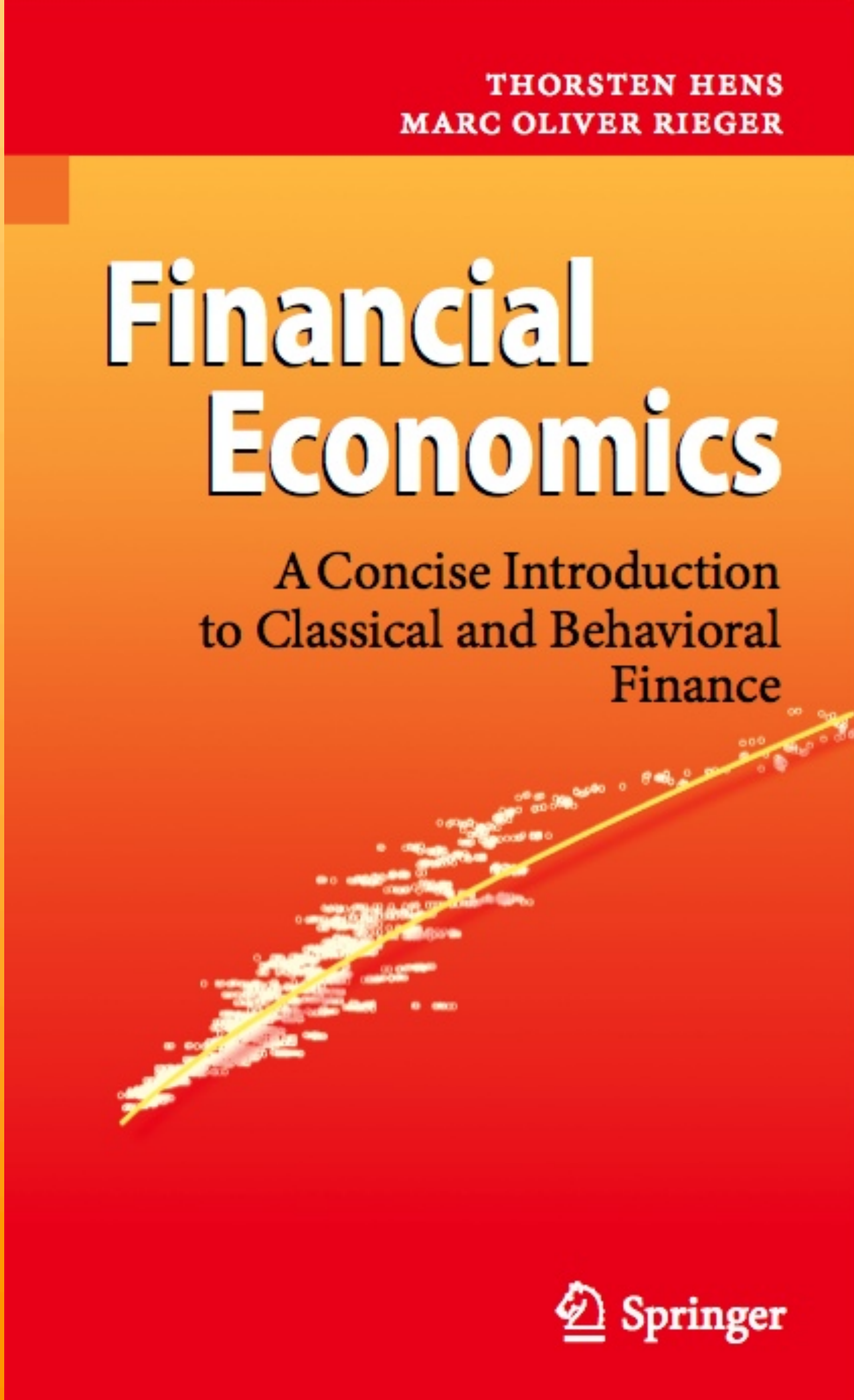


HENS · RIEGER

Financial Economics

Financial economics is a fascinating topic where ideas from economics, mathematics and, most recently, psychology are combined to understand financial markets. This book gives a concise introduction into this field and includes for the first time recent results from behavioral finance that help to understand many puzzles in traditional finance. The book is tailor made for master and PhD students and includes tests and exercises that enable the students to keep track of their progress. Parts of the book can also be used on a bachelor level. Researchers will find it particularly useful as a source for recent results in behavioral finance and decision theory.

The text book to this class is available at www.springer.com

On the book's homepage at www.financial-economics.de there is further material available to this lecture, e.g. corrections and updates.

Financial Economics

A Concise Introduction to Classical and Behavioral Finance
Chapter 3

Thorsten Hens and Marc Oliver Rieger

Swiss Banking Institute, University of Zurich / BWL, University of Trier

August 6, 2010

Two-Period Model: Mean-Variance Approach

千里之行始于足下

"A journey of a thousand miles starts with the first step."
Chinese proverb

Overview on assumptions

- Two-period model:
  - First we invest into assets
  - Then the assets pay off
- Mean-variance preferences:
  - done in practice
  - some shortcomings

CAPM

- Foundation: Markowitz mean-variance analysis (1952)
- Markowitz recommends the use of an expected return-variance of return rule,

  > *... both as a hypothesis to explain well-established investment behavior and as a maxim to guide one's own action.*

- Nobel prize: 1990 to Markowitz and Sharpe
- Main point: excess returns are explained by covariance to market portfolio.

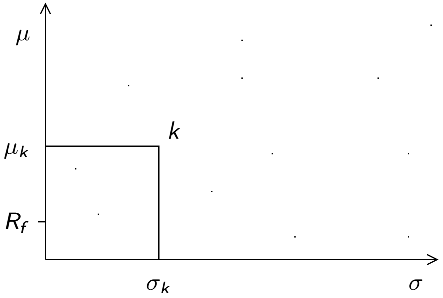
Mathematics of the CAPM

We start with an intuitive approach before we discuss more formal derivations:

$k = 1, 2, \ldots, K$	assets
$R_k := A_k / q_k$	gross return of asset k
q_k	first period market price
A_k	second period payoff
$\mu_k := \mu(R_k)$	expected return
$\sigma_k^2 := \mathrm{var}\,(R_k)$	variance

Geometric Intuition for the CAPM

Assets can be represented in a two-dimensional diagram.



Attractiveness of a single asset is characterized by mean and standard deviation.
Risk free-asset has an expected return of R_f with a zero standard deviation.

Diversification – History

There was a time when diversification as a means of reducing risk was not universally accepted. Markowitz' portfolio theory and their risk diversification was very controversial.

> *To suppose that safety-first consists of having a small gamble in a large number of different [companies] . . . strikes me as a travesty of investment policy.*
> *— J. M. Keynes [Keynes, 1988].*

Later the impact of the idea of diversification made such criticism look queer.

Diversification – Introduction

If we combine two risky assets k and j we obtain an expected portfolio return of $\mu_\lambda := \lambda\mu_k + (1-\lambda)\mu_j$, where λ is the portion of wealth invested in asset k. The portfolio variance is

$$\sigma_\lambda^2 := \lambda^2\sigma_k^2 + (1-\lambda)^2\sigma_j^2 + 2\lambda(1-\lambda)\,\mathrm{cov}_{k,j}\,.$$

How much one can gain by combining risky assets depends on covariance:

smaller covariance
$\rightsquigarrow$ higher *diversification* potential of mixing risky assets.

Correlation

It is convenient to standardize the covariance with the standard deviation.

The *correlation*

$$\mathrm{corr}_{k,j} := \mathrm{cov}_{k,j} / (\sigma_k \sigma_j)$$

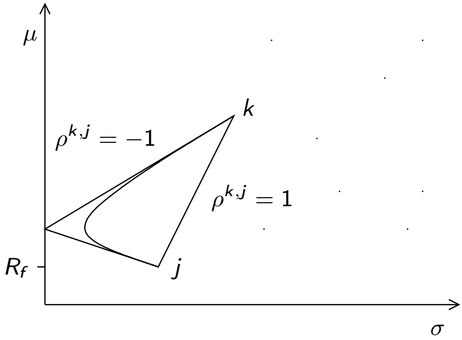
takes values between -1 (perfectly negatively correlated) and $+1$ (perfectly positively correlated).

Further explanation of possible correlation values can be found in the text book on page 97.

Correlation

When the risky assets are perfectly negatively correlated, i.e., when $\mathrm{corr}_{k,j} = -1$ the portfolio may even achieve an expected return higher than the riskfree rate without bearing additional risk.

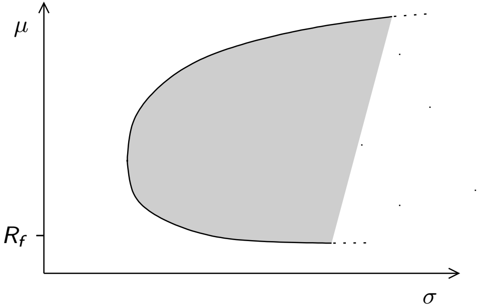
Diversification of two assets



Minimum-variance opportunity set

- Investors can build portfolios from risky and riskfree assets but also portfolios from other portfolios etc.
- The set of possible μ-σ-combinations offered by portfolios of risky assets that yield minimum variance for a given rate of return is called minimum-variance opportunity set or portfolio bound.

Minimum-variance opportunity set



Optimal investments

Choosing an optimal portfolio is to pick a portfolio with the highest expected returns for a given level of risk. This is similar to the following optimization problem:

$$\min_{\lambda_k,\lambda_j} \sum_k \sum_j \lambda_k \operatorname{cov}_{k,j} \lambda_j$$

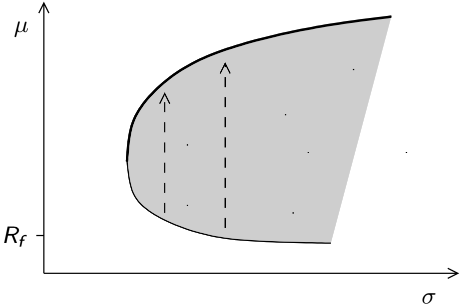
$$\text{such that } \sum_k \lambda_k \mu_k = \text{const} \text{ and } \sum_k \lambda_k = 1,$$

where λ_k denote the proportion of money invested in asset k.

Efficient Frontier

- Solution of optimization problem gives the mean-variance opportunity set or the portfolio bound.
- Efficient portfolios focus on that part of the mean-variance efficient set that is not dominated by lower risk and higher return: upper part of the portfolio bound.

Efficient Frontier



Optimal Portfolio

- If an investor combines a risky asset (or a portfolio of risky assets) with a riskless security, he must choose a point on the line connecting both assets.
- This is a straight line, since covariance is zero and therefore standard deviation σ_λ is a *linear* function of the portfolio weights.

Capital Market Line

- Best portfolio combination: when the line achieves its highest possible slope.
- Defines the Capital Market Line (CML).
- Its slope is called Sharpe ratio, $(\mu_\lambda - R_f)/\sigma_\lambda$.
- Point at which the CML touches the efficient frontier is the tangent portfolio.

Optional Excursion: Mathematical Analysis of the Minimum-Variance Opportunity Set*

- It is sometimes said that the minimum-variance opportunity set is convex.
- This is not always the case: In the case of two assets, the opportunity set is *only* convex if their correlation is $+1$.
- We don't need convexity to prove existence of a tangent portfolio, we only need that the opportunity set is closed and certain properties of the efficient frontier that we summarize later.

Opportunity set is closed and connected

Lemma

If we have finitely many assets, the minimum-variance opportunity set is closed and connected.

The proof can be found in the text book on page 101f.

Infinitely many assets

What about if we have infinitely many assets? In this case the opportunity set does not have to be closed.

Example

Perfectly correlated assets with $\mu_k = 1 - 1/k$ *and* $\sigma_k = 1$. *The opportunity set is given by* $\{(\mu, 1) | \, \mu \in [0, 1)\}$ *and is obviously not closed.*

We better stick to the case of finitely many assets.

Efficient frontier can be discontinuous

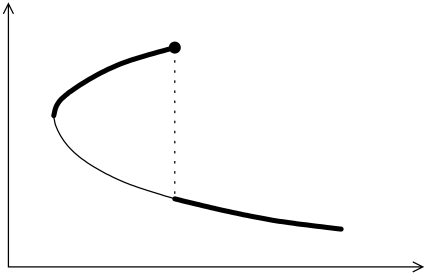
Lemma

If we have finitely many assets, the efficient frontier can be described as the graph of a function $f : [a, b]$*, where* $0 \leq a \leq b < \infty$*. Moreover there exists a point* $c \in [a, b]$ *such that* f *is concave and increasing on* $[a, c]$ *and decreasing on* $[c, b]$*.*

The proof can be found in the text book on page 102.

Efficient frontier can be discontinuous

Example: two assets, correlation less than 1.



Existence of a tangent portfolio

Proposition

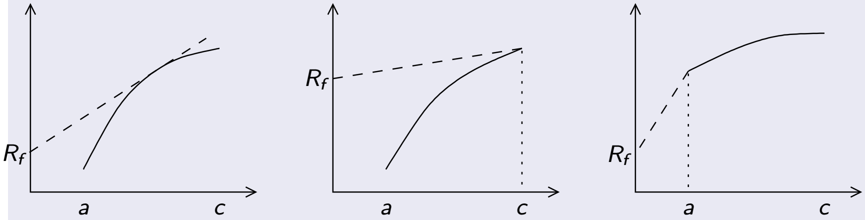
If we have finitely many assets, and at least one asset has a mean which is not lower than the return R_f of the risk-free asset, then a tangent portfolio exists.

The proof can be found in the text book on page 102.

Existence of a tangent portfolio

Proof.

Now, we have to distinguish three cases:



In all three cases, the constructed line cannot lie below other points of the efficient frontier, since f is decreasing for values larger than c, but the tangent line is increasing (or at least horizontal), since $f(c) \geq R_f$. □

Two-Fund Separation Theorem (1)

- Optimal asset allocation of risky assets and a riskless security depends on investor's preferences

$$U^i(\mu_\lambda, \sigma_\lambda^2) := \mu_\lambda - \frac{\rho^i}{2}\sigma_\lambda^2,$$

  where ρ^i is a risk aversion parameter of investor i. The higher this parameter, the higher is the slope of the utility function.
- The higher the risk aversion, the higher is the required expected return for a unit risk (required risk premium).
- Different investors have different risk-return preferences. Investors with higher (lower) level of risk aversion choose portfolios with a low (high) level of expected return and variance, i.e., their portfolios move down (up) the efficient frontier.

Two-Fund Separation Theorem (2)

The *Separation Theorem* of Tobin (1958) states that agents should diversify between the risk free asset (e.g., money) and a single optimal portfolio of risky assets.

Different attitudes toward risk result in different combinations of the risk free asset and the optimal portfolio of risky assets.

- More conservative investors will choose to put a higher fraction of their wealth into the risk free asset
- more aggressive investors decide to borrow capital on the money market and invest it in the Tangent Portfolio

This property is known as Two-Fund Separation.

Computing the Tangent Portfolio (1)

According to the Two-Fund Separation an investor with utility

$$U^i(\mu_{\boldsymbol{\lambda}}, \sigma^2_{\boldsymbol{\lambda}}) = \mu_{\boldsymbol{\lambda}} - \frac{\rho^i}{2}\sigma^2_{\boldsymbol{\lambda}}$$

has to decide how to split his wealth between the optimal portfolio of risky assets with a certain variance-covariance structure (Tangent Portfolio) and the riskless asset.

Further information on how to compute the tangent portfolio can be found in the text book on page 106f.

Market Equilibrium

Market Equilibrium

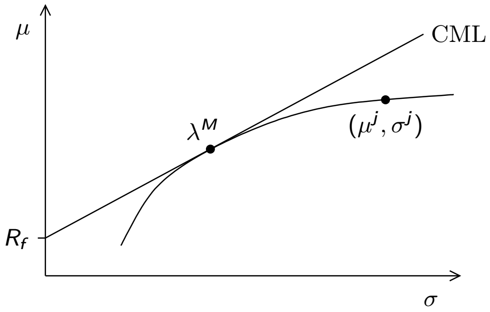
We want to study market equilibria, therefore we make the following observation:
If individual portfolios satisfy the Two-Fund-Separation, then by setting demand equal to supply the sum of the individual portfolios must be proportional to the vector of market capitalization $\boldsymbol{\lambda}^M$:

$$\sum_i \lambda_k^i = \Big(\sum_i (1 - \lambda_0^i)\Big)\lambda_k^T = \lambda_k^M.$$

Hence, in equilibrium, the normalized *Tangent Portfolio* will be identical to the *Market Portfolio*.

Derivation of the SML (1)

Compare the slopes of the Capital Market Line and a curve *j* that is obtained by mixing a portfolio of any asset *j* with the market portfolio. By the tangency property of $\boldsymbol{\lambda}^M$ these two slopes must be equal!



Derivation of the SML (2)

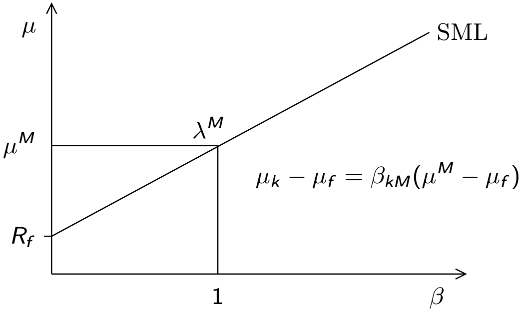
The slope of the Capital Market Line is

$$\frac{R_f - \mu^M}{-\sigma_M}.$$

A detailed derivation of the SML can be found in the text book on page 107f.

Derivation of the SML (3)

The result is the Security Market Line:



Derivation of the SML (4)

The difference to the mean-variance analysis is the risk measure:

- In the CAPM the asset's risk is captured by the factor β instead of the standard deviation of asset's returns.
- It measures the sensitivity of asset j returns to changes in the returns of the market portfolio. This is the so called systematic risk.

Market Neutral Strategies

The Capital Asset Pricing Model has many applications for investment managers and corporate finance.
Example: *Market Neutral Strategy* followed by some hedge funds.

- This strategy aims a zero exposure to market risk.
- To exclude the impact of market movements, it takes simultaneous long and short positions on risky assets.
- These assets have the same Beta (as measure for market risk) but different market prices.
- Under the assumption that market prices will eventually return to their fundamental value defined by the CAPM, hedge fund managers take long positions in underpriced assets and short positions in overpriced assets.

We will discuss later the potential risks of this strategy.

Empirical Validity of the CAPM

- An advisor should apply the same expectations when giving recommendations to different clients
- Hence, following the two-fund separation property, he should recommend the same portfolio of risky assets.
- Canner, Mankiw and Weil [Canner et al., 1997] showed that this simple rule is however not followed by advisors.
- An application example of the CAPM for investment managers can be found in the text book on page 108f.
- Further empirical validity of the CAPM can be found in the text book on page 109 f.

Empirical Validity of the CAPM

- One of the nice properties of the SML is that it suggests a linear relation between the Beta and the excess returns.
- Many studies found that market risk, the Beta, indeed explains the excess returns of assets. But more factors are needed to get a really good fit.
- Most famous additional factors are *value*, *size* and *momentum*. Investing in value stocks give significantly higher returns – even with lower Beta – than investing in glamour stocks.
- Also, investing in small cap stocks has this feature.
- Finally, investing in stocks that have gone up is increasing returns in the short run and the reverse is true in the long run. [Fama and French, 1992], [Fama and French, 1998] and [Lakonishok et al., 1994]

Heterogeneous beliefs and the Alpha

Reasons for Trading

So far we have mentioned two motives for trade:

Smoothing intertemporal consumption	Risk diversification
Fixed income markets	Reinsurance markets, stock markets and other markets which allow diversifying risks.

Diversification motive is best served by mutual funds (ETFs).
But what about, e.g., hedge funds?

Hedge funds (1)

Hedge funds claim to offer

- returns as high as those of stocks
- with a volatility as low as that of bonds

a clear violation of the CAPM!

Hedge funds (2)

- Hedge funds claim to generate the "Alpha": excess returns that cannot be explained by market risk.
- The *Alpha* has become a magic selling word. Banks offer Alpha funds, hedge funds call themselves "*AlphaSwiss*", or "*Alpha Lake*". Analysts write about the *future of the Alpha*, or the *pure Alpha* etc.
- Do banks and hedge funds sell dreams like a perpetuum mobile that do not exist?

Heterogeneous beliefs

We show:

- Lack of theory can be removed by extending the standard CAPM towards heterogeneous beliefs
- In a CAPM with heterogeneous beliefs every investor who holds beliefs different to the average market belief, *sees some Alpha*
- However, the sum of these Alphas is zero: the hunt for Alphas is a *zero-sum game*.
- It becomes more and more difficult for the active managers to outperform each other.
- Long-run outcome of the zero-sum game is consistent with the efficient market hypothesis and the CAPM based on homogeneous beliefs.

Definition of the Alpha

- The *Alpha* is a departure from the Security Market Line, SML. Not to be mixed up with the parameter α of the risk-aversion!
- According to the SML excess return of any asset is proportional to excess return of the market portfolio with proportionality factor Beta

$$\mu(R_k) - R_f = \beta_{k,M}(\mu(R^M) - R_f),$$

where

$$\beta_{k,M} := \frac{\mathrm{cov}\left(R_k, R^M\right)}{\mathrm{var}\left(R^M\right)}.$$

Definition of the Alpha

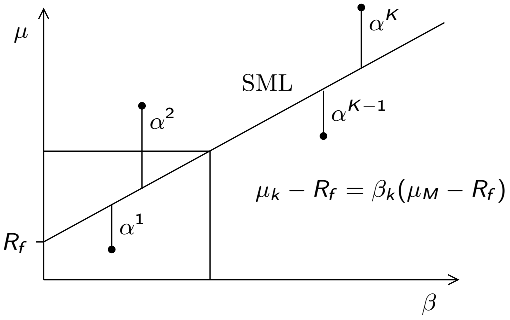
Define the Alpha of asset k as the gap between the claimed excess return and the theoretically justified return:

$$\alpha_{k,M} := \mu(R_k) - R_f - \beta_{k,M}(\mu(R^M) - R_f),$$

where

$$\beta_{k,M} := \frac{\mathrm{cov}\left(R_k, R^M\right)}{\mathrm{var}\left(R^M\right)}.$$

Definition of the Alpha



CML and SML

We need to check the desirability of positive Alpha in the mean standard-deviation diagram and not in the mean-Beta diagram. Clearly, the SML in the mean-Beta diagram is the image of the CML in the mean-standard-deviation diagram and vice versa.

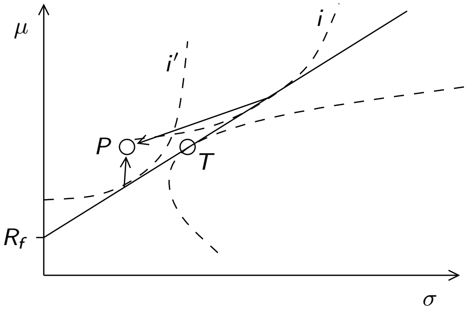
$$\begin{aligned}
\text{SML:}\quad & \mu(\lambda R^M + (1-\lambda)R_f) \\
&= R_f + \frac{\operatorname{cov}\left(\lambda R^M + (1-\lambda)R_f, R^M\right)}{\operatorname{var}\left(R^M\right)}(\mu(R^M) - R_f) \\
&= R_f + \lambda(\mu(R^M) - R_f). \\
\text{CML:}\quad & \mu(\lambda R^M + (1-\lambda)R_f) \\
&= R_f + \frac{\sigma(\lambda)R^M + (1-\lambda)R_f}{\sigma(R^M)}(\mu(R^M) - R_f) \\
&= R_f + \lambda(\mu(R^M) - R_f).
\end{aligned}$$

Alpha opportunities

But:

- Is a point above the SML indeed also a point above the CML
- If so, is any point above the CML also an improvement for the agent?

Alpha opportunities



Switching to portfolio P improves i' but not i. However, both can improve T by investing *some* wealth in P.

Alpha opportunities

- Not every point above the SML is an outright improvement of the agents' portfolio.
- However, adding *some* of it to the agent's portfolio makes the agent better off.
- Actually we show that the Alpha is the *direction* in which the mean-variance utility of the agent has its steepest increase!

Proof

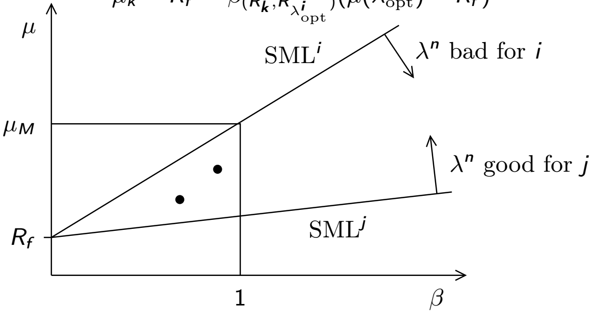
The proof can be found in the textbook on page 113f.

One practical caveat

- Adding any amount of Alpha opportunity to improve a benchmark portfolio may make a suboptimal portfolio worse.
- Hence general selling initiatives that are typical in large banks, in which all clients are suggested to add the same Alpha opportunity computed on the basis of a benchmark portfolio, may be bad for many clients with suboptimal portfolios.
- It would be better to first move the suboptimal portfolios towards the benchmark portfolios.

One practical caveat

$$\mu_k - R_f = \beta_{(R_k, R_{\lambda^i_{\text{opt}}})}(\mu(\lambda^i_{\text{opt}}) - R_f)$$



A counterargument

A natural counterargument to this observation:

> *Alpha opportunities improve the efficient frontier, therefore they should always improve the overall quality of portfolios, shouldn't they?*

This line of argument is right and wrong!
It is important to understand the different notions of "improvement" here.

A counterargument

A simple example:

- If you decided for a nice menu in a restaurant and now the set of available items is suddenly enlarged by a wonderful red wine for a reasonable price, this is obviously an improvement.
- However, your particular dinner, let's say fish and white wine, is probably not improved if you add a little bit of red wine to it: the red wine would fit neither to the fish nor to the white wine. The better approach is to choose a completely new menu.

CAPM with Heterogeneous Beliefs

- Standard CAPM investors differ with respect to initial endowments and risk aversion, but share same beliefs about the expected returns and covariance of returns.
- Now we allow the investors to also differ with respect to their beliefs on the assets' expected returns.
- However, we keep the assumption that investors agree on the covariances of the assets.

SML with heterogeneous beliefs

Proposition

In the CAPM with heterogeneous beliefs the Security Market Line holds for the average beliefs, i.e., for all assets $k = 1, \ldots, K$,

$$\bar{\mu}_k - R_f = \beta_{k,M}(\bar{\mu}^M - R_f),$$

where as usual

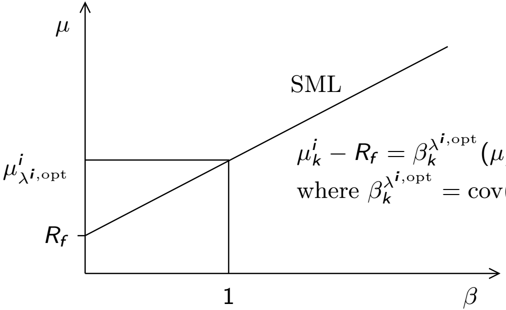
$$\beta_{k,M} := \mathrm{cov}\left(R_k, R^M\right) \Big/ \mathrm{var}\left(R^M\right), \quad \bar{\mu}^M := \sum_{i=1}^{I} a^i \mu^i,$$

$$a^i := \frac{r^i}{\rho^i} \Big/ \sum_{j=1}^{I} \frac{r^j}{\rho^j}, \qquad r^i = w_f^i \Big/ \sum_{j=1}^{I} w_f^i,$$

$$w_f^i = (1 - \lambda_0^i) W_0^i.$$

The proof can be found in the text book on page 118ff.

Individual SML



$$\mu^i_k - R_f = \beta_k^{\lambda^{i,\mathrm{opt}}}(\mu_{\lambda^{i,\mathrm{opt}}} - R_f)$$

$$\text{where } \beta_k^{\lambda^{i,\mathrm{opt}}} = \mathrm{cov}(R_k, R\lambda^{i,\mathrm{opt}})/\sigma^2(R\lambda^{i,\mathrm{opt}})$$

Is underdiversification always bad?

Underdiversified portfolios do not need to be worse than well diversified portfolios:

- Based on 78'000 households portfolios observed from 1991 to 1996 [Ivković et al., 2005] find that the more wealthy have more underdiversified portfolios achieving a positive Alpha to the market.

Obviously, there are some people who detect Alpha opportunities well!

Zero Sum Game

In utility terms the CAPM is clearly not a zero sum game since it still involves trade to share risks which is beneficial to *all* investors.
So how is it with Alpha opportunities?

Proposition

Defining the Alpha as the excess return that agent i sees in asset k over and above the return seen by the market, the weighted average of the individual investors' Alphas is zero. The weights are given as in the security market line.

The proof can be found in the text book on page 122.

A different Alpha

A different way of defining Alphas is to define them with respect to the true average returns.
To this end let $\hat{\mu}_k$, $k = 1, \ldots, K$, denote the true average return of the assets. Define:

- $\hat{\mu}^M := \sum_{k=1}^K \lambda_k^M \hat{\mu}^k$
- the Alpha of asset k as the realized average return compared to the expected average return based on market expectations:

$$\hat{\alpha}_{k,M} := (\hat{\mu}_k - R_f) - \beta_{k,M} \left(\hat{\mu}^M - R_f \right), \quad k = 1, \ldots, K.$$

- the Alpha of the portfolio of investor i:

$$\hat{\alpha}^i := \sum_{k=1}^K \lambda_k^i \hat{\alpha}_{k,M}.$$

Zero Sum Game again

Proposition

Defining the Alpha of asset k as the excess return that asset k realizes over and above the return justified by the security market line, the weighted average of the individual investors' Alphas is zero. The weights are given by the relative wealth of the investors.

The proof can be found in the text book on page 123.

Active or passive?

- Active investor optimizes his portfolio given his beliefs, invests in his Tangent Portfolio.
- Passive investor invests in the market portfolio as if he shared the average belief of the investors who shall be active and who shall be passive.

Active or passive?

We assume that

- active asset management is costly
- being passive is for free

Every investor has the choice to "passify" if he discovers himself to be a loser of the zero sum game.

- If more and more unskilled investors drop out, the remaining investors will have an ever harder task.
- Eventually, only the best active manager determines asset prices, which is a conclusion in the line of the efficient market hypothesis.

Active or passive?

One can show that the agent should be active if and only if:

$$U^i_{\hat{\mu}}(\mu^i) - U^i_{\hat{\mu}}(\bar{\mu}) = \frac{1}{2\rho^i}\left(\|\hat{\mu} - \bar{\mu}\|^2 - \left\|\hat{\mu} - \mu^i\right\|^2\right) > C^i,$$

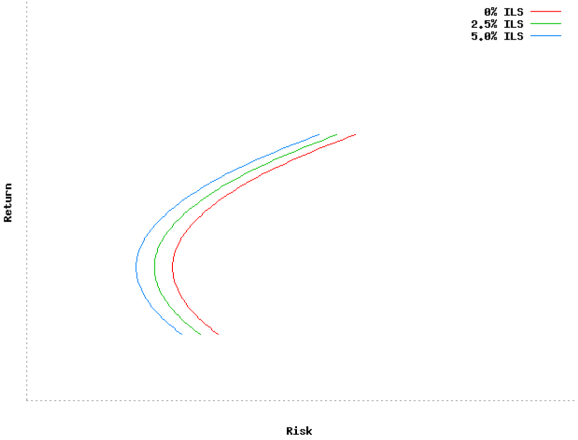
where $\|\mathbf{x}\|^2 := \mathbf{x}' COV^{-1}\mathbf{x}$. Here, $\|\hat{\mu} - \bar{\mu}\|^2$ is the market inefficiency term and $\left\|\hat{\mu} - \mu^i\right\|^2$ measures the deviation of expectations from reality.

This result shows that the investor should be active

- the less efficient the market,
- the more skilled the investor,
- the smaller his costs to be active and
- the less risk averse he is.

Alternative Betas

Some banks sell their products by showing that including their product enlarges the efficient frontier:



Alternative Betas

Mean-variance diversification benefits from investing in Insurance Linked Securities.

Is the enlargement argument sufficient for investing in the product?

- Yes, if investors only care about mean and standard deviation.
- But if the investors are already quite loaded with the underlying risk of the product then they should take this into consideration.

Example

Consider the Goldman Sachs Commodity Index (GSCI) which is related to the oil price.

- For the CEO of a car producer it may make sense to invest in the GSCI since this may compensate him for a smaller bonus if the demand for cars drops due to a rise in oil prices.
- But for the CEO of a solar technical firm the opposite may be true.

Higher Moment Betas

- The CAPM ignores higher moments of the return distribution.
- Yet agents may not only care for mean and variance.
- Some investments that look very attractive in the mean-variance framework may lose their attraction once higher moments are taken into account.

Value- and size-puzzle (1)

Apply Prospect Theory to standard data on value and size portfolios. Due to skewness and fat tails, the deep value and the small cap returns are not more attractive than the stock market index from a Prospect Theory perspective.

Value- and size-puzzle (2)

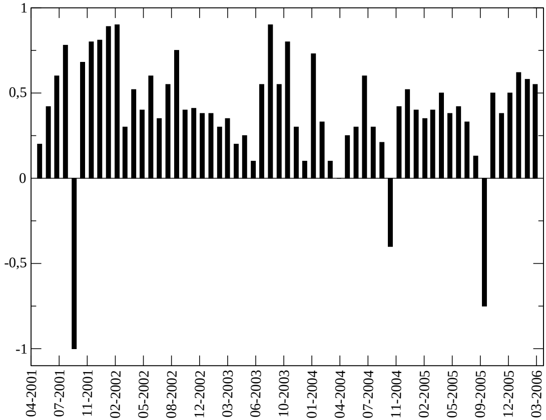
	MV		CPT		CPT (exp.)	
	statistic	p-value	statistic	p-value	statistic	p-value
Equity	0.380		-1.590		-1.496	
Bond	**0.329**	**0.007**	**-0.788**	**0.008**	-1.105	0.240
Small	0.384	0.140	**2.290**	**0.030**	2.172	0.933
2	0.357	0.317	1.053	0.085	-1.981	0.888
3	0.384	0.215	0.654	0.085	-1.749	0.749
4	0.387	0.212	0.278	0.066	-1.509	0.514
5	0.394	0.180	0.197	0.070	-1.411	0.377
6	0.400	0.153	**0.101**	**0.043**	-1.441	0.413
7	0.402	0.142	**0.076**	**0.033**	-1.416	0.347
8	0.403	0.140	**-0.006**	**0.020**	-1.342	0.233
9	0.404	0.116	**-0.552**	**0.035**	-1.322	0.224
Large	0.376	0.457	-1.767	0.741	-1.427	0.279
Growth	0.308	0.821	-2.673	0.863	-2.012	0.920
2	0.410	0.104	-1.352	0.410	-1.286	0.129
3	0.392	0.219	-1.299	0.251	-1.503	0.516
4	0.336	0.591	-0.695	0.158	-1.484	0.465
5	0.420	0.075	**0.502**	**0.039**	-0.985	0.059
6	0.403	0.137	0.176	0.147	-1.380	0.336
7	0.419	0.076	-0.018	0.101	-1.234	0.273
8	**0.447**	**0.027**	**2.083**	**0.003**	-1.163	0.233
9	**0.449**	**0.026**	**1.905**	**0.008**	-1.098	0.203
Value	0.383	0.174	-0.050	0.202	-1.422	0.436

Why ILS are not so popular. . .

Probability weighting in Prospect Theory may explain why investors are reluctant to invest in Insurance Linked Securities (ILS):

- The return distribution of ILS is very fat tailed to the left: every now and then a real catastrophe happens and investors have to face huge losses.
- A Prospect Theory investor exaggerates these small probability events and may hence not invest into ILS.

Why ILS are not so popular...



Behavioral CAPM

- Idea: find alternative concept for risk and return (instead of mean and variance).
- Use a piecewise quadratic value function for prospect theory:

$$v(\Delta x) = \begin{cases} \Delta x - \frac{\alpha^+}{2}(\Delta x)^2, & \text{if } \Delta x \geq 0, \\ \beta\left(\Delta x - \frac{\alpha^-}{2}(\Delta x)^2\right), & \text{if } \Delta x < 0, \end{cases},$$

where $\Delta x = x - RP$.

- The overall prospect utility then is

$$PT_u(\Delta x) = \sum_s P_s u(\Delta x_s),$$

where we ignored probability weighting for simplicity.

- Special case for $\alpha^+ = \alpha^-$ and $\beta = 1$: mean-variance

Risk and reward in prospect theory

- *Risk* as negative part of PT-utility;
- *Reward* as positive part of PT-utility.

In the case of standard PT this means:

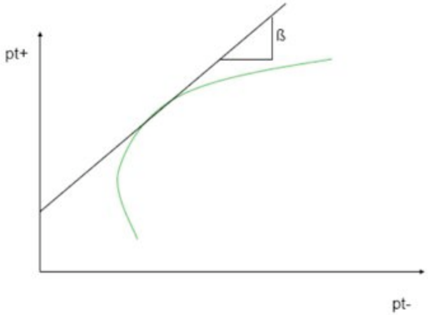
$$pt^{+}(c) = \sum_{c_s > RP} p_s \nu(c_s),$$
$$pt^{-}(c) = \sum_{c_s < RP} p_s \nu(c_s),$$

thus the PT-utility can be written as

$$PT_u(c) = pt^{+}(c) - \beta pt^{-}(c).$$

Risk-reward diagram

Investments can then be described within a risk-reward diagram (very similar to mean-variance):



The derivation of B-CAPM can be found in the text book on page 130ff.

References

References I

Canner, N., Mankiw, N., and Weil, D. (1997).
An Asset Allocation Puzzle.
The American Economic Review, 87(1):181–191.

Fama, E. and French, K. (1992).
The Cross-Section of Expected Stock Returns.
The Journal of Finance, 47(2):427–465.

Fama, E. and French, K. (1998).
Value versus Growth: The International Evidence.
The Journal of Finance, 53(6):1975–1999.

Ineichen, A. (2005).
The Critique of Pure Alpha.
UBS Global Equity Research (March).

References II

Ivković, Z., Sialm, C., and Weisbenner, S. (2005).
Portfolio Concentration and Performance of Individual Investors.

Keynes, J. (1988).
Memorandum for the Estates Committee, Kings College, Cambridge.
In *The Collected Writings of John Maynard Keynes*.

Lakonishok, J., Shleifer, A., and Vishny, R. (1994).
Contrarian Investment, Extrapolation, and Risk.
The Journal of Finance, 49(5):1541–1578.