Two-stage Benchmarking of Time-Series Models for Small Area Estimation

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Small Area Conference, Trier, 2011

What is benchmarking?

Y_{dt} - target characteristic in area d at time t, $\begin{matrix} d = 1,2,\ldots,D & \text{Areas} \\ t = 1,2,\ldots & \text{Time} \end{matrix}$,

y_{dt} - direct survey estimate,

$\hat{Y}_{dt}^{model}$ - estimate obtained under a model.

Benchmarking: modify model based estimates to satisfy:

$\sum_{d=1}^{D} b_{dt} \hat{Y}_{dt}^{\text{model}} = B_t$; $t = 1,2,\ldots$ (B_t **known**, **e.g.,** $B_t = \sum_{d=1}^{D} b_{dt} y_{dt}$).

b_{dt} **fixed** coefficients (relative size, scale factors,**…**).

Condition: B_t sufficiently close to true value $\sum_{d=1}^{D} b_{dt} Y_{dt}$.

- $\hat{Y}_{dt}^{model}$ not necessarily a linear estimator.

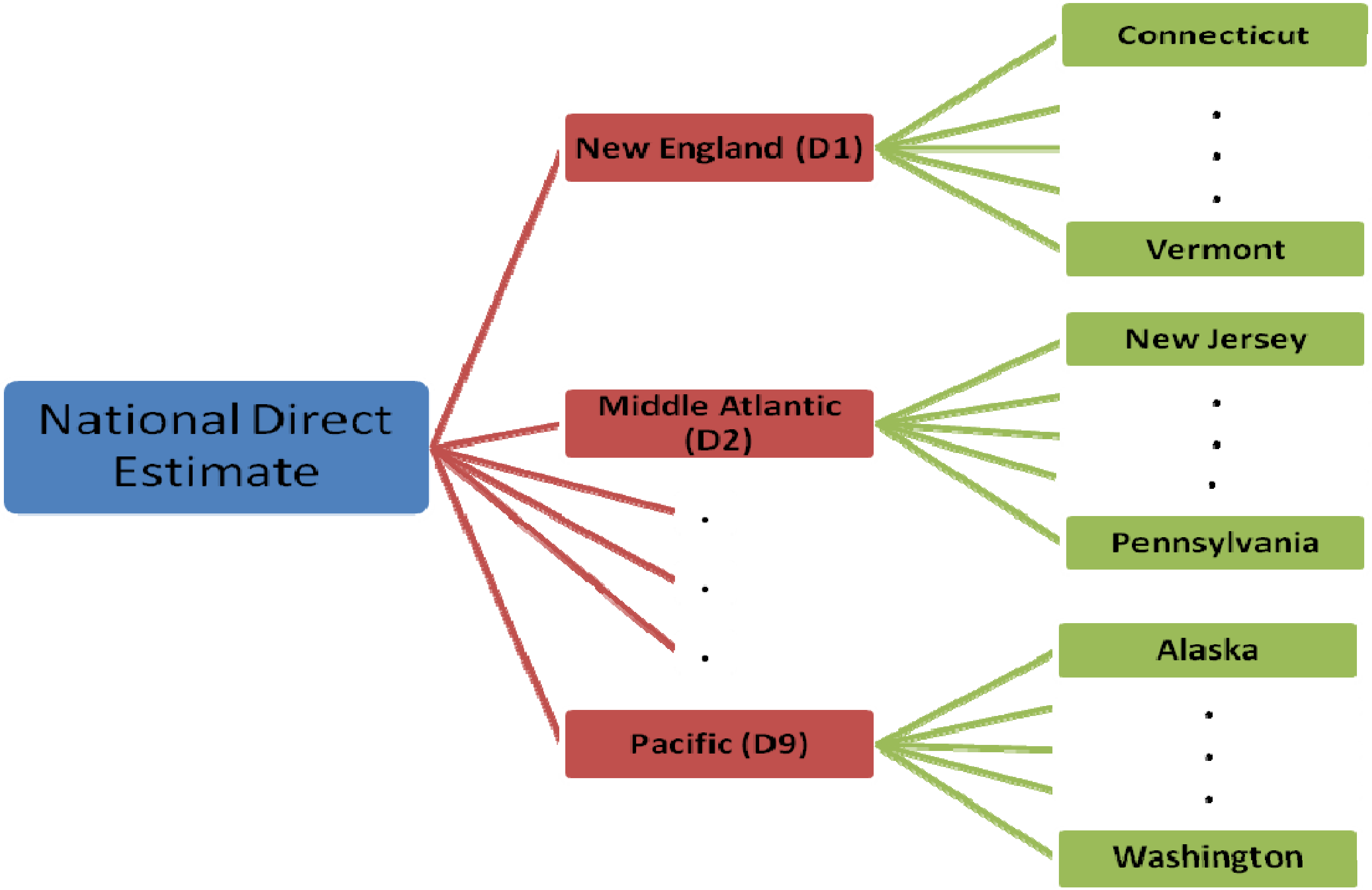
Problem considered in present presentation

Develop a **two-stage** benchmarking procedure for hierarchical time series models fitted to survey estimates.

First stage: benchmark concurrent model-based estimators at **higher level** of hierarchy to **reliable aggregate** of corresponding survey estimates.

Second stage: benchmark concurrent model-based estimates at **lower level** of hierarchy to **first stage benchmarked estimate** of higher level to which they belong.

Example: Labour Force estimates in the U.S.A



Why benchmark?

1- Time series models reflect **historical** behavior of the series. Slow in adapting to changes $\Rightarrow$ benchmarking provides some **protection** against **abrupt changes** affecting the areas in a given hierarchy.

2- The published benchmarked estimates at each level sum up to the published estimate at the higher level. Required by official statistical bureaus.

3- Another way of **'borrowing strength'** across areas.

Why not benchmark second level areas in one step?

1- May not be feasible in a real time production system**:** For **U.S.A.-CPS** our proposed procedure requires joint modeling of all the areas that need to be benchmarked, $\Rightarrow$ state-space model of order **700**.

2- Delay in processing data for one second level area could hold up all the area estimates.

3- When **1**st – level hierarchy composed of homogeneous **2**nd level areas, benchmarking more effectively tailored to **1**st – level characteristics.

Apply cross-sectional benchmarking at every time t?

Pro-rata (ratio) benchmarking,

$$\hat{Y}_{d,\boldsymbol{R}}^{bmk} = \hat{Y}_d^{\text{model}} \times \mathrm{B} / \sum_{k=1}^{D} b_k \hat{Y}_k^{\text{model}});\quad B = \sum_{d=1}^{D} b_d y_d \,.$$

Limitations:

1- Adjusts all the small area model-based estimates exactly the same way, irrespective of their precision,

2- Benchmarked estimates **not consistent:** if sample size in area $\boldsymbol{d}$ increases but sample sizes in other areas unchanged, $\hat{Y}_{d,\boldsymbol{R}}^{bmk}$ does not converge to true population value $\boldsymbol{Y_d}$.

Limitations of independent pro-rata benchmarking (cont.)

3- Does not lend itself to simple variance estimation.

4- If applied independently at every time point $\Rightarrow$ ignores inherent time series relationships between the benchmarks $B_t = \sum_{d=1}^{D} b_{dt} y_{dt}$ $\Rightarrow$ may add extra **roughness** to benchmarked estimates and the corresponding estimated trend.

- Possibly similar problem with all cross-sectional benchmarking procedures when applied to a time series.

Additive cross-sectional benchmarking

$$\hat{Y}_{d,A}^{bmk} = \hat{Y}_d^{\text{model}} + a_d\left(\sum_{k=1}^{D} b_k y_k - \sum_{k=1}^{D} b_k \hat{Y}_k^{\text{model}}\right); \ \sum_{d=1}^{D} b_d a_d = 1.$$

Coefficients $\{a_d\}$ measure **precision** (next slide); distribute **difference** between benchmark and aggregate of model-based estimates between the areas.

- If $a_d \underset{n_d \to \infty}{\to} 0 \Rightarrow \hat{Y}_{d,A}^{bmk} \to \hat{Y}_d^{\text{model}} \to Y_d \Rightarrow$ **consistent**.

Bad news? $\underset{n_d \to \infty}{\text{Plim}}(\hat{Y}_{d,A}^{bmk} - y_d) = 0 \Rightarrow$ Area **d** accurate estimate not contributing to benchmarking in other areas.

- '**Easy**' to estimate variance of $\hat{Y}_{d,A}^{bmk}$.

Examples of additive cross-sectional benchmarking

Wang *et al*. (2008) minimize $\sum_{d=1}^{D} \varphi_d E(Y_d - \hat{Y}_{d,A}^{bmk})^2$ under **F-H**

s.t. $\sum_{d=1}^{D} b_d y_d = \sum_{d=1}^{D} b_d \hat{Y}_{d,A}^{bmk}$. **Sol**: $\tilde{a}_d = \varphi_d^{-1} b_d / \sum_{k=1}^{D} \varphi_k^{-1} b_k^2$.

$\{\varphi_d\}$ represent precision of direct or model-based estimators.

$\varphi_d = [Var(\hat{Y}_d^{\text{model}})]^{-1} \rightarrow$ **Battese *et al*. 1988**.

$\varphi_d = b_d [\text{cov}(\hat{Y}_d^{\text{model}}, \sum_{k=1}^{D} \hat{Y}_k^{\text{model}})]^{-1} \rightarrow$ **Pfeffermann & Barnard 1991**.

$\varphi_d = [Var(y_d)]^{-1} \rightarrow$ **Isaki *et al*. 2000**.

- In practice, model parameters replaced by estimates.

Examples of additive cross-sectional benchmark. (cont.)

Datta *et al*. (2011) minimize $\sum_{d=1}^{D} \varphi_d E[(Y_d - \hat{Y}_{d,A}^{bmk})^2 \mid data]$ and obtain solution of ***Wang et al***., with $\hat{Y}_d^{\text{model}} = E(Y_d \mid data)$.

- Solution general - not restricted to particular model.

You and Rao (2002) propose "self benchmarked" estimators for unit-level model by modifying the estimator of β. Approach applied by **Wang *et al*. (2008)** to area-lave model.

Ugarte *et al*. (2009) benchmark the **BLUP** under unit-level model to **synthetic estimator** for all areas under regression model with heterogeneous variances.

First-stage time series benchmarking

Pfeffermann & Tiller (2006) consider the following model for unemployment census division series obtained from **CPS**.

Let $Y_t = (Y_{1t},...,Y_{Dt})'$ = true **division totals**, $y_t = (y_{1t},\ldots,y_{Dt})'$= **direct estimates**, $e_t = (e_{1t},\ldots,e_{Dt})'$= **sampling errors**.

$$y_t = Y_t + e_t; E(e_t) = 0,\ E\left(e_\tau e_t'\right) = \boldsymbol{\Sigma}_{\tau t} = \mathbf{Diag}[\sigma^2_{1,\tau,t},\ldots,\sigma^2_{D,\tau,t}].$$

- Division sampling errors independent between divisions but **highly** auto-correlated within a division and heteroscedastic. (**4 in**, **8 out**, **4 in** rotation pattern)

Time series model for division d

Totals Y_{dt} assumed to evolve independently between divisions according to basic structural model (**BSM, Harvey 1989**). Model accounts for **stochastic trend**, **stochastically varying seasonal effects** and **random irregular terms**.

Model written: $Y_{dt} = z'_{dt}\alpha_{dt}$; $\alpha_{dt} = T_d \alpha_{d,t-1} + \eta_{dt}$. (**state-space**)

Errors η_{dt} mutually independent white noise, $E(\eta_{dt}\eta'_{dt}) = Q_d$.

- **ARIMA**, **regression with random coefficients** and **unit & area level** models can all be expressed in **state-space** form.

Combining the separate division models

$$y_t = Y_t + e_t = Z_t\alpha_t + e_t$$ (**measurement eq.**) **;** $y_t = (y_{1t},...,y_{Dt})'$,

$$\alpha_t = \tilde{T}\alpha_{t-1} + \eta_t$$ (**state eq.**) **;** $\alpha_t = (\alpha'_{1t},...,\alpha'_{Dt})'$,

$Z_t = \mathrm{I}_D \otimes z'_{dt}$, $\tilde{T} = \mathrm{I}_D \otimes T_d$ **;** $\otimes$- block diagonal

$$E(\eta_t) = 0,\ E(\eta_t\eta'_t) = Q = \mathrm{I}_D \otimes Q_d,\ E(\eta_\tau\eta'_t) = 0,\ \tau \neq t.$$

Benchmark constraints:

$$\sum_{d=1}^{D} b_{dt}y_{dt} = \sum_{d=1}^{D} b_{dt}z'_{dt}\alpha_{dt} \overset{\text{MODEL}}{=} \sum_{d=1}^{D} b_{dt}Y_{dt},\ t = 1,2,...$$

But in truth, $\sum_{d=1}^{D} b_{dt}y_{dt} = \sum_{d=1}^{D} b_{dt}z'_{dt}\alpha_{dt} + \sum_{d=1}^{D} b_{dt}e_{dt}$.

Adding benchmark equations to model

Add $\sum_{d=1}^{D} b_{dt} y_{dt} = \sum_{d=1}^{D} b_{dt} z'_{dt} \alpha_{dt} + \sum_{d=1}^{D} b_{dt} e_{dt}$ to **measurement eq.**

$$\tilde{y}_t = \tilde{Z}_t \alpha_t + \tilde{e}_t \;\; ; \quad \tilde{y}_t = (y'_{t,} \sum_{d=1}^{D} b_{dt} y_{dt})' ,$$

$$\tilde{Z}_t = \begin{bmatrix} Z_t \\ b_{1t} z'_{1t}, \ldots, b_{Dt} z'_{Dt} \end{bmatrix}, \; \tilde{e}_t = \left(e'_{t,} \sum_{d=1}^{D} b_{dt} e_{dt} \right)' .$$

- State equations $\alpha_t = \tilde{T} \alpha_{t-1} + \eta_t$ **unchanged**.

Set up random coefficients regression model

$$\begin{pmatrix} \tilde{T}\tilde{\alpha}_{t-1}^{bmk} \\ \tilde{\boldsymbol{y}}_{\boldsymbol{t}} \end{pmatrix} = \begin{pmatrix} I \\ \tilde{\boldsymbol{Z}}_{\boldsymbol{t}} \end{pmatrix} \alpha_t + \begin{pmatrix} u_{t|t-1}^{bmk} \\ \tilde{e}_t \end{pmatrix}, \; u_{t|t-1}^{bmk} = \tilde{T}\tilde{\alpha}_{t-1}^{bmk} - \alpha_t;$$

$$Var\begin{pmatrix} u_{t|t-1}^{bmk} \\ \tilde{e}_t \end{pmatrix} = \begin{bmatrix} P_{t|t-1}^{bmk} & \boldsymbol{C}_{\boldsymbol{t}}^{\boldsymbol{bmk}} \\ \boldsymbol{C}_{\boldsymbol{t}}^{\boldsymbol{bmk}\prime} & \tilde{\boldsymbol{\Sigma}}_{\boldsymbol{tt}} \end{bmatrix} = \tilde{\boldsymbol{V}}_{\boldsymbol{t}}. \quad \tilde{\Sigma}_{tt} = E(\tilde{e}_t \tilde{e}_t') = \begin{bmatrix} \Sigma_{tt} & h_{tt} \\ h_{tt}' & v_{tt} \end{bmatrix};$$

$$h_{tt} = Cov(e_t, \sum_{d=1}^{D} b_{dt} e_{dt}) \; ; \; v_{tt} = Var(\sum_{d=1}^{D} b_{dt} e_{dt}).$$

$C_t^{bmk} = E(u_{t|t-1}^{bmk} \tilde{e}_t') = \sum_{\tau=1}^{t-1} D_\tau \boldsymbol{\Sigma}_{\boldsymbol{\tau t}} \rightarrow$ linear combination of covariance matrices of sampling errors.

Imposing benchmark constraints

Impose, $\sum_{d=1}^{D} b_{dt} y_{dt} = \sum_{d=1}^{D} b_{dt} z'_{dt} \alpha_{dt} \;\Leftrightarrow\; \sum_{d=1}^{D} b_{dt} e_{dt} = 0$ when estimating the state vector under **RCR** model. **Define**,

$$\tilde{e}_{t,0} = (e'_t, 0)', \; \tilde{\Sigma}_{tt,0} = E(\tilde{e}_{t,0}\tilde{e}'_{t,0}), \; C^{bmk}_{t,0} = E(u^{bmk}_{t|t-1}\tilde{e}'_{t,0}), \; \tilde{V}_{t,0} = \begin{bmatrix} P^{bmk}_{t|t-1} & C^{bmk}_{t,0} \\ C^{bmk\prime}_{t,0} & \tilde{\Sigma}_{tt,0} \end{bmatrix}$$

$$\tilde{\alpha}^{bmk}_{t} = \left[(\mathrm{I}, \tilde{Z}'_t)\tilde{V}^{-1}_{t,0} \begin{pmatrix} \mathrm{I} \\ \tilde{Z}_t \end{pmatrix} \right]^{-1} (\mathrm{I}, \tilde{Z}'_t)\tilde{V}^{-1}_{t,0} \begin{pmatrix} \tilde{T}\tilde{\alpha}^{bmk}_{t-1} \\ \tilde{y}_t \end{pmatrix} \rightarrow$$ **'standard' GLS**.

Benchmarked predictor for division d: $\hat{Y}^{bmk}_{dt} = z'_{dt}\tilde{\alpha}^{bmk}_{dt}$.

Variance of benchmarked estimator

Setting $\sum_{d=1}^{D} b_{dt} y_{dt} = \sum_{d=1}^{D} b_{dt} z'_{dt} \alpha_{dt} \Leftrightarrow \sum_{d=1}^{D} b_{dt} e_{dt} = 0$ only for computing benchmarked predictor but **not** when computing $Var(\tilde{\alpha}_t^{bmk} - \alpha_t)$.

- $Var(\hat{Y}_{dt}^{bmk} - Y_{dt})$ accounts for variances and auto-covariances of division sampling errors, variances and auto-covariances of benchmark errors, $\sum_{d=1}^{D} b_{dt} e_{dt} = \sum_{d=1}^{D} b_{dt} y_{dt} - \sum_{d=1}^{D} b_{dt} z'_{dt} \alpha_{dt}$, and their covariances with division sampling errors, and variances of model components.

Alternative expression for benchmarked predictor

Denote by $\hat{\alpha}_{t,u}$ the state predictor **without imposing** the constraint $\sum_{d=1}^{D} b_{dt} e_{dt} = 0$ at time t (but **imposing constraints** in **previous** time points).

Define, $\Lambda_{ft} = Var[\sum_{d=1}^{D} b_{dt} z'_{dt} (\hat{\alpha}_{dt,u} - \alpha_{dt})]$;

$\delta_{dft} = Cov[(\hat{\alpha}_{dt,u} - \alpha_{dt}), \sum_{d=1}^{D} b_{dt} z'_{dt} (\hat{\alpha}_{dt,u} - \alpha_{dt})]$.

The benchmarked predictor of total in division d is,

$$\hat{Y}_{dt}^{bmk} = z'_{dt} \hat{\alpha}_{dt,u} + z'_{dt} \delta_{dft} \Lambda_{ft}^{-1} (\sum_{d=1}^{D} b_{dt} y_{dt} - \sum_{d=1}^{D} b_{dt} z'_{dt} \hat{\alpha}_{dt,u})$$

- $\sum_{d=1}^{D} b_{dt} z'_{dt} \delta_{dft} \Lambda_{ft}^{-1} = 1$.

Properties of division benchmarked predictor

$$\hat{Y}_{dt}^{bmk} = z'_{dt}\hat{\alpha}_{dt,u} + z'_{dt}\delta_{dft}\Lambda_{ft}^{-1}(\sum_{d=1}^{D} b_{dt}y_{dt} - \sum_{d=1}^{D} b_{dt}z'_{dt}\hat{\alpha}_{dt,u}).$$

$\hat{Y}_{dt}^{bmk}$ member of **cross-sectional benchmarked predictors**,

$\hat{Y}_{d,A}^{bmk} = \hat{Y}_{d}^{\text{model}} + \tilde{a}_d(\sum_{k=1}^{D} b_k y_k - \sum_{k=1}^{D} b_k \hat{Y}_{k}^{\text{model}})$ **(Wang *et al*. 2008)**

$\tilde{a}_d = \varphi_d^{-1} b_d / \sum_{k=1}^{D} \varphi_k^{-1} b_k^2$. In present case,

$\hat{Y}_{dt}^{model} = z'_{dt}\hat{\alpha}_{dt,u} \rightarrow$ **un-benchmarked** predictor at time ***t*;**

$\varphi_{dt} = b_{dt} / z'_{dt}\delta_{dft} = [cov(\hat{Y}_{dt}^{model}, \sum_{k=1}^{D} b_{kt}\hat{Y}_{kt}^{model})]^{-1} \rightarrow$ **Pfeffermann & Barnard (1991).**

Properties of division benchmarked predictor (cont.)

(a)- unbiasedness: if $E(\tilde{\alpha}_{t-1}^{bmk} - \alpha_{t-1}) = 0 \Rightarrow E(\tilde{T}\tilde{\alpha}_{t-1}^{bmk} - \alpha_t) = 0$

$\Rightarrow E(\tilde{\alpha}_t^{bmk} - \alpha_t) = 0.$

- To warrant unbiasedness under model, suffices to initialize at time $t = 1$ with unbiased predictor.

(b)- Consistency: $\underset{n_d \to \infty}{\text{Plim}}(y_{dt} - Y_{dt}) = 0$ **&** $\underset{n_d \to \infty}{\text{Plim}}(\hat{Y}_{dt}^{bmk} - y_{dt}) = 0$

(by **GLS**) $\Rightarrow \underset{n_d \to \infty}{\text{Plim}}(\hat{Y}_{dt}^{bmk} - Y_{dt}) = 0$ (even if model misspecified).

- $n_d \to \infty \Rightarrow$ area **d** not helping benchmarking other areas.

Second-stage benchmarking

Suppose **S** **'states'** in Division **d** and similar model;

Direct $\rightarrow$ $y_{ds,t} = Y_{ds,t} + e_{ds,t}$; $E(e_{ds,t}) = 0$, $Cov(e_{ds,\tau}, e_{ds*,t}) = \delta_{s,s*}\sigma^2_{ds,\tau t}$

Total $\rightarrow$

$$Y_{ds,t} = z'_{ds,t}\alpha_{ds,t};$$

$$\alpha_{ds,t} = T_{ds}\alpha_{ds,t-1} + \eta_{ds,t},\ E(\eta_{ds,t}) = 0,\ E(\eta_{ds,t}\eta'_{ds,t*}) = \delta_{t,t*}Q_{ds,t}$$

Benchmark: $\sum_{s=1}^{S} b_{ds,t}y_{ds,t} = \sum_{s=1}^{S} b_{ds,t}z'_{ds,t}\hat{\alpha}_{ds,t} = \hat{Y}^{bmk}_{dt}$

Benchmark error: $r^{bmk}_{dt} = (\hat{Y}^{bmk}_{dt} - Y_{dt}) = (z'_{dt}\hat{\alpha}^{bmk}_{dt} - \sum_{s=1}^{S} b_{ds,t}z'_{ds,t}\alpha_{ds,t})$

- No longer simple linear combination of sampling errors.

Benchmarking of State estimates (cont.)

$$\tilde{y}_t^d = (y_{d1,t},...,y_{dS,t},\hat{Y}_{dt}^{bmk})' = (y_t'^d,\hat{Y}_{dt}^{bmk})'$$

$$\tilde{e}_t^d = (e_{d1,t},...,e_{dS,t},r_{dt}^{bmk})' = (e_t'^d,r_{dt}^{bmk})'.$$

$$\alpha_t^d = (\alpha'_{d1,t},...,\alpha'_{dS,t})',\ \tilde{T}^d = \mathrm{I}_S \otimes T_{ds},\ Z_t^d = \mathrm{I}_S \otimes z'_{ds,t},...$$

$$\tilde{Z}_t^d = \begin{bmatrix} Z_t^d \\ b_{d1,t}z'_{d1,t},...,b_{dS,t}z'_{dS,t} \end{bmatrix}.$$

Combined model:

$$\tilde{y}_t^d = \tilde{Z}_t^d \alpha_t^d + \tilde{e}_t^d;\ \alpha_t^d = \tilde{T}^d \alpha_{t-1}^d + \eta_t^d$$

$$E(\eta_t^d \eta_t'^d) = \mathrm{I}_S \otimes Q_{ds,t} = Q_d;\ E(\tilde{e}_\tau^d \tilde{e}_t'^d) = \tilde{\Sigma}_{\tau t}^d.$$

Benchmarking of State estimates (cont.)

RCR Model:

$$\begin{pmatrix} \tilde{T}^d \tilde{\alpha}_{t-1}^{d,bmk} \\ \tilde{y}_t^d \end{pmatrix} = \begin{pmatrix} I \\ \tilde{Z}_t^d \end{pmatrix} \alpha_t^d + \begin{pmatrix} \tilde{u}_{t|t-1}^{d,bmk} \\ \tilde{e}_t^d \end{pmatrix}; \; \tilde{u}_{t|t-1}^{d,bmk} = \tilde{T}^d \tilde{\alpha}_{t-1}^{d,bmk} - \alpha_t^d$$

$$Var\begin{pmatrix} \tilde{u}_{t|t-1}^{d,bmk} \\ \tilde{e}_t^d \end{pmatrix} = \begin{bmatrix} P_{t|t-1}^{d,bmk} & C_{dt}^{bmk} \\ C_{dt}^{bmk\prime} & \tilde{\Sigma}_{tt}^d \end{bmatrix} = \tilde{V}_t^d$$

$$C_{dt}^{bmk} = E(\tilde{u}_{t|t-1}^{d,bmk} \tilde{e}_t^{\prime d}); \; \tilde{\Sigma}_{tt}^d = E(\tilde{e}_t^d \tilde{e}_t^{\prime d}) = \begin{bmatrix} \Sigma_{tt}^d & h_{tt}^d \\ h_{tt}^{\prime d} & v_{tt}^d \end{bmatrix}. \quad \tilde{e}_t^d = (e_t^{\prime d}, \boldsymbol{r}_{dt}^{bmk})'$$

- $\boldsymbol{r}_{dt}^{bmk} = (\hat{Y}_{dt}^{bmk} - Y_{dt})$ correlated with model errors, $\tilde{u}_{t|t-1}^{d,bmk}$, and

State sampling errors, $e_{ds,t'}$ in complicated way (see paper).

Computation of State benchmarked predictors

Impose $\sum_{s=1}^{S} b_{ds,t} y_{ds,t} = \sum_{s=1}^{S} b_{ds,t} z'_{ds,t} \alpha_{ds,t} \Leftrightarrow \boldsymbol{r_{dt}^{bmk} = 0}$. Define,

$$\tilde{e}_t^d = (e_t'^d, \mathbf{o})', \; C_{dt,\mathbf{0}}^{bmk} = E(\tilde{u}_{t|t-1}^{d,bmk} \tilde{e}_{t,\mathbf{0}}'^d), \; \tilde{\Sigma}_{tt,\mathbf{0}}^d = E(\tilde{e}_{t,\mathbf{0}}^d \tilde{e}_{t,\mathbf{0}}'^d), \; \tilde{V}_{t,\mathbf{0}}^d = \begin{bmatrix} P_{t|t-1}^{d,bmk} & C_{dt,\mathbf{0}}^{bmk} \\ C_{dt,\mathbf{0}}^{bmk\prime} & \tilde{\Sigma}_{tt,\mathbf{0}}^d \end{bmatrix}.$$

$$\tilde{\boldsymbol{\alpha}}_t^{d,bmk} = \left[(\mathrm{I}, \tilde{Z}_t'^d)(\tilde{V}_{t,\mathbf{0}}^d)^{-1} \begin{pmatrix} \mathrm{I} \\ \tilde{Z}_t^d \end{pmatrix} \right]^{-1} (\mathrm{I}, \tilde{Z}_t'^d)(\tilde{V}_{t,\mathbf{0}}^d)^{-1} \begin{pmatrix} \tilde{T}^d \tilde{\alpha}_{t-1}^{d,bmk} \\ \tilde{y}_t^d \end{pmatrix} \rightarrow \textbf{GLS}.$$

Benchmarked predictor for State (d,s): $\underline{\hat{Y}_{ds,t}^{bmk} = z'_{ds,t} \tilde{\boldsymbol{\alpha}}_{ds,t}^{d,bmk}}$.

Variance of benchmarked predictors

Matrices $C_{dt,\mathbf{0}}^{bmk} = E(\tilde{u}_{t|t-1}^{d,bmk}\tilde{e}_{t,\mathbf{0}}'^{d})$ **&** $\tilde{\Sigma}_{tt,\mathbf{0}}^{d} = E(\tilde{e}_{t,\mathbf{0}}^{d}\tilde{e}_{t,\mathbf{0}}'^{d})$ only used for computing benchmarked predictors.

- True ***Var*** $(\hat{Y}_{ds,t}^{bmk} - Y_{ds,t})$ accounts for variances and auto-covariances of State sampling errors, $e_{ds,t}$, variances and auto-covariances of division benchmark prediction errors, $\boldsymbol{r_{dt}^{bmk}}$, and their covariances with State sampling errors, and variances of model components, $\eta_{ds,t}$.

Empirical results

Total unemployment, CPS-USA, Jan1990 - Dec2009.

First level- Census divisions, **Second level-** States

1. Compare **smoothness** of time series benchmarking and independent prorating;

$$\boldsymbol{R^{bmk}_{t/t\text{-}1}} = |1 - R^{bmk}_{t/t\text{-}1,\mathbf{pr}}| - |1 - R^{bmk}_{t/t\text{-}1,\mathbf{model}}| \;/\; |1 - R^{bmk}_{t/t\text{-}1,\mathbf{pr}}| + |1 - R^{bmk}_{t/t\text{-}1,\mathbf{model}}|$$

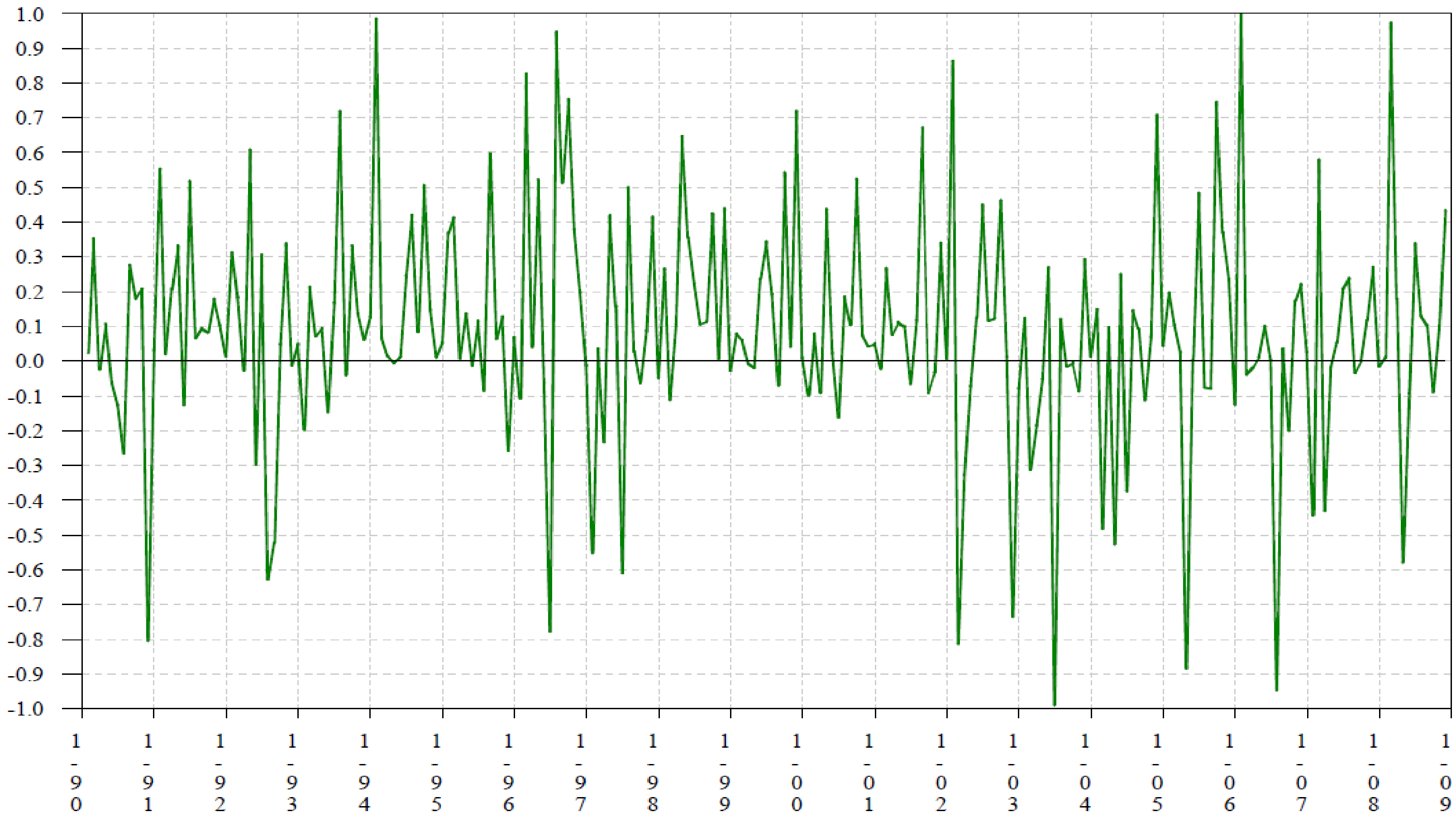
$R^{bmk}_{t/t\text{-}1,\ldots} \rightarrow$ month to month ratio of benchmarked predictor

2. Illustrate **consistency** of benchmarked predictors;

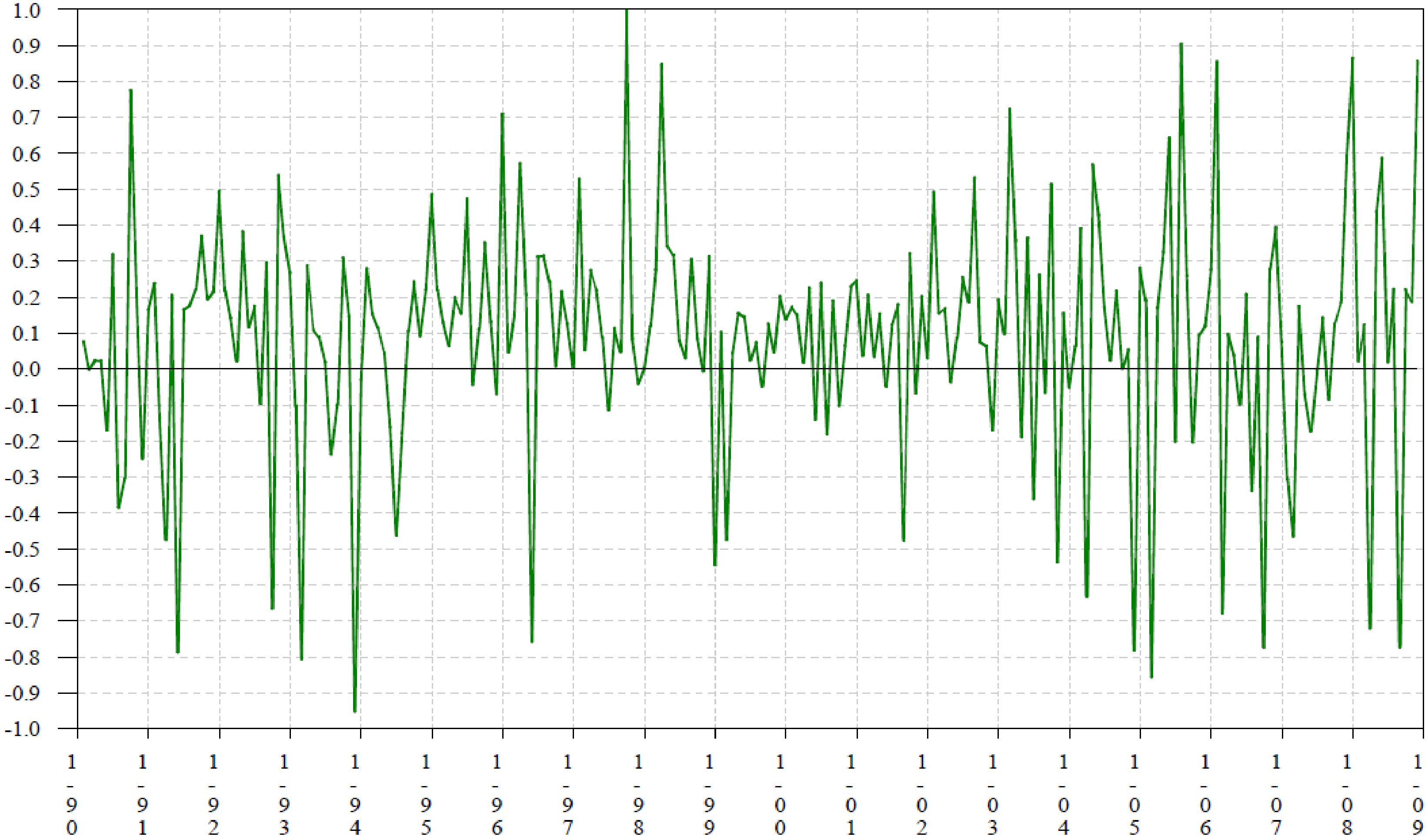
3. Illustrate **robustness**;

4. Illustrate **variance reduction**.

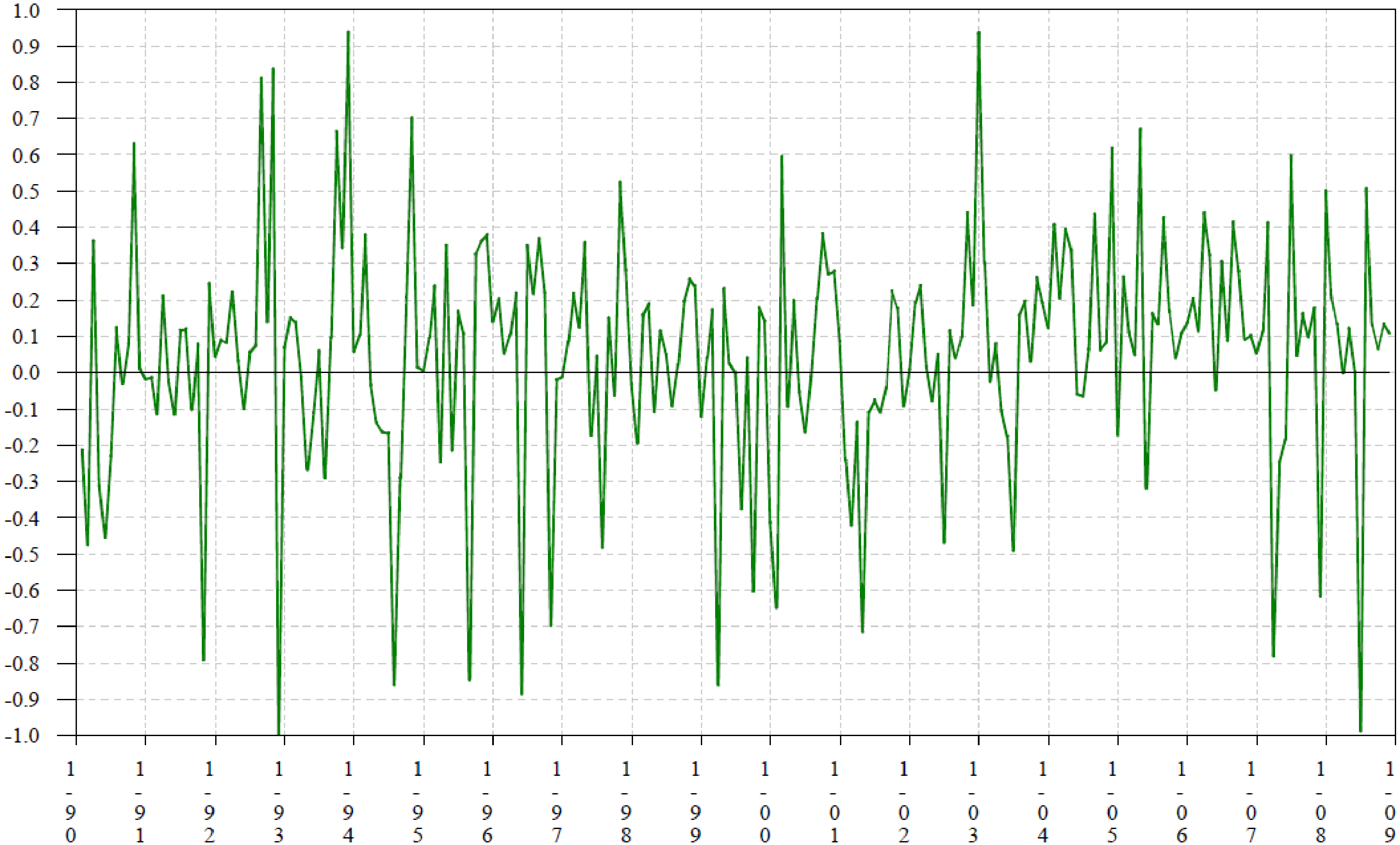
Ratios $R^{bmk}_{t/t\text{-}1}$ when estimating totals, New Hampshire



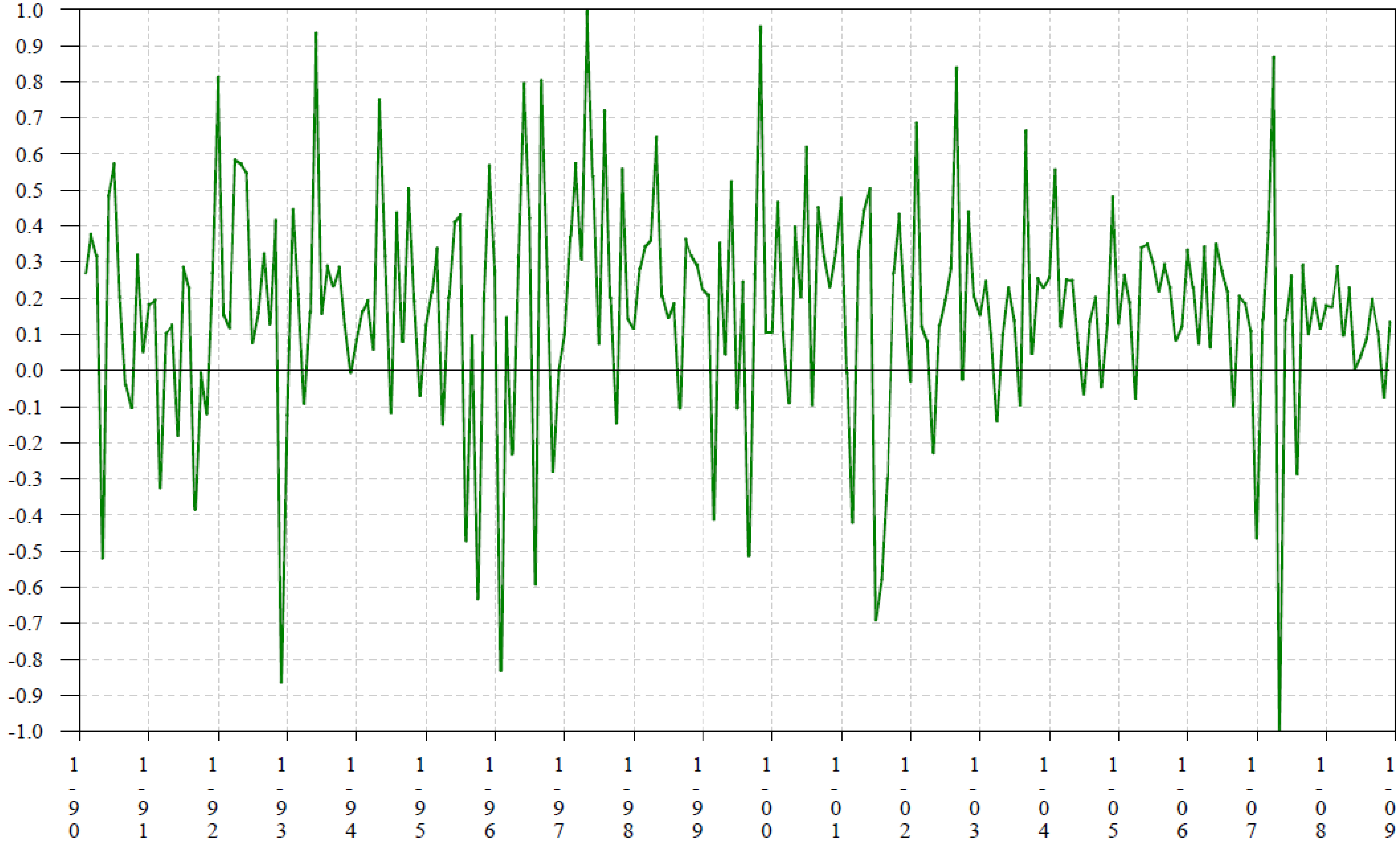
Ratios $R^{bmk}_{t/t\text{-}1}$ when estimating trends, New Hampshire



Ratios $R^{bmk}_{t/t\text{-}1}$ when estimating totals, New Mexico



Ratios $R^{bmk}_{t/t\text{-}1}$ when estimating trends, New Mexico

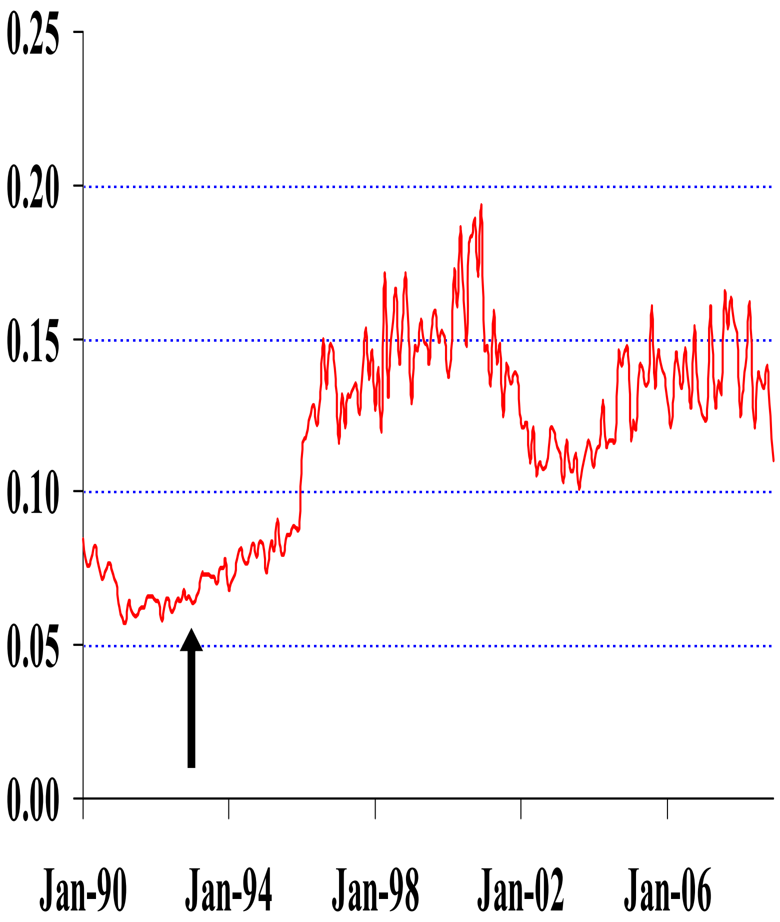


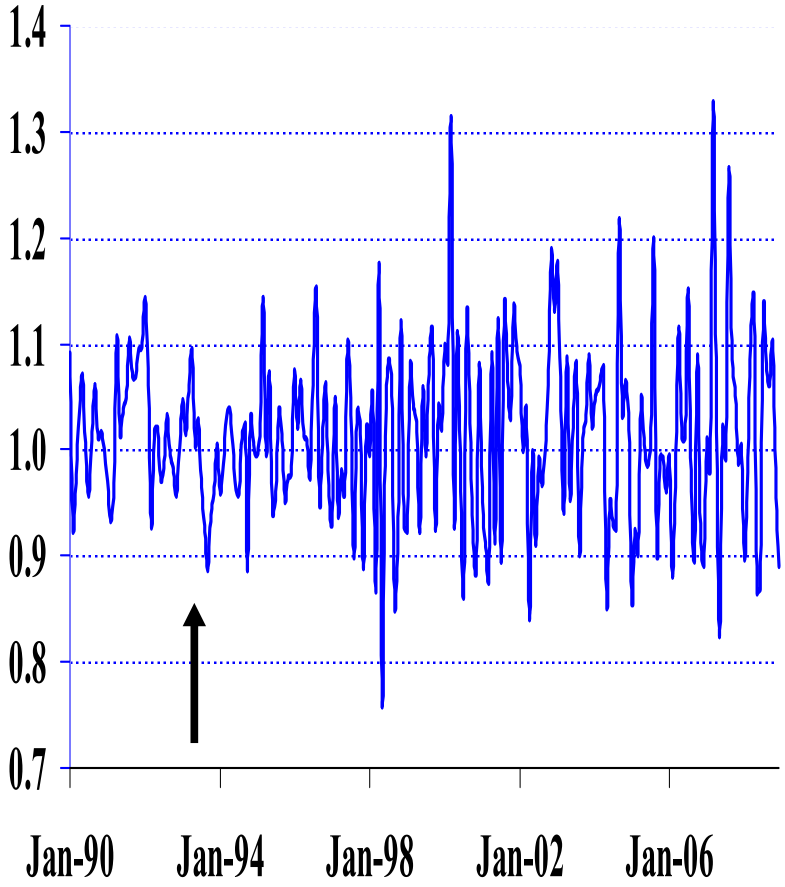
Distribution of $\frac{1}{227}\sum_{t} \mathrm{I}[R^{bmk}_{t/t\text{-}1} > 0]$ over States, 1990-2008.

	0.4-	0.4-0.5	0.5-0.6	0.6-0.7	0.7+	Total
Estimate total	**3**	**7**	**24**	**14**	**5**	**53**
Estimate trend	**1**	**6**	**7**	**11**	**28**	**53**

$[S.E.(y_{ds,t}) / y_{ds,t}]$ (left) & $[\hat{Y}^{bmk}_{ds,t} / y_{ds,t}]$ (right)

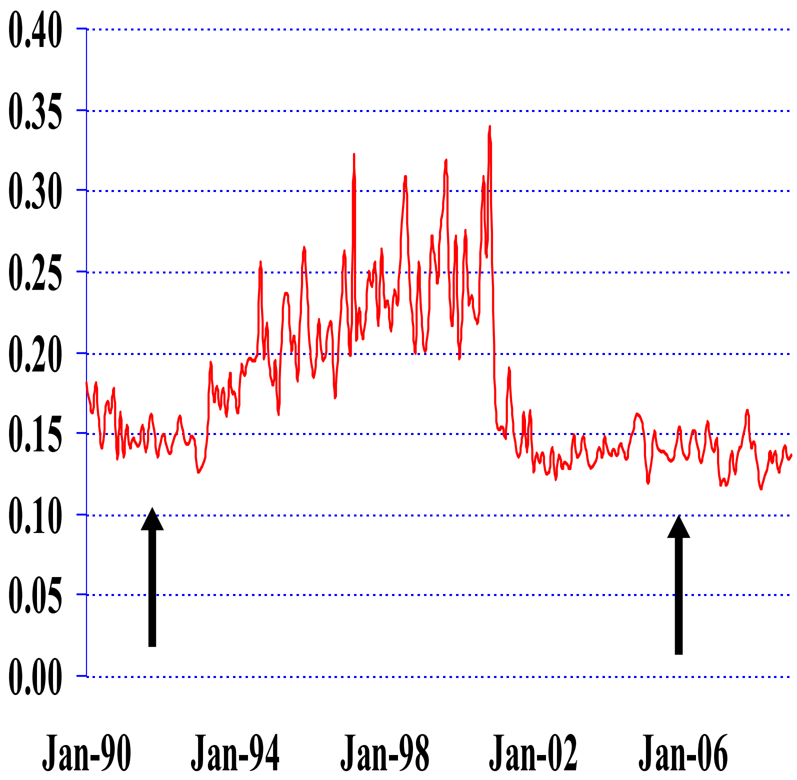
Massachusetts

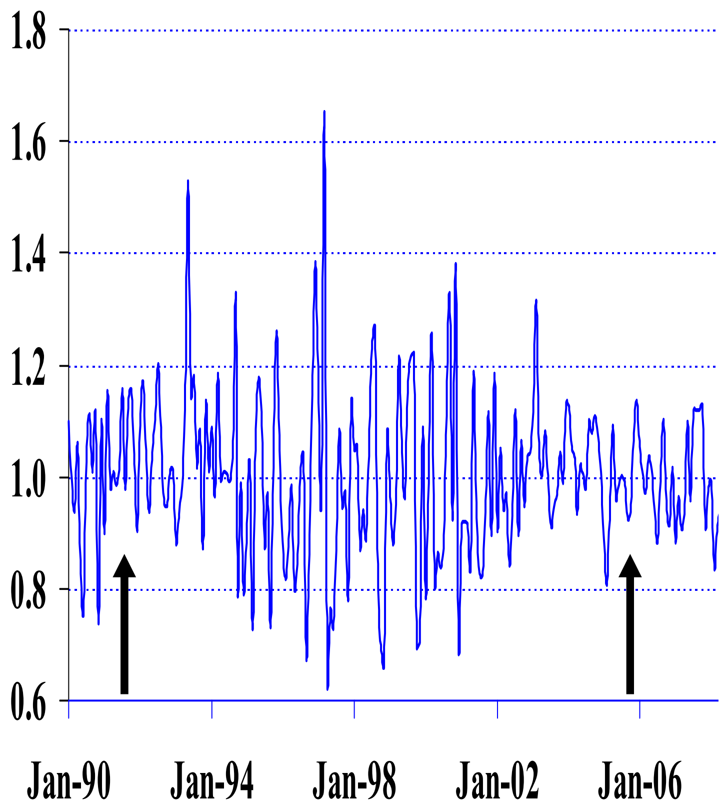




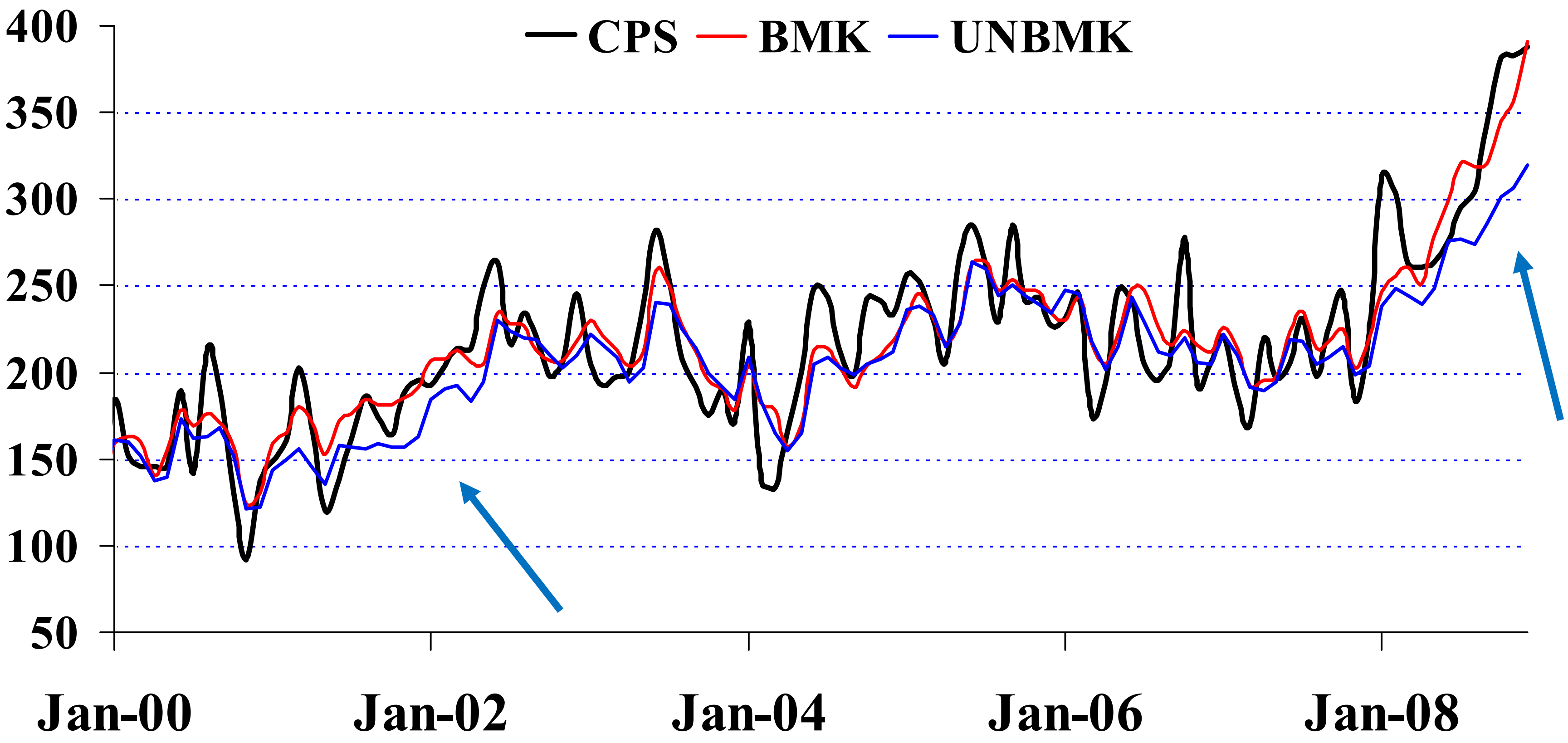
$[S.E.(y_{ds,t}) / y_{ds,t}]$ (left) & $[\hat{Y}^{bmk}_{ds,t} / y_{ds,t}]$ (right)

New Hampshire

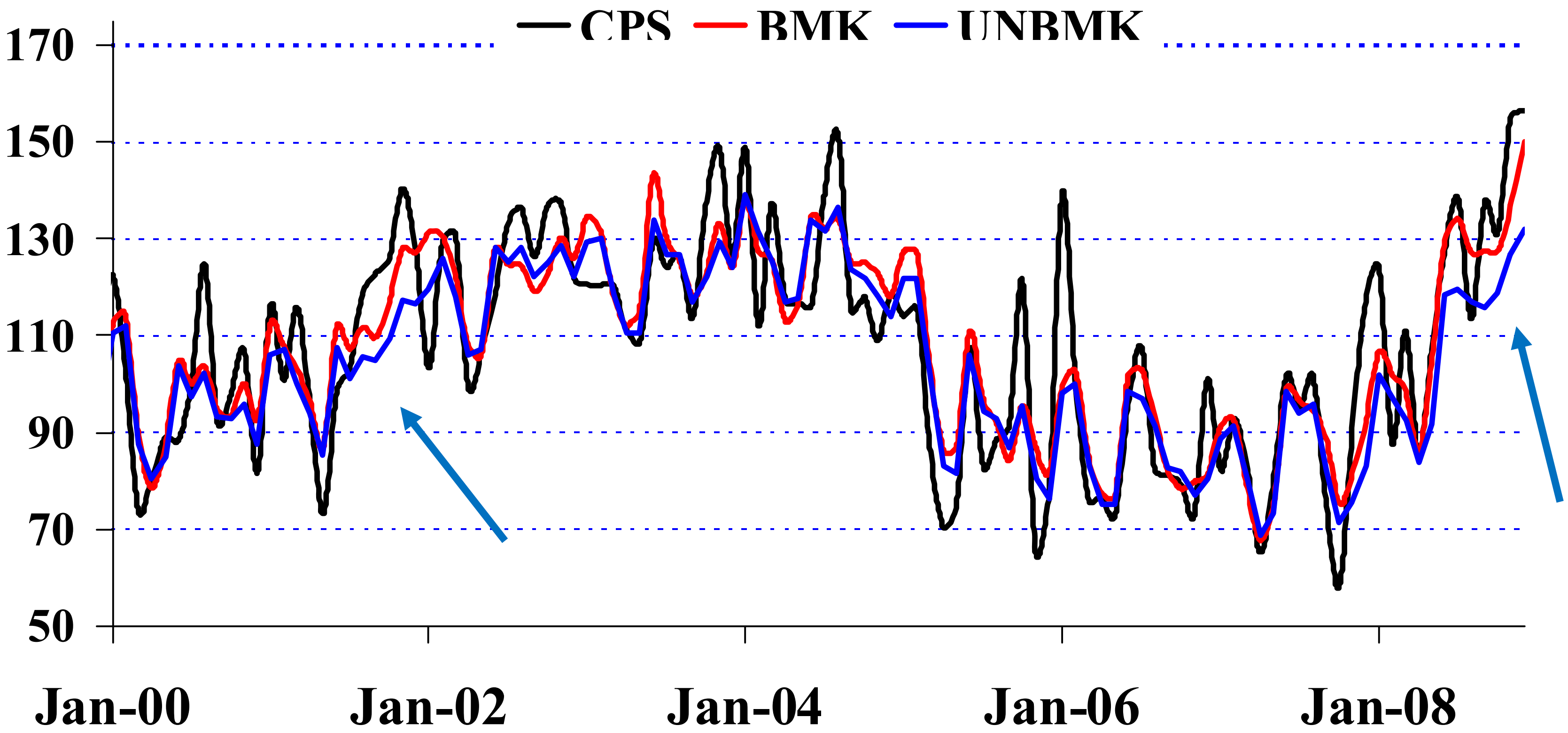




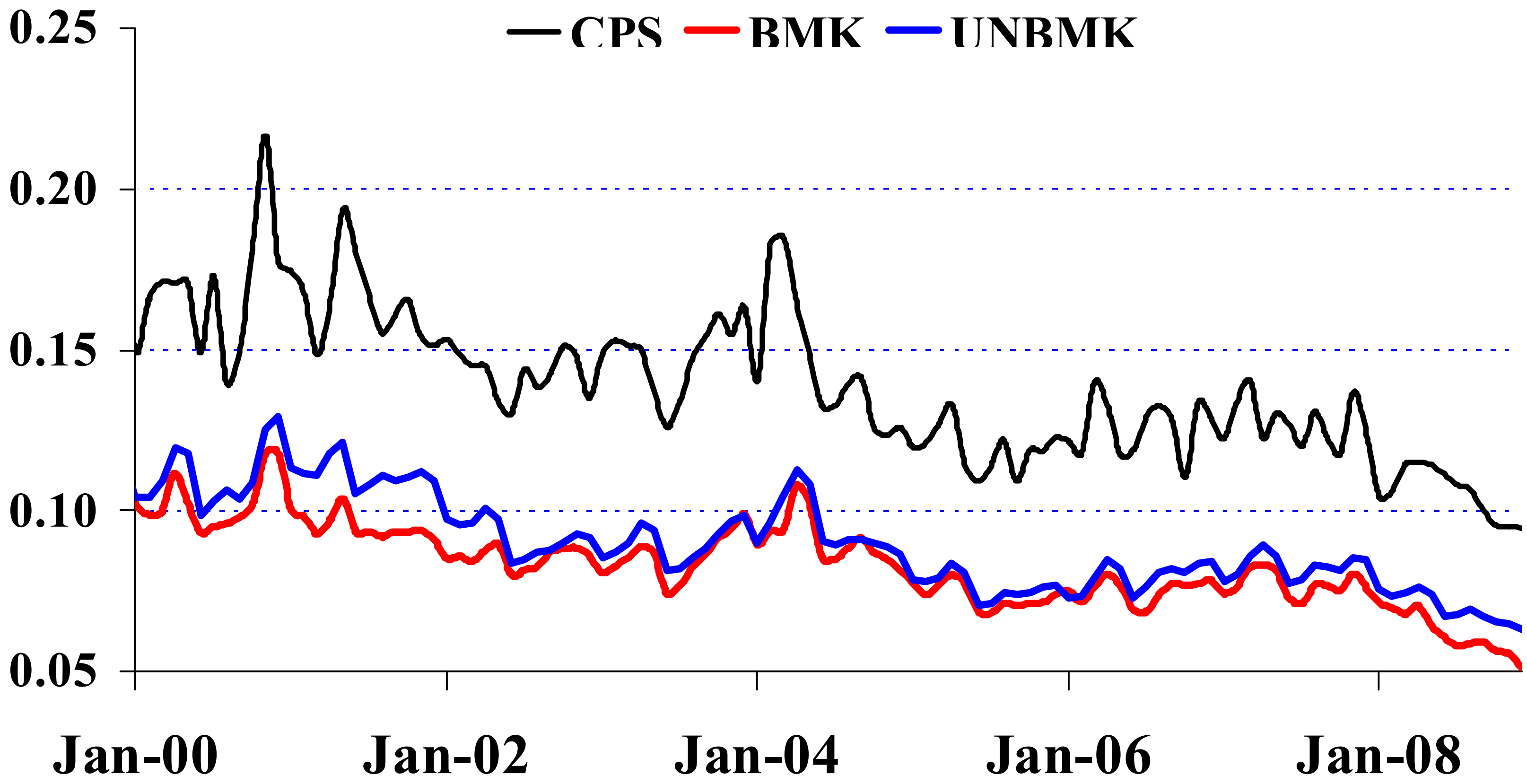
Direct, Benchmarked and Unbenchmarked estimates of Total Unemployment, Georgia (numbers in 000's).



Direct, Benchmarked and Unbenchmarked estimates of Total Unemployment, Alabama (numbers in 000's).



Relative Std errors of Direct, Benchmarked and Unbenchmarked est. of Total Unemployment, Georgia



Relative Std errors of Direct, Benchmarked and Unbenchmarked est. of Total Unemployment, Alabama