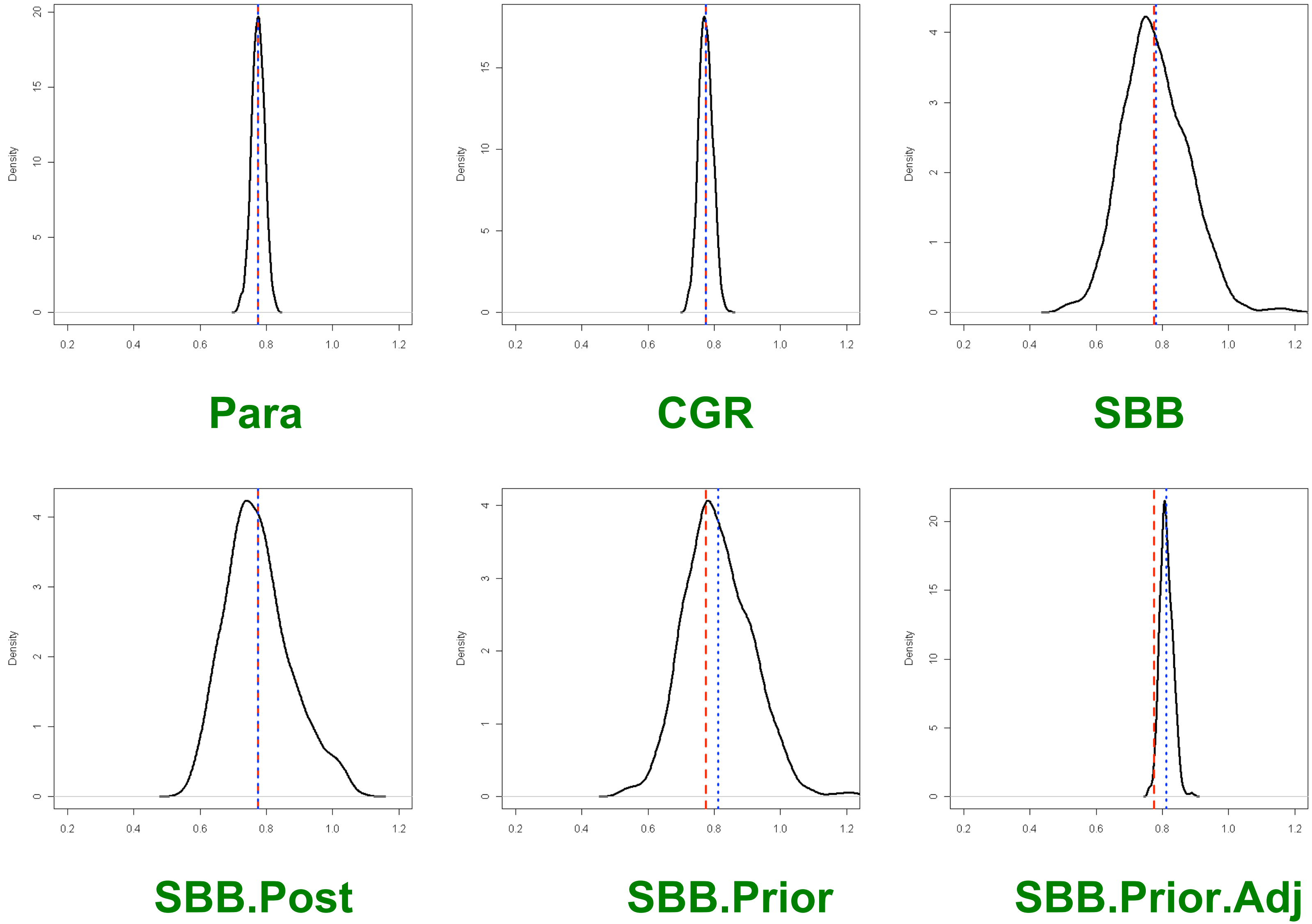


SBB.Prior

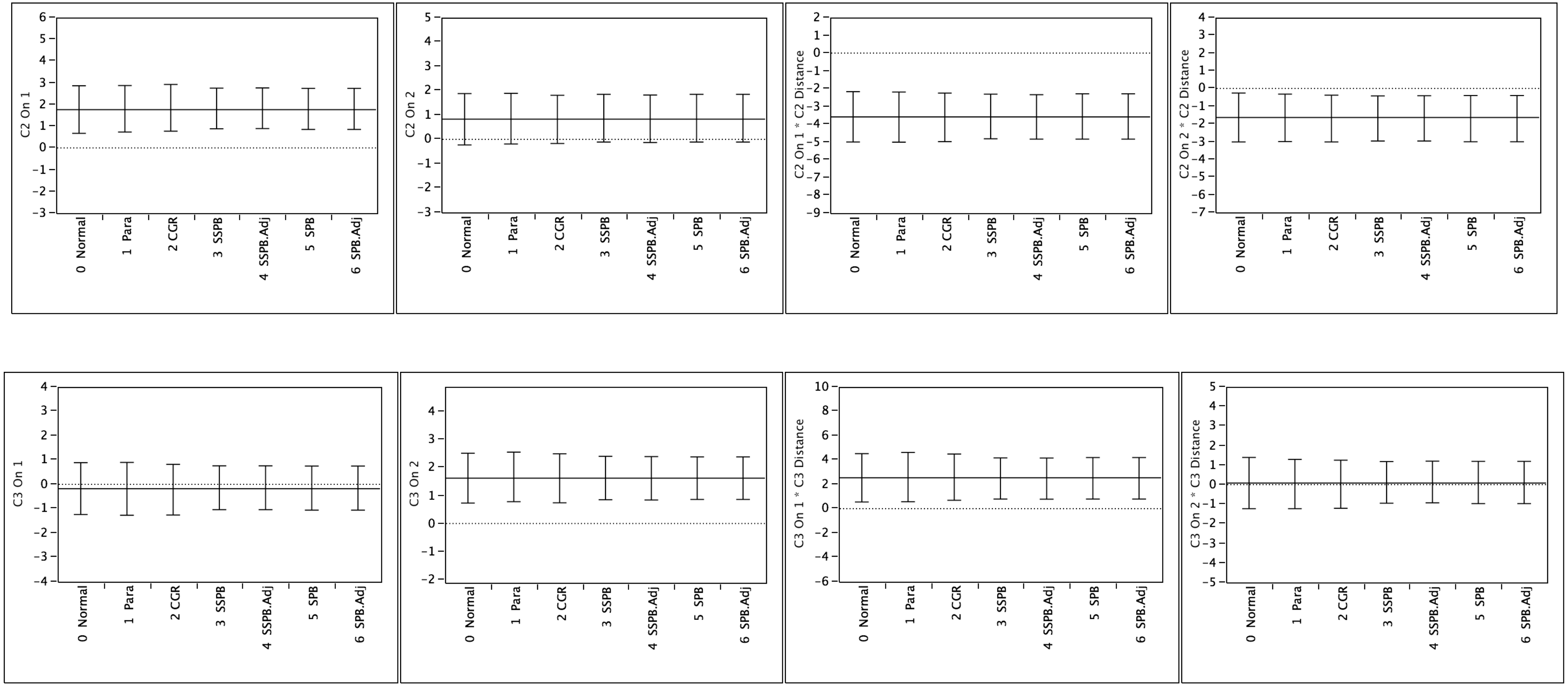


SBB.Prior.Adj

Red: $\hat{\sigma}_e^2 = 0.775$ Blue: $\bar{\hat{\sigma}}_e^{2*} = B^{-1}\sum_{b=1}^{B}\hat{\sigma}_e^{2*(b)}$



Daily Rainfall: 95% CIs for 8 Effect Variables

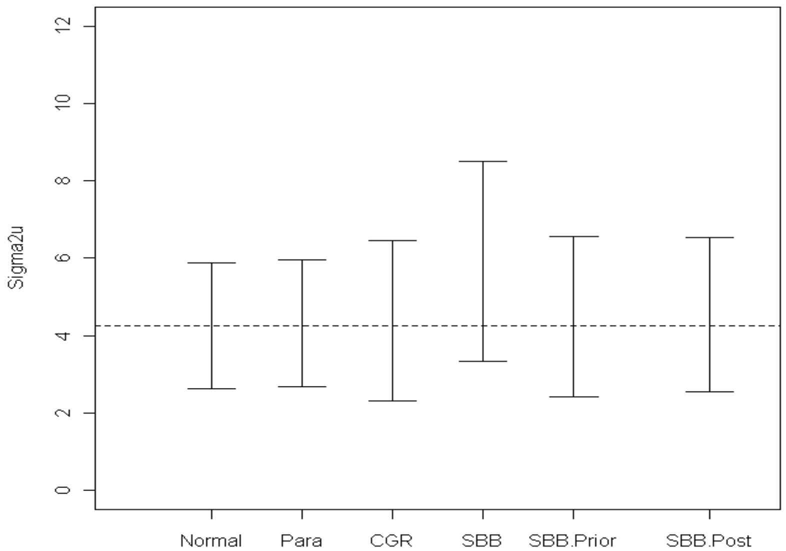


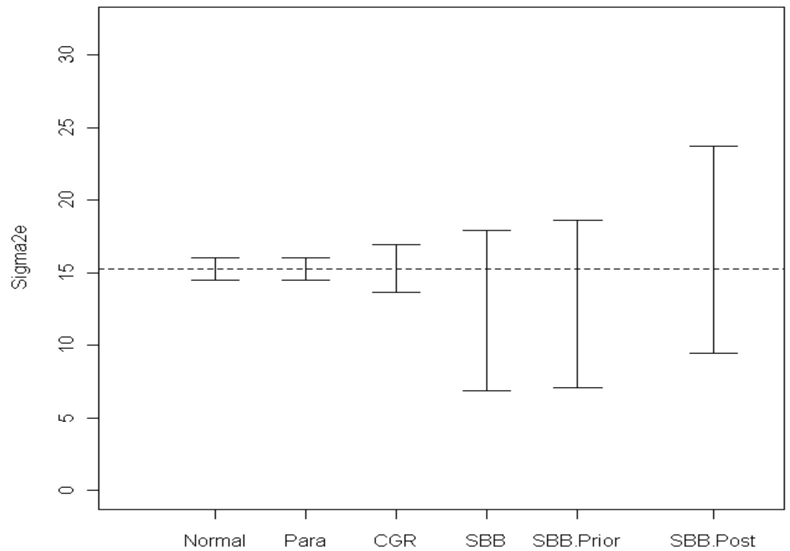
- Again, bootstrap does not change conclusions about fixed effects parameters

Daily Rainfall: 95% CIs for Variance Components

$\hat{\sigma}_u^2 = 4.246$

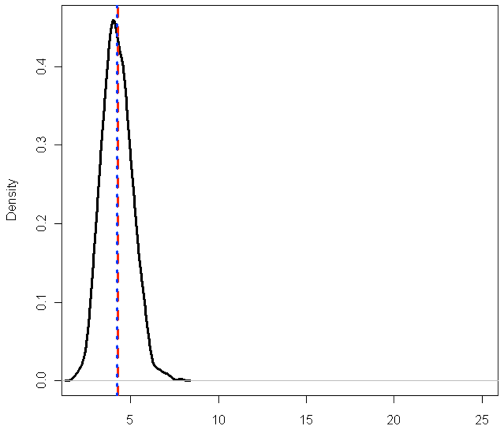
$\hat{\sigma}_e^2 = 15.269$



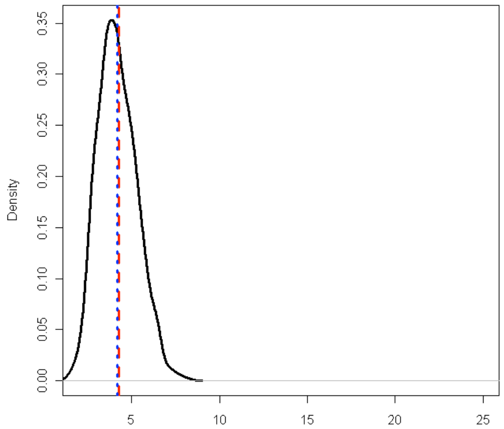


- Normal, Para and CGR CIs for σ_e^2 seem overly optimistic ....

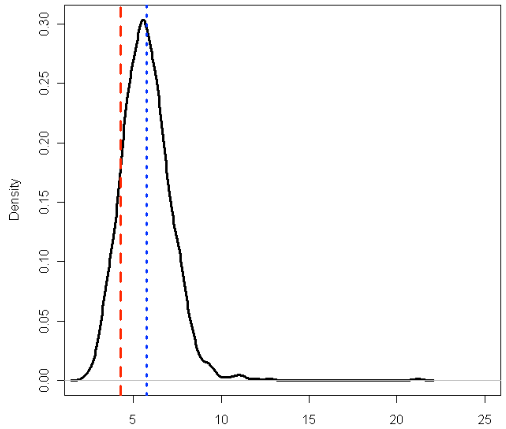
$$\text{Red}: \hat{\sigma}_u^2 = 4.246 \qquad \text{Blue}: \bar{\hat{\sigma}}_u^{2*} = B^{-1}\sum_{b=1}^{B}\hat{\sigma}_u^{2*(b)}$$



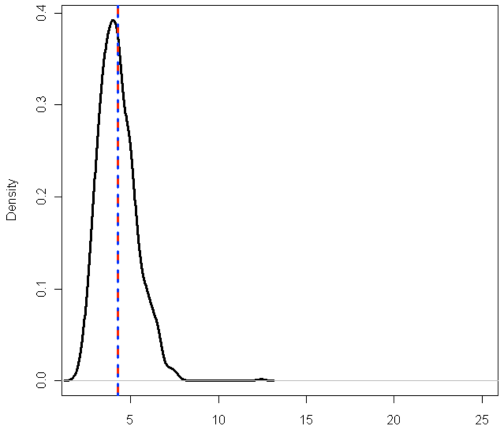
Para



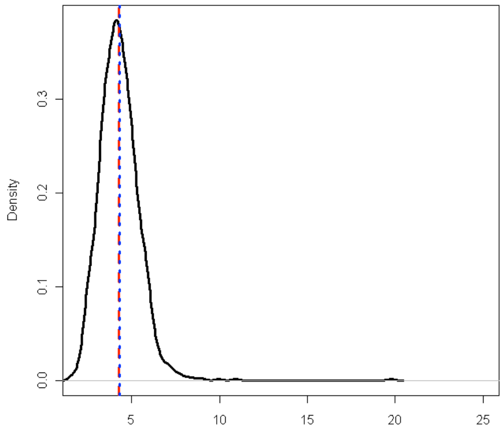
CGR



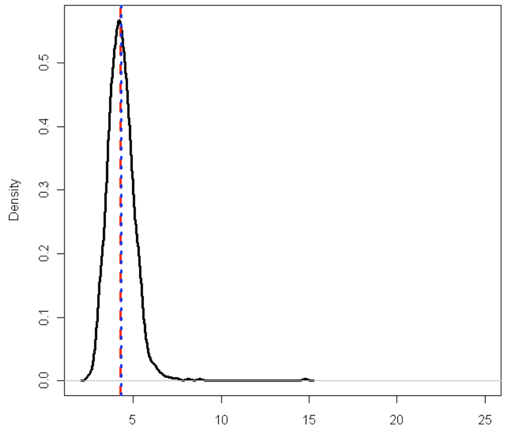
SBB



SBB.Post

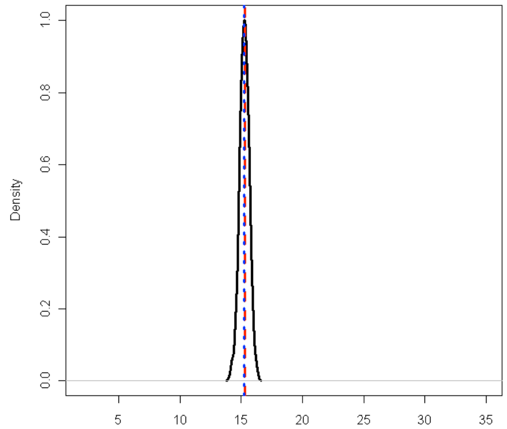


SBB.Prior

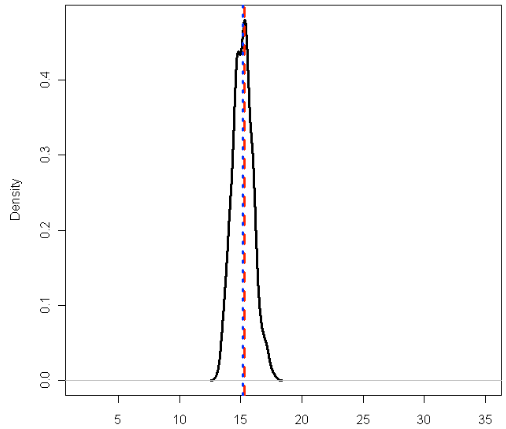


SBB.Prior.Adj

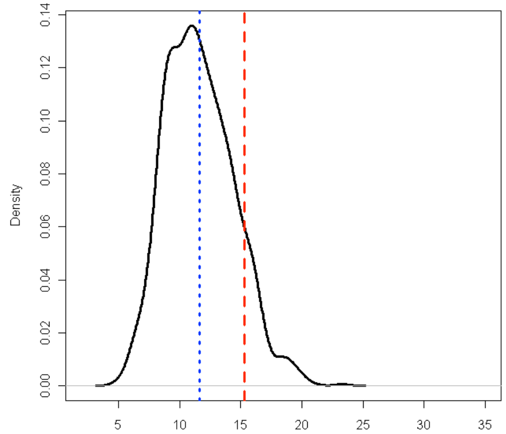
Red: $\hat{\sigma}_e^2 = 15.269$ Blue: $\bar{\hat{\sigma}}_e^{2*} = B^{-1}\sum_{b=1}^{B} \hat{\sigma}_e^{2*(b)}$



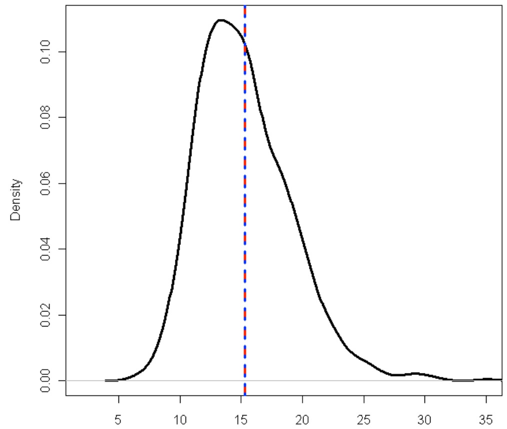
Para



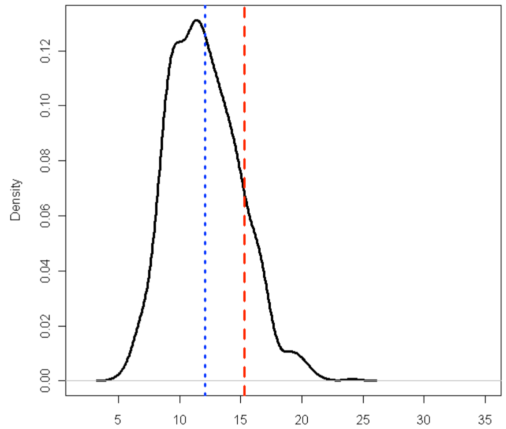
CGR



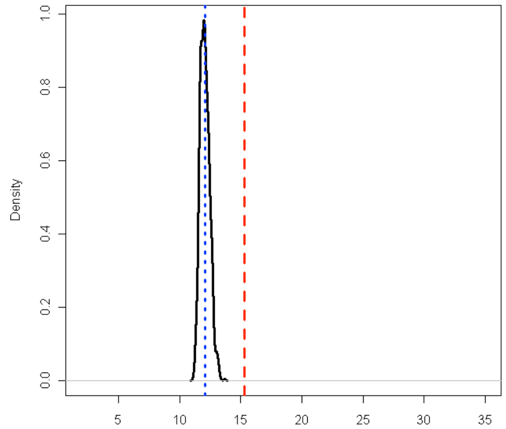
SBB



SBB.Post



SBB.Prior



SBB.Prior.Adj

Concluding Remarks

- The semiparametric block bootstrap methods are simple to implement and are free of both the distribution and the dependence assumptions of the parametric bootstrap

- Their main assumption is that the marginal model is correct

- They lead to **consistent** estimators of confidence intervals (theoretical results are not shown here)

- **A robust alternative**: Protection against within group heterogeneity

- Empirical evaluations confirm these conclusions

- Satisfactory performance when applied to real data

References

1. Beare, S., Chambers, R., Peak, S. and Ring, J.M. (2010). Accounting for Spatiotemporal Variation of Rainfall Measurements when Evaluating Ground-Based Methods of Weather Modification. Working Papers Series 17-10, Centre for Statistical and Survey Methodology, The University of Wollongong, Australia. Available from: http://cssm.uow.edu.au/publications

2. Carpenter, J.R., Goldstein, H., Rasbash, J. (2003). A Novel Bootstrap Procedure for Assessing the Relationship between Class Size and Achievement. *Journal of the Royal Statistical Society*. Series C (Applied Statistics), **52 (4)**, pp. 431-443.

3. Efron, B. (1979). Bootstrap Methods: Another Look at the Jackknife. The Annals of Statistics, **7 (1)**, 1 – 26.

4. Efron, B. and Tibshirani, R.J. (1993): *An introduction to the bootstrap*. Chapman & Hall.

5. Rasbash, J., Browne, W, Goldstein, H., Yang, M., Plewis, I., Healy, M., Woodhouse, G., Draper, D., Langford, I. and Lewis, T. (2000) A User's Guide to MLwiN. London: Institute of Education.