Discretization of Advection Dominated Optimal Control Problems

Prof. Dr. M. Heinkenschloss
Rice University, Houston USA

Optimization problems governed by advection dominated partial differential equations (PDEs) arise in many science and engineering applications. The discretization of these problems leads to many interesting questions that arise from the presence of the advection term in the governing PDE. In particular, there can be significant differences between the behavior of a discretization scheme when it is applied to a single advection dominated PDE and when it is applied to an advection dominated PDE constrained problem: the order of convergence in the optimization context may be significantly lower.

In this talk I will present results for the streamline upwind Petrov Galerkin (SUPG) discretization and for the symmetric interior penalty upwind discontinuous Galerkin (SIPG) discretization. I will present theoretical and numerical results that show that for advection-dominated problems the convergence properties of the SIPG discretization can be superior to the convergence properties of the SUPG discretization. For example, for a small diffusion parameter the SIPG method is optimal in the interior of the domain. This is in sharp contrast to SUPG discretizations, for which it is known that boundary layers can pollute the numerical solution of optimal control problems everywhere even into domains where the solution is smooth. For the SIPG discretization the size of the numerical boundary layer is controlled not by the mesh size but rather by the size of the diffusion parameter and, consequently, the boundary layers are too "weak" to pollute the SIPG solution into domains of smoothness in optimal control problems. I will demonstrate that the favorable convergence property of the SIPG method is due to the weak treatment of boundary conditions which is natural for discontinuous Galerkin methods, while for SUPG methods strong imposition of boundary conditions is more conventional.

This talk is based on joint work with Dmitriy Leykekhman (U. of Connecticut)