Extending GMRES to Nonlinear Optimization: Application to Tensor Approximation

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A new algorithm is presented for computing a canonical rank-R tensor approximation that has minimal distance to a given tensor in the Frobenius norm, where the canonical rank-R tensor consists of the sum of R rank-one components. Each iteration of the method consists of three steps. In the first step, a tentative new iterate is generated by a stand-alone one-step process, for which we use alternating least squares (ALS). In the second step, an accelerated iterate is generated by a nonlinear generalized minimal residual (GMRES) approach, recombining previous iterates in an optimal way, and essentially using the stand-alone one-step process as a preconditioner. In the third step, a line search is performed for globalization. The resulting nonlinear GMRES (N-GMRES) optimization algorithm is applied to dense and sparse tensor decomposition test problems. The numerical tests show that ALS accelerated by N-GMRES may significantly outperform both stand-alone ALS and a standard nonlinear conjugate gradient optimization method, especially when highly accurate stationary points are desired for difficult problems. The proposed N-GMRES optimization algorithm is based on general concepts and may be applied to other nonlinear optimization problems. We briefly discuss how updates in the steepest descent direction can be used as a natural preconditioner to make the N-GMRES optimization algorithm applicable to a broad range of smooth nonlinear optimization problems.